Thomaston Public Schools

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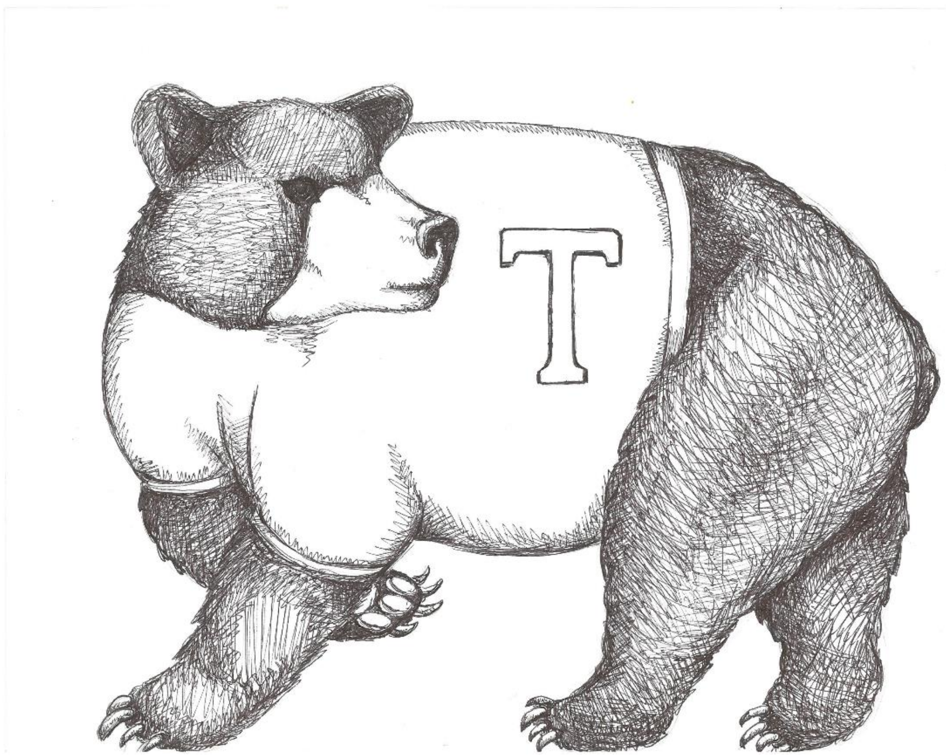

Thomaston Public Schools Curriculum
Thomaston High School
Grade 8: Mathematics 2015

Learn to Live, Live to Learn

Acknowledgements

Curriculum Writer(s):

Mark Olsen

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

______*Alisha DiCorpo*________________________________
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(Math Curriculum Grade 8)

Grade 8 Mathematics

<u>Board of Education Mission Statement:</u>

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:

The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main Resource used when writing this curriculum:

Course Description:

Sequence of Grade 8 Modules Aligned with the Standards
Module 1: Integer Exponents and Scientific Notation
Module 2: The Concept of Congruence
Module 3: Similarity
Module 4: Linear Equations
Module 5: Examples of Functions from Geometry
Module 6: Linear Functions
Module 7: Introduction to Irrational Numbers Using Geometry

Summary of Year

Eighth grade mathematics is about (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Key Areas of Focus for Grade 8: Linear algebra

CCLS Major Emphasis Clusters

Expressions and Equations

Work with radicals and integer exponents.
Understand the connections between proportional relationships, lines, and linear equations.
Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

Define, evaluate, and compare functions.

Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.
Understand and apply the Pythagorean Theorem.

Rationale for Module Sequence in Grade 8

This year begins with students extending the properties of exponents to integer exponents in Module 1. They use the number line model to support their understanding of the rational numbers and the number system. The number system is revisited at the end of the year (in Module 7) to develop the real number line through a detailed study of irrational numbers.

In Module 2, students study congruence by experimenting with rotations, reflections, and translations of geometrical figures. Their study of congruence culminates with an introduction to the Pythagorean Theorem in which the teacher guides students through the “square-within-a-square” proof of the theorem. Students practice the theorem in real-world applications and mathematical problems throughout the year. (In Module 7, students learn to prove the Pythagorean Theorem on their own and are assessed on that knowledge in that module.)

The experimental study of rotations, reflections, and translations in Module 2 prepares students for the more complex work of understanding the effects of dilations on geometrical figures in their study of similarity in

Module 3. They use similar triangles to solve unknown angle, side length and area problems. Module 3 concludes with revisiting a proof of the Pythagorean Theorem from the perspective of similar triangles.

In Module 4, students use similar triangles learned in Module 3 to explain why the slope of a line is well-defined. Students learn the connection between proportional relationships, lines, and linear equations as they develop ways to represent a line by different equations ($y = mx + b$, $y - y1 = m (x - x1)$, etc.). They analyze and solve linear equations and pairs of simultaneous linear equations. The equation of a line provides a natural transition into the idea of a function explored in the next two modules.

Students are introduced to functions in the context of linear equations and area/volume formulas in Module 5. They define, evaluate, and compare functions using equations of lines as a source of linear functions and area and volume formulas as a source of non-linear functions.

In Module 6, students return to linear functions in the context of statistics and probability as bivariate data provides support in the use of linear functions.

By Module 7 students have been using the Pythagorean Theorem for several months. They are sufficiently prepared to learn and explain a proof of the theorem on their own. The Pythagorean Theorem is also used to motivate a discussion of irrational square roots (irrational cube roots are introduced via volume of a sphere). Thus, as the year began with looking at the number system, so it concludes with students understanding irrational numbers and ways to represent them (radicals, non-repeating decimal expansions) on the real number line.

Curriculum Map / Pacing Guide

Note: Adjustments should be made to accommodate testing schedules as they are made available. Pacing is based on the testing of the 2014-2015 school year.

<table>
<tr><th></th><th>Grade 6</th><th>Grade 7</th><th>Grade 8</th><th></th></tr>
<tr><td>20 days</td><td rowspan="2">M1:
Ratios and Unit Rates
(35 days)</td><td rowspan="2">M1:
Ratios and Proportional Relationships
(30 days)</td><td>M1: Integer Exponents and the Scientific Notation
(20 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td>M2:
The Concept of Congruence
(25 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td>M2:
Arithmetic Operations Including Dividing by a Fraction
(25 days)</td><td>M2:
Rational Numbers
(30 days)</td><td>M3:
Similarity
(25 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td>M3:
Rational Numbers
(25 days)</td><td rowspan="2">M3:
Expressions and Equations
(35 days)</td><td rowspan="2">M4:
Linear Equations
(40 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td rowspan="3">M4:
Expressions and Equations
(45 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td>M4:
Percent and Proportional Relationships
(25 days)</td><td>M5: Examples of Functions from Geometry (15 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td>M5:
Statistics and Probability
(25 days)</td><td>M6:
Linear Functions
(20 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td>M5:
Area, Surface Area, and Volume Problems
(25 days)</td><td rowspan="2">M6:
Geometry
(35 days)</td><td rowspan="2">M7:
Introduction to Irrational Numbers Using Geometry
(35 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td>M6:
Statistics
(25 days)</td><td>20 days</td></tr>
</table>

Approx. test date for Grades 6-8

Key:	Number	Geometry	Ratios and Proportions	Expressions and Equations	Statistics and Probability	Functions

Integer Exponents and Scientific Notation

Overview

In Module 1, students' knowledge of operations on numbers will be expanded to include operations on numbers in integer exponents. Module 1 also builds on students' understanding from previous grades with regard to transforming expressions. Students were introduced to exponential notation in Grade 5 as they used whole number exponents to denote powers of ten **(5.NBT.A.2)**. In Grade 6, students expanded the use of exponents to include bases other than ten as they wrote and evaluated exponential expressions limited to whole-number exponents **(6.EE.A1)**. Students made use of exponents again in Grade 7 as they learned formulas for the area of a circle **(7.G.B.4)** and volume **(7.G.B.6)**.

In this module, students build upon their foundation with exponents as they make conjectures about how zero and negative exponents of a number should be defined and prove the properties of integer exponents **(8.EE.A.1)**. These properties are codified into three laws of exponents. They make sense out of very large and very small numbers, using the number line model to guide their understanding of the relationship of those numbers to each other **(8.EE.A.3)**.

Having established the properties of integer exponents, students learn to express the magnitude of a positive number through the use of scientific notation and to compare the relative size of two numbers written in scientific notation **(8.EE.A.3)**. Students explore use of scientific notation and choose appropriately sized units as they represent, compare, and make calculations with very large quantities, such as the U.S. national debt, the number of stars in the universe, and the mass of planets; and very small quantities, such as the mass of subatomic particles **(8.EE.A.4)**.

The Mid-Module Assessment follows Topic A. The End-of-Module Assessment follows Topic B.

Real Numbers

Unit 1

Subject: Mathematics

Grade/Course: Grade 8

Pacing: 30 days

Unit of Study: Unit 1: Real Numbers

Priority Standards:

Work with radicals and integer exponents

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Foundational Standards

Understand the place value system.

5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Focus Standards for Mathematical Practice

MP.2 Reason abstractly and quantitatively.
Students use concrete numbers to explore the properties of numbers in exponential form and then prove that the properties are true for all positive bases and all integer exponents using symbolic representations for bases and exponents. As lessons progress, students use symbols to represent integer exponents and make sense of those quantities in problem situations. Students refer to symbolic notation in order to contextualize the requirements and limitations of given statements (e.g., letting □, □ represent positive integers, letting □, □ represent all integers, both with respect to the properties of exponents).

MP.3 Construct viable arguments and critique the reasoning of others.
Students reason through the acceptability of definitions and proofs (e.g., the definitions of □ 0 and □ −□ for all integers □ and positive integers □). New definitions, as well as proofs, require students to analyze situations and break them into cases. Further, students examine the implications of these definitions and proofs on existing properties of integer exponents. Students keep the goal of a logical argument in mind while attending to details that develop during the reasoning process.

MP.6 Attend to precision.
Beginning with the first lesson on exponential notation, students are required to attend to the definitions provided throughout the lessons and the limitations of symbolic statements, making sure to express what they mean clearly. Students are provided a hypothesis, such as □ < □, for positive integers □, □, and then asked to evaluate whether a statement, like $-2 < 5$, contradicts this hypothesis.

MP.7 Look for and make use of structure.
Students understand and make analogies to the distributive law as they develop properties of exponents. Students will know □ □ · □ □ = □ □+□ as an analog of □□ + □□ = (□ + □)□ and (□ □) □ = □ □ × □ as an analog of □ × (□ × □) = (□ × □) × □.

MP.8 Look for and express regularity in repeated reasoning.
While evaluating the cases developed for the proofs of laws of exponents, students identify when a statement must be proved or if it has already been proven. Students see the use of the laws of exponents in application problems and notice the patterns that are developed in problems.

"Unwrapped" Standards"

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor

spreading). Interpret scientific notation that has been generated by technology.

Concepts - What Students Need to Know	**Skills - What Students Need to Be Able to Do (DOK)**
Properties of integer exponents.	know (L1) apply (L4)
Equivalent numerical expressions.	generate (L2)
Very large or very small quantities.	estimate (L2) - using numbers in scientific notation
Operations with numbers expressed in scientific notation.	perform (calculate) (L1)
Scientific notation.	use (L1) - for measurements
Units of appropriate size.	choose (L3)

Essential Questions	**Big ideas**
How is the best numerical representation for a given situation determined? Which computational method (mental math, estimation, paper and pencil, calculator, technology) should be used to solve a problem? When is it appropriate to use estimation and/or approximation? How is a reasonable estimate made? Why are things measured? How does what is measured influence how it is measured? How exact does a measurement have to be?	Number sense develops through experience. The relationships between operations and their properties promote computational fluency. In certain situations, an estimate is as useful as an exact answer. Measurements describe the attributes of objects and events. All measurements have some degree of uncertainty.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary Scientific Notation Order of Magnitude Familiar Terms and Symbols Exponential Notation Base, Exponent, Power Integer Whole Number Expanded Form (of decimal numbers) Square and Cube (of a number) Equivalent Fractions Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences IXL Math	Type: Mid-Module Assessment Task Administered: After Topic A Format: Constructed response with rubric Standards Addressed: 8.EE.A.1 Type: End-of-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addresses: 8.EE.A.3, 8.EE.A.4

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task is detailed in Unit 6.
Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 6.

Instructional Resources
Suggested Tools and Representations Graphing calculator ixl.com (Math) flippedmath.com (Grade 8 Math) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/grade-8-mathematics (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Grade 8 Mathematics propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course

	might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______

	is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are “just right” for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day’s sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond. Assess by multiple means, including “show and tell” rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice

	with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities. Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Scientific Notation** - The scientific notation for a finite decimal is the representation of that decimal as the product of a decimal ⍰ and a power of 10, where ⍰ satisfies the property that it is at least 1, but smaller than 10, or in symbolic notation, 1 ≤ ⍰ < 10. **Order of Magnitude** - The order of magnitude of a finite decimal is the exponent in the power of 10 when that decimal is expressed in scientific notation.	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model

Familiar Terms and Symbols Exponential Notation Base, Exponent, Power Integer Whole Number Expanded Form (of decimal numbers) Square and Cube (of a number) Equivalent Fractions	students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **Provide Multiple Means of Engagement**	and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.

	Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises

		for practicing skip- counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

The Concept of Congruence

Overview

In this module, students learn about translations, reflections, and rotations in the plane and, more importantly, how to use them to precisely define the concept of congruence. Up to this point, “congruence” has been taken to mean, intuitively, “same size and same shape.” Because this module begins with a serious study of geometry, this intuitive definition must be replaced by a precise definition. This module is a first step; its goal is to provide the needed intuitive background for the precise definitions that are introduced in this module for the first time.

Translations, reflections, and rotations are examples of rigid motions, which are, intuitively, rules of moving points in the plane in such a way that preserves distance. For the sake of brevity, these three rigid motions will be referred to exclusively as the basic rigid motions. Initially, the exploration of these basic rigid motions is done via hands-on activities using an overhead projector transparency, but with the availability of geometry software, the use of technology in this learning environment is inevitable, and some general guidelines for this usage will be laid out at the end of Lesson 2. What needs to be emphasized is that the importance of these basic rigid motions lies not in the fun activities they bring but in the mathematical purpose they serve in clarifying the meaning of congruence.

Throughout Topic A, on the definitions and properties of the basic rigid motions, students verify experimentally their basic properties and, when feasible, deepen their understanding of these properties using reasoning. In particular, what students learned in Grade 4 about angles and angle measurement **(4.MD.C.5)** will be put to good use here. They learn that the basic rigid motions preserve angle measurements, as well as segment lengths.

Topic B is a critical foundation to the understanding of congruence. All the lessons of Topic B demonstrate to students the ability to sequence various combinations of rigid motions while maintaining the basic properties of individual rigid motions. Lesson 7 begins this work with a sequence of translations. Students verify experimentally that a sequence of translations have the same properties as a single translation. Lessons 8 and 9 demonstrate sequences of reflections and translations and sequences of rotations. The concept of sequencing a combination of all three rigid motions is introduced in Lesson 10; this paves the way for the study of congruence in the next topic.

In Topic C, which introduces the definition and properties of congruence, students learn that congruence is just a sequence of basic rigid motions. The fundamental properties shared by all the basic rigid motions are then inherited by congruence: Congruence moves lines to lines and angles to angles, and it is both distance- and angle-preserving (Lesson 11). In Grade 7, students used facts about supplementary, complementary, vertical, and adjacent angles to find the measures of unknown angles **(7.G.B.5)**. This module extends that knowledge to angle relationships that are formed when two parallel lines are cut by a transversal. In Topic C, on angle relationships related to parallel lines, students learn that pairs of angles are congruent because they are angles that have been translated along a transversal, rotated around a point, or reflected across a line.

Students use this knowledge of angle relationships in Lessons 13 and 14 to show why a triangle has a sum of interior angles equal to 180° and why the measure of each exterior angle of a triangle is the sum of the measures of the two remote interior angles of the triangle.

Optional Topic D introduces the Pythagorean theorem. Students are shown the “square within a square” proof of the Pythagorean theorem. The proof uses concepts learned in previous topics of the module, i.e., the concept of congruence and concepts related to degrees of angles. Students begin the work of finding the length of a leg or hypotenuse of a right triangle using $a^2 + b^2 = c^2$. Note that this topic will not be assessed until Module 7.

The Concept of Congruence

Unit 2

Subject: Mathematics

Grade/Course: Grade 8

Pacing: 25 days

Unit of Study: Unit 2: The Concept of Congruence

Priority Standards:

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.

Foundational Standards

Geometric measurement: understand concepts of angle and measure angles.

4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one degree angle,” and can be used to measure angles.

b. An angle that turns through ⍰ one-degree angles is said to have an angle measure of ⍰ degrees.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Focus Standards for Mathematical Practice

MP.2 Reason abstractly and quantitatively.
This module is rich with notation that requires students to decontextualize and contextualize throughout. Students work with figures and their transformed images using symbolic representations and need to attend to the meaning of the symbolic notation to contextualize problems. Students use facts learned about rigid motions in order to make sense of problems involving congruence.

MP.3 Construct viable arguments and critique the reasoning of others.
Throughout this module, students construct arguments around the properties of rigid motions. Students make assumptions about parallel and perpendicular lines and use properties of rigid motions to directly or indirectly prove their assumptions. Students use definitions to describe a sequence of rigid motions to prove or disprove congruence. Students build a logical progression of statements to show relationships between angles of parallel lines cut by a transversal, the angle sum of triangles, and properties of polygons like rectangles and parallelograms.

MP.5 Use appropriate tools strategically.
This module relies on students’ fundamental understanding of rigid motions. As a means to this end, students use a variety of tools but none as important as an overhead transparency. Students verify experimentally the

properties of rigid motions using physical models and transparencies. Students use transparencies when learning about translation, rotation, reflection, and congruence in general. Students determine when they need to use the transparency as a tool to justify conjectures or when critiquing the reasoning of others.

MP.6 Attend to precision.
This module begins with precise definitions related to transformations and statements about transformations being distance- and angle-preserving. Students are expected to attend to the precision of these definitions and statements consistently and appropriately as they communicate with others. Students describe sequences of motions precisely and carefully label diagrams so that there is clarity about figures and their transformed images. Students attend to precision in their verbal and written descriptions of rays, segments, points, angles, and transformations in general.

<table>
<tr><td colspan="2" align="center">“Unwrapped” Standards

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:
a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.</td></tr>
<tr><td>Concepts (What Students Need to Know)</td><td>Skills (What Students Need to Be Able to Do)</td></tr>
</table>

The properties of rotations, reflections, and translations.	verify (L4) - experimentally
Congruence of two-dimensional figures.	understand (L2)
A sequence that exhibits the congruence between two-dimensional figures.	describe (L2)
The angle-sum theorem.	prove (L4)
The exterior angle-sum theorem.	prove (L4)
Theorems resulting from cutting parallel lines by a transversal.	prove (L4)
Angle-angle theorem for triangle similarity.	prove (L4)
Proof of the Pythagorean Theorem and its converse.	explain (L3)
Pythagorean Theorem.	apply (L2)

Essential Questions	**Big ideas**
Why do the theorems and postulates of Geometry need to be proven? How do geometric models describe spatial relationships? What is rigid motion? How is it used in geometry?	Analyzing geometric relationships develops reasoning and justification skills. Geometry and spatial sense offer ways to interpret and reflect on our physical environment. Rigid motion, or isometries, (rotation, reflection and translation) preserves distance and angle measures.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary New Terms Transformation Basic Rigid Motion Translation Rotation Reflection Image of a Point Image of a Figure Sequence (Composition) of Transformations Vector Congruence Transversal Familiar Terms and Symbols Ray Line Line Segment Angle Parallel Lines Perpendicular Lines Supplementary Angles Complementary Angles Vertical Angles Adjacent Angles Triangle Quadrilateral Area Perimeter Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences IXL Math	Type: Mid-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addressed: 8.G.A.1 Type: End-of-Module Assessment Task Administered: After Topic C Format: Constructed response with rubric Standards Addresses: 8.G.A.2, 8.G.A.5

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task is detailed in Unit 6.

Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 6.

Instructional Resources
Suggested Tools and Representations Transparency or patty paper Wet or dry erase markers for use with transparency Optional: geometry software Composition of Rigid Motions: http://youtu.be/O2XPy3ZLU7Y ASA: http://www.youtube.com/watch?v=-yIZdenw5U4 ixl.com (Math) flippedmath.com (Grade 8 Math) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/grade-8-mathematics (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Grade 8 Mathematics propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **Provide Multiple Means of Representation** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to

	calculate and explain. For example, a student searching to define “multiplication” may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…” Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are “just right” for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day’s sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond. Assess by multiple means, including “show and tell”

	rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus

	on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities. Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Transformation** - a rule, to be denoted by □, that assigns each point □ of the plane a unique point which is denoted by □(□). **Basic Rigid Motion** - a rotation, reflection, or translation of the plane. **Translation** - a basic rigid motion that moves a figure along a given vector. **Rotation** - a basic rigid motion that moves a figure around a point, □ degrees. **Reflection** - a basic rigid motion that moves a figure across a line. **Image of a point, image of a figure** - the location of a point or figure after it has been transformed. **Sequence (Composition) of Transformations** - more than one transformation. **Vector** - a directed segment **Congruence** - a sequence of basic	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>**

rigid motions (rotations, reflections, translations) of the plane. **Transversal** - Given a pair of lines ⍰ and ⍰ in a plane, a third line ⍰ is a transversal if it intersects ⍰ at a single point and intersects ⍰ at a single but different point. **Familiar Terms and Symbols** Ray Line Line Segment Angle Parallel Lines Perpendicular Lines Supplementary Angles Complementary Angles Vertical Angles Adjacent Angles Triangle Quadrilateral Area Perimeter	**<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.

	the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Similarity

Overview

In Module 3, students learn about dilation and similarity and apply that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. The module begins with the definition of dilation, properties of dilations and compositions of dilations. The instruction regarding dilation in Module 3 is structured similarly to the instruction regarding concepts of basic rigid motions in Module 2. One overarching goal of this module is to replace the common idea of “same shape, different sizes” with a definition of similarity that can be applied to geometric shapes that are not polygons, such as ellipses and circles.

In this module, students describe the effect of dilations on two-dimensional figures in general and using coordinates. Building on prior knowledge of scale drawings **(7.G.A.1)**, Module 3 demonstrates the effect dilation has on a figure when the scale factor is greater than zero but less than one (shrinking of figure), equal to one (congruence) and greater than one (magnification of figure). Once students understand how dilation transforms figures in the plane, they examine the effect that dilation has on points and figures in the coordinate plane. Beginning with points, students learn the multiplicative effect that dilation has on the coordinates of an ordered pair. Then students apply the knowledge about points to describe the effect dilation has on figures in the coordinate plane, in terms of their coordinates.

Additionally, Module 3 demonstrates that a two-dimensional figure is similar to another if the second can be obtained from a dilation followed by congruence. Knowledge of basic rigid motions is reinforced throughout the module, specifically when students describe the sequence that exhibits a similarity between two given figures. In Module 2, students used vectors to describe the translation of the plane. Module 3 begins in the same way, but once figures are bound to the coordinate plane, students will describe translations in terms of units left or right and units up or down. When figures on the coordinate plane are rotated, the center of rotation is the origin of the graph. In most cases, students will describe the rotation as having center □ and degree □, unless the rotation can be easily identified, i.e., a rotation of 90° or 180°. Reflections remain reflections across a line, but when possible, students should identify the line of reflection as the □-axis or □-axis.

It should be noted that congruence, together with similarity, is the fundamental concept in planar geometry. It is a concept defined without coordinates. In fact, it is most transparently understood when introduced without the extra conceptual baggage of a coordinate system. This is partly because a coordinate system picks out a preferred point (the origin), which then centers most discussions of rotations, reflections, and translations at or in reference to that point. These discussions are further restricted to only the “nice” rotations, reflections, or translations that are easy to do in a coordinate plane. Restricting to “nice” transformations is a huge mistake mathematically because it is antithetical to the main points that must be made about congruence: that rotations, translations, and reflections are abundant in the plane; that for every point in the plane, there are an infinite number of rotations up to 360°, that for every line in the plane there is a reflection, and that for every directed line segment there is a translation. It is this abundance that helps students realize that every congruence transformation (i.e., the act of "picking up a figure" and moving it to another location) can be accomplished through a sequence of translations, rotations, and reflections and further, that similarity is a dilation followed by a congruence transformation.

In Grades 6 and 7, students learned about unit rate, rates in general **(6.RP.A.2)**, and how to represent and use proportional relationships between quantities **(7.RP.A.2, 7.RP.A.3)**. In Module 3, students apply this knowledge of proportional relationships and rates to determine if two figures are similar, and if so, by what scale factor one can be obtained from the other. By looking at the effect of a scale factor on the length of a segment of a given figure, students will write proportions to find missing lengths of similar figures.

Module 3 provides another opportunity for students to learn about the Pythagorean Theorem and its applications in these extension lessons. With the concept of similarity firmly in place, students are shown a proof of the Pythagorean Theorem that uses similar triangles.

Similarity

Unit 3

Subject: Mathematics

Grade/Course: Grade 8

Pacing: 25 days

Unit of Study: Unit 3: Similarity

Priority Standards:

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.2 Recognize and represent proportional relationships between quantities.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Focus Standards for Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.
Many times in this module, students are exposed to the reasoned logic of proofs. Students are called on to make conjectures about the effect of dilations on angles, rays, lines, and segments, and then they must evaluate the validity of their claims based on evidence. Students also make conjectures about the effect of dilation on circles, ellipses, and other figures. Students are encouraged to participate in discussions and evaluate the claims of others.

MP.4 Model with mathematics.
This module provides an opportunity for students to apply their knowledge of dilation and similarity in real-world applications. Students will use shadow lengths and a known height to find the height of trees, the distance across a lake, and the height of a flagpole.

MP.6 Attend to precision.
To communicate precisely, students will use clear definitions in discussions with others and in their own reasoning with respect to similar figures. Students will use the basic properties of dilations to prove or disprove claims about a pair of figures. Students will incorporate their knowledge about basic rigid motions as it relates to similarity, specifically in the description of the sequence that is required to prove two figures are similar.

MP.8 Look for and express regularity in repeated reasoning.
Students will look at multiple examples of dilations with different scale factors. Then students explore dilations to determine what scale factor to apply to return a figure dilated by a scale factor to its original size.

“Unwrapped” Standards

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-

dimensional figures, describe a sequence that exhibits the similarity between them. 8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. 8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse. 8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
The effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	describe (L2)
A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.	understand (L2)
A sequence that exhibits the similarity between two similar two-dimensional figures.	describe (L2)
The angle-sum theorem.	prove (L4)
The exterior angle-sum theorem.	prove (L4)
Theorems resulting from cutting parallel lines by a transversal.	prove (L4)
Angle-angle theorem for triangle similarity.	prove (L4)
Proof of the Pythagorean Theorem and its converse.	explain (L3)
Pythagorean Theorem.	apply (L2)

Essential Questions	**Big ideas**
When and why are proportional comparisons made? Why do the theorems and postulates of Geometry need to be proven? How do geometric models describe spatial relationships? What is rigid motion? How is it used in geometry?	Proportional relationships express how quantities change in relationship to each other. Analyzing geometric relationships develops reasoning and justification skills. Geometry and spatial sense offer ways to interpret and reflect on our physical environment. Rigid motion, or isometries, (rotation, reflection and translation) preserves distance and angle measures.

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Performance Task
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Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 6.

Instructional Resources
Suggested Tools and Representations Compass (required) Transparency or patty paper Wet or dry erase markers for use with transparency Geometry software (optional) Ruler Protractor Video that demonstrate Pythagorean Theorem proof using similar triangles: http://www.youtube.com/watch?v=QCyvxYLFSfU ixl.com (Math) flippedmath.com (Grade 8 Math) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/grade-8-mathematics (See Appendix A for an example.)

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to define “multiplication” may model groups of 6 with drawings or concrete objects and write the number sentence to match.
Teach students how to ask questions (such as “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…”
Couple number sentences with models.
Enlarge sprint print for visually impaired learners.
Use student boards to work on one calculation at a time.
Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students.
Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones.
Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses.
Adjust wait time for interpreters of deaf and hard-of-hearing students.
Select numbers and tasks that are “just right” for learners.
Model each step of the algorithm before students begin.
Give students a chance to practice the next day’s sprint beforehand.
Give students a few extra minutes to process the information before giving the signal to respond.
Assess by multiple means, including “show and tell” rather than written.

	Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior.

	Allow "everyday" and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities. Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Dilation** - a transformation that shrinks or magnifies a figure by multiplying each coordinate of the figure by a scale factor. **Congruence** - a finite composition of basic rigid motions - reflections, rotations, translations - of the plane. Two figures in a plane are congruent if there is a congruence that maps one figure onto the other figure. **Similar** - two figures in the plane are similar if there exists a similarity transformation taking one figure to the other. **Similarity Transformation** - a composition of a finite number of basic rigid motions or dilations. The scale factor of a similarity	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..."

transformation is the product of the scale factors of the dilations in the composition; if there are no dilations in the composition, the scale factor is defined to be 1. **Similarity** - an example of a transformation. **Familiar Terms and Symbols** Scale Drawing Angle-Preserving	videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g.,	Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word

	dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games.

		Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Linear Equations

Overview

In Module 4, students extend what they already know about unit rates and proportional relationships **(6.RP.A.2, 7.RP.A.2)** to linear equations and their graphs. Students understand the connections between proportional relationships, lines, and linear equations in this module **(8.EE.B.5, 8.EE.B.6)**. Also, students learn to apply the skills they acquired in Grades 6 and 7, with respect to symbolic notation and properties of equality **(6.EE.A.2, 7.EE.A.1, 7.EE.B.4)** to transcribe and solve equations in one variable and then in two variables.

In Topic A, students begin by transcribing written statements using symbolic notation. Then, students write linear and non-linear expressions leading to linear equations, which are solved using properties of equality **(8.EE.C.7b)**. Students learn that not every linear equation has a solution. In doing so, students learn how to transform given equations into simpler forms until an equivalent equation results in a unique solution, no solution, or infinitely many solutions **(8.EE.C.7a**). Throughout Topic A, students must write and solve linear equations in real-world and mathematical situations.

In Topic B, students work with constant speed, a concept learned in Grade 6 **(6.RP.A.3)**, but this time with proportional relationships related to average speed and constant speed. These relationships are expressed as linear equations in two variables. Students find solutions to linear equations in two variables, organize them in a table, and plot the solutions on a coordinate plane **(8.EE.C.8a)**. It is in Topic B that students begin to investigate the shape of a graph of a linear equation. Students predict that the graph of a linear equation is a line and select points on and off the line to verify their claim. Also in this topic is the standard form of a linear equation, $ax + by = c$, and when $a \neq 0$ and $b \neq 0$, a non-vertical line is produced. Further, when $a = 0$ or $b = 0$, then a vertical or horizontal line is produced.

In Topic C, students know that the slope of a line describes the rate of change of a line. Students first encounter slope by interpreting the unit rate of a graph **(8.EE.B.5)**. In general, students learn that slope can be determined using any two distinct points on a line by relying on their understanding of properties of similar triangles from Module 3 **(8.EE.B.6)**. Students verify this fact by checking the slope using several pairs of points and comparing their answers. In this topic, students derive $y = mx$ and $y = mx + b$ for linear equations by examining similar triangles. Students generate graphs of linear equations in two variables first by completing a table of solutions, then by using information about slope and y-intercept. Once students are sure that every linear equation graphs as a line and that every line is the graph of a linear equation, students graph equations using information about x- and y-intercepts. Next, students learn some basic facts about lines and equations, such as why two lines with the same slope and a common point are the same line, how to write equations of lines given slope and a point, and how to write an equation given two points. With the concepts of slope and lines firmly in place, students compare two different proportional relationships represented by graphs, tables, equations, or descriptions. Finally, students learn that multiple forms of an equation can define the same line.

Simultaneous equations and their solutions are the focus of Topic D. Students begin by comparing the constant speed of two individuals to determine which has greater speed **(8.EE.C.8c).** Students graph simultaneous linear equations to find the point of intersection and then verify that the point of intersection is in fact a solution to each equation in the system **(8.EE.C.8a)**. To motivate the need to solve systems algebraically, students graph systems of linear equations whose solutions do not have integer coordinates. Students learn to solve systems of linear equations by substitution and elimination **(8.EE.C.8b).** Students understand that a system can have a unique solution, no solution, or infinitely many solutions, as they did with linear equations in one variable. Finally, students apply their knowledge of systems to solve problems in real-world contexts, including converting temperatures from Celsius to Fahrenheit.

Optional Topic E is an application of systems of linear equations **(8.EE.C.8b)**. Specifically, this system generates Pythagorean triples. First, students learn that a Pythagorean triple can be obtained by multiplying any known triple by a positive integer **(8.G.B.7).** Then, students are shown the Babylonian method for finding a triple that requires the understanding and use of a system of linear equations.

Linear Equations

Unit 4

Subject: Mathematics

Grade/Course: Grade 8

Pacing: 40 days

Unit of Study: Unit 4: Linear Equations

Priority Standards:

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.2 Understand the concept of a unit rate ⍰/⍰ associated with a ratio ⍰: ⍰ with ⍰ ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract ⍰ from 5” as 5 − ⍰.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

c. Evaluate expression at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas ⍰ = ⍰ ^{3} and ⍰ = 6⍰ ^{2} to find the volume and surface area of a cube with side length ⍰ = ½ .

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. For example, if total cost ⍰ is proportional to the number ⍰ of items purchased at a constant price ⍰, the relationship between the total cost and the number of items can be expressed as ⍰ = ⍰⍰.

d. Explain what a point (⍰, ⍰) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, ⍰) where ⍰ is the unit rate.

Use properties of operations to generate equivalent expressions.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Focus Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Students analyze given constraints to make conjectures about the form and meaning of a solution to a given situation in onevariable and two-variable linear equations, as well as in simultaneous linear equations. Students are systematically guided to understand the meaning of a linear equation in one variable, the natural occurrence of linear equations in two variables with respect to proportional relationships, and the natural emergence of a system of two linear equations when looking at related, continuous proportional relationships.

MP.2 Reason abstractly and quantitatively.
Students decontextualize and contextualize throughout the module as they represent situations symbolically and make sense of solutions within a context. Students use facts learned about rational numbers in previous grade levels to solve linear equations and systems of linear equations.

MP.3 Construct viable arguments and critique the reasoning of others.
Students use assumptions, definitions, and previously established facts throughout the module as they solve linear equations. Students make conjectures about the graph of a linear equation being a line, then proceed to prove this claim. While solving linear equations, they learn that they must first assume that a solution exists, then proceed to solve the equation using properties of equality based on the assumption. Once a solution is found, students justify that it is in fact a solution to the given equation, thereby verifying their initial assumption. This process is repeated for systems of linear equations.

MP.4 Model with mathematics.
Throughout the module, students represent real-world situations symbolically. Students identify important quantities from a context and represent the relationship in the form of an equation, a table, and a graph. Students analyze the various representations and draw conclusions and/or make predictions. Once a solution or prediction has been made, students reflect on whether the solution makes sense in the context presented. One example of this is when students determine how many buses are needed for a field trip. Students must interpret their fractional solution and make sense of it as it applies to the real world.

MP.7 Look for and make use of structure.
Students use the structure of an equation to make sense of the information in the equation. For example, students write equations that represent the constant rate of motion for a person walking. In doing so, they interpret an equation such as $d = 3/5\ t$ as the total distance a person walks, d, in t amount of time, at a rate of 3/5 . Students look for patterns or structure in tables and show that a rate is constant.

“Unwrapped” Standards

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

8.EE.C.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
Proportional relationships.	graph (L2)
The unit rate as the slope of the graph.	interpret (L2)
Two different proportional relationships represented in different ways.	compare (L2)
The slope is the same between any two distinct points on a non-vertical line in the coordinate plane.	explain (L2)
The equation y = mx.	derive (L3)
The equation y = mx + b.	derive (L3)
Linear equations in one variable with one solution, infinitely many solutions, or no solutions.	identify (L1)
An equation.	transform ((L4) - into simpler forms
Linear equations with rational number coefficients.	solve (L2)
Solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy	understand (L2)

both equations simultaneously. Systems of two linear equations in two variables. Real-world and mathematical problems leading to two linear equations in two variables.	 Solve (L2) - algebraically Estimate (L2) - graphically Solve (L2) - by inspection Solve (L2)

Essential Questions	**Big ideas**
When and how is mathematics used in solving real world problems? When and why is it necessary to follow set rules/procedures/properties when manipulating numeric or algebraic expressions? What characteristics of problems would determine how to model the situation and develop a problem solving strategy?	Mathematics can be used to solve real world problems and can be used to communicate solutions. Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations. Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities. Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. Multiple representations may be used to model given real world relationships.

<table>
<tr><th colspan="3">Assessments</th></tr>
<tr><td>Common Formative Pre-Assessments</td><td>Progress Monitoring Checks – “Dipsticks”</td><td>Common Formative Mid and or Post-Assessments Resources</td></tr>
<tr><td>Pretest vocabulary
New terms
Slope
Solution to a System of Linear Equations
System of Linear Equations
Familiar Terms and Symbols
Coefficient
Equation
Like Terms
Linear Expressions
Solution
Term
Unit Rate
Variable

Conduct opening exercise

Use exit ticket as pre-assessment and post where applicable</td><td>Post-test vocabulary

Opening Exercise - Give again and reflect on results from first administration prior to the unit.

Exploratory Challenge

Exit Ticket

Student Conferences

IXL Math</td><td>Type: Mid-Module Assessment Task
Administered: After Topic B
Format: Constructed response with rubric
Standards Addressed: 8.EE.C.7

Type: End-of-Module Assessment Task
Administered: After Topic D
Format: Constructed response with rubric
Standards Addresses: 8.EE.B.5, 8.EE.B.6, 8.EE.B.7, 8.EE.B.8</td></tr>
</table>

Performance Task
Performance taskss are to be created with teacher input throughout the year. A sample of a performance task is included in Unit 6.
Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 6.

Instructional Resources
Suggested Tools and Representations Scientific calculator Online graphing calculator (for example: https://www.desmos.com/calculator) Graph paper Straight-edge Graphing calculator ixl.com (Math) flippedmath.com (Grade 8 Math) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/grade-8-mathematics (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Grade 8 Mathematics propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-

	pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are "just right" for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day's sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond. Assess by multiple means, including "show and tell" rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking

	process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities.

	Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Slope** - a number that describes the “steepness” or “slant” of a line. It is the constant rate of change. **Solution to a System of Linear Equations** - a pair of numbers from the domain of the variables that, when each number from the pair is substituted into all instances of its corresponding variable, makes the equation a true number sentence. **System of Linear Equations** - also referred to as simultaneous linear equations, is the set of at least two linear equations. Example: $\{ \square + \square = 15\ 3\square - 7\square = -2$ is a system of linear equations. **Familiar Terms and Symbols** Coefficient Equation Like Terms Linear Expressions Solution Term Unit Rate Variable	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>**

	<u>Provide Multiple Means of Action and Expression</u> First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.

	the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Examples of Functions from Geometry

Overview

In Module 5, Topic A, students learn the concept of a function and why functions are necessary for describing geometric concepts and occurrences in everyday life. The module begins by explaining the important role functions play in making predictions. For example, if an object is dropped, a function allows us to determine its height at a specific time. To this point, our work has relied on assumptions of constant rates; here, students are given data that show that objects do not always travel at a constant speed. Once we explain the concept of a function, we then provide a formal definition of function. A function is defined as an assignment to each input, exactly one output **(8.F.A.1)**. Students learn that the assignment of some functions can be described by a mathematical rule or formula. With the concept and definition firmly in place, students begin to work with functions in real-world contexts. For example, students relate constant speed and other proportional relationships **(8.EE.B.5)** to linear functions. Next, students consider functions of discrete and continuous rates and understand the difference between the two. For example, we ask students to explain why they can write a cost function for a book, but they cannot input 2.6 into the function and get an accurate cost as the output.

Students apply their knowledge of linear equations and their graphs from Module 4 **(8.EE.B.5, 8.EE.B.6)** to graphs of linear functions. Students know that the definition of a graph of a function is the set of ordered pairs consisting of an input and the corresponding output **(8.F.A.1)**. Students relate a function to an input-output machine: a number or piece of data, known as the input, goes into the machine, and a number or piece of data, known as the output, comes out of the machine. In Module 4, students learned that a linear equation graphs as a line and that all lines are graphs of linear equations. In Module 5, students inspect the rate of change of linear functions and conclude that the rate of change is the slope of the graph of a line. They learn to interpret the equation $y = mx + b$ **(8.EE.B.6)** as defining a linear function whose graph is a line **(8.F.A.3)**. Students will also gain some experience with nonlinear functions, specifically by compiling and graphing a set of ordered pairs, and then by identifying the graph as something other than a straight line.

Once students understand the graph of a function, they begin comparing two functions represented in different ways **(8.EE.C.8)**, similar to comparing proportional relationships in Module 4. For example, students are presented with the graph of a function and a table of values that represent a function and are asked to determine which function has the greater rate of change **(8.F.A.2)**. Students are also presented with functions in the form of an algebraic equation or written description. In each case, students examine the average rate of change and know that the one with the greater rate of change must overtake the other at some point.

In Topic B, students use their knowledge of volume from previous grade levels **(5.MD.C.3, 5.MD.C.5)** to learn the volume formulas for cones, cylinders, and spheres **(8.G.C.9)**. First, students are reminded of what they already know about volume, that volume is always a positive number that describes the hollowed-out portion of a solid figure that can be filled with water. Next, students use what they learned about the area of circles **(7.G.B.4)** to determine the volume formulas for cones and cylinders. In each case, physical models will be used to explain the formulas, beginning with a cylinder seen as a stack of circular disks that provide the height of the cylinder. Students consider the total area of the disks in three dimensions, understanding it as volume of a cylinder. Next, students make predictions about the volume of a cone that has the same dimensions as a cylinder. A demonstration shows students that the volume of a cone is one-third the volume of a cylinder with the same dimension, a fact that will be proved in Module 7. Next, students compare the volume of a sphere to its circumscribing cylinder (i.e., the cylinder of dimensions that touches the sphere at points but does not cut off any part of it). Students learn that the formula for the volume of a sphere is two-thirds the volume of the cylinder that fits tightly around it. Students extend what they learned in Grade 7 **(7.G.B.6)** about how to solve real-world and mathematical problems related to volume from simple solids to include problems that require the formulas for cones, cylinders, and spheres.

Examples of Functions from Geometry

Unit 5

Subject: Mathematics

Grade/Course: Grade 8

Pacing: 15 days

Unit of Study: Unit 5: Examples of Functions from Geometry

Priority Standards:

Define, evaluate, and compare functions.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.A.3 Interpret the equation □ = □□ + □ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function □ = $□^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9) which are not on a straight line.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Foundational Standards

Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using □ unit cubes is said to have a volume of □ cubic units.

5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole–number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volume of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to real world problems. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.6 Solve real-world and mathematical problems involving area, volume, and surface area of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Focus Standards for Mathematical Practice

MP.2 Reason abstractly or quantitatively.
Students examine, interpret, and represent functions symbolically. They make sense of quantities and their relationships in problem situations. For example, students make sense of values as they relate to the total cost of items purchased or a phone bill based on usage in a particular time interval. Students use what they know about rate of change to distinguish between linear and nonlinear functions. Further, students contextualize information gained from the comparison of two functions.

MP.6 Attend to precision.
Students use notation related to functions, in general, as well as notation related to volume formulas. Students are expected to clearly state the meaning of the symbols used in order to communicate effectively and precisely to others. Students attend to precision when they interpret data generated by functions. They know when claims are false; for example, calculating the height of an object after it falls for −2 seconds. Students also understand that a table of values is an incomplete representation of a continuous function, as an infinite number of values can be found for a function.

MP.8 Look for and express regularity in repeated reasoning.
Students will use repeated computations to determine equations from graphs or tables. While focused on the details of a specific pair of numbers related to the input and output of a function, students will maintain oversight of the process. As students develop equations from graphs or tables, they will evaluate the reasonableness of their equation as they ensure that the desired output is a function of the given input.

"Unwrapped" Standards

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9) which are not on a straight line.

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Concepts (What Students Need to Know)	Skills (What Students Need to Be Able to Do)
A function is a rule that assigns to each input exactly one output.	understand (L2)
Properties of two functions each represented in a different way.	compare (L2)
The equation ▯ = ▯▯ + ▯.	interpret (L2)
The formulas for the volumes of cones, cylinders, and spheres.	know (L1)
Real-world and mathematical problems.	solve (L2) - using formulas

Essential Questions	Big ideas
What problem solving strategy will work best for a real world problem?	There can be different strategies to solve a problem, but some are more effective and efficient than others are.
How does explaining my process help me to understand a problem's solution better?	A problem solver understands what has been done, knows why the process was appropriate, and can support it with reasons and evidence.
How do I use algebraic expressions to analyze or solve problems?	Algebraic expressions and equations generalize relationships from specific cases.
How is thinking algebraically different from thinking arithmetically?	Real world situations can be represented symbolically and graphically.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary Function Input Output Familiar Terms and Symbols Area Linear Equation Nonlinear Equation Rate of Change Solids Volume Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences IXL Math	Type: End-of-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addresses: 8.F.A.1, 8.F.A.2, 8.F.A.3, 8.G.C.9

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task detailed in Unit 6.
Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 6.

Instructional Resources

Suggested Tools and Representations

3D solids: cones, cylinders, and spheres.
ixl.com (Math)
flippedmath.com (Grade 8 Math)

Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/grade-8-mathematics (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Grade 8 Mathematics propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

	Provide Multiple Means of Representation Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **Provide Multiple Means of Action and Expression** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_______ is _____ hundreds, _____ tens, and _____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-

hearing students.

Select numbers and tasks that are “just right” for learners.

Model each step of the algorithm before students begin.

Give students a chance to practice the next day’s sprint beforehand.

Give students a few extra minutes to process the information before giving the signal to respond.

Assess by multiple means, including “show and tell” rather than written.

Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?”

Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?”

Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.

Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process.

Teach in small chunks so students get a lot of practice with one step at a time.

Know, use, and make the most of Deaf culture and sign language.

Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!”

	Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities. Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Function** - a rule that assigns to each input exactly one output. **Input** - The number or piece of data that is put into a function. **Output** - The number or piece of data that is the result of an input of a function. **Familiar Terms and Symbols** Area Linear Equation Nonlinear Equation Rate of Change Solids Volume	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...”

	videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g.,	Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word

	dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games.

		Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Linear Functions

Overview

In Grades 6 and 7, students worked with data involving a single variable. This module introduces students to bivariate data. Students are introduced to a function as a rule that assigns exactly one value to each input. In this module, students use their understanding of functions to model the relationships of bivariate data. This module is important in setting a foundation for students’ work in Algebra I.

Topic A examines the relationship between two variables using linear functions **(8.F.B.4).** Linear functions are connected to a context using the initial value and slope as a rate of change to interpret the context. Students represent linear functions by using tables and graphs and by specifying rate of change and initial value. Slope is also interpreted as an indication of whether the function is increasing or decreasing and as an indication of the steepness of the graph of the linear function **(8.F.B.5).** Nonlinear functions are explored by examining nonlinear graphs and verbal descriptions of nonlinear behavior.

In Topic B, students use linear functions to model the relationship between two quantitative variables as students move to the domain of statistics and probability. Students make scatter plots based on data. They also examine the patterns of their scatter plots or given scatter plots. Students assess the fit of a linear model by judging the closeness of the data points to the line **(8.SP.A.1, 8.SP.A.2)**.

In Topic C, students use linear and nonlinear models to answer questions in context **(8.SP.A.1, 8.SP.A.2)**. They interpret the rate of change and the initial value in context **(8.SP.A.3).** They use the equation of a linear function and its graph to make predictions. Students also examine graphs of nonlinear functions and use nonlinear functions to model relationships that are nonlinear. Students gain experience with the mathematical practice of “modeling with mathematics” (MP.4).

In Topic D, students examine bivariate categorical data by using two-way tables to determine relative frequencies. They use the relative frequencies calculated from tables to informally assess possible associations between two categorical variables **(8.SP.A.4).**

Linear Functions

Unit 6

Subject: Mathematics

Grade/Course: Grade 8

Pacing: 20 days

Unit of Study: Unit 6: Linear Functions

Priority Standards:

Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Foundational Standards

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Define, evaluate, and compare functions.

8.F.B.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.B.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.B.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Focus Standards for Mathematical Practice

MP.2 Reason abstractly and quantitatively.
Students reason quantitatively by symbolically representing the verbal description of a relationship between two bivariate variables. They attend to the meaning of data based on the context of problems and the possible linear or nonlinear functions that explain the relationships of the variables.

MP.4 Model with mathematics.
Students model relationships between variables using linear and nonlinear functions. They interpret models in the context of the data and reflect on whether or not the models make sense based on slopes, initial values, or the fit to the data.

MP.6 Attend to precision.
Students evaluate functions to model a relationship between numerical variables. They evaluate the function by assessing the closeness of the data points to the line. They use care in interpreting the slope and the y-intercept in linear functions.

MP.7 Look for and make use of structure.

Students identify pattern or structure in scatter plots. They fit lines to data displayed in a scatter plot and determine the equations of lines based on points or the slope and initial value.

"Unwrapped" Standards

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (□,□) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
A function to model a linear relationship between two quantities.	construct (L3)
The rate of change and initial value of a linear function.	determine (L2) interpret (L2) - in terms of the situation it models interpret (L2) - in terms of its graph or a table of values
A graph depicting the functional relationship between two quantities.	analyze (L4)
A function that has been described verbally.	graph (L2)
Scatterplots for bivariate measurement data to investigate patterns of association between two quantities.	construct (L3) interpret (L2)
Patterns seen in scatterplots.	describe (L2)
Straight lines are widely used to model relationships between two quantitative variables.	know (L1)
A straight line for scatter plots that suggest a linear association.	construct (L3) assess (L3) - by judging the closeness of the data points to the line
Problems in the context of bivariate measurement data.	solve (L2)
The slope and intercept.	interpret (L2)
Patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.	know (L1)
A two-way table summarizing data on two categorical variables collected from the same subjects.	construct (L3) interpret (L2)
An association between the two variables	describe (L2) - using relative frequencies calculated for rows or columns.

Essential Questions	Big ideas
How are patterns described? How are patterns used to show a relationship? How can patterns be used to make predictions? Why is data collected and analyzed? How are algebraic expressions used to analyze or solve problems?	Patterns and relationships can be represented numerically, graphically, symbolically, and verbally. Patterns provide insights into potential relationships. The way that data is collected, organized and displayed influences interpretation. Algebraic expressions and equations generalize relationships from specific cases.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary New Terms Association Column relative frequency Row relative frequency Two-way table Familiar Terms and Symbols Categorical variable Intercept Initial value Numerical variable Scatter plot Slope Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences IXL Math	Type: Mid-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addressed: 8.F.B.4, 8.F.B.5, 8.SP.A.1, 8.SP.A.2 Type: End-of-Module Assessment Task Administered: After Topic D Format: Constructed response with rubric Standards Addresses: 8.F.B.4, 8.F.B.5, 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task is detailed below. An engaging scenario which can accompany the tasks is included in the "Engaging Learning Experiences" section, also below. **"Ride The Wild Side"** Task 1: Write down the components of a good roller coaster. Find data about a roller coaster at a theme park on the internet. Superimpose a picture of the roller coaster on a graph and analyze the slopes of its descents and ascents. Task 2: Create your own roller coaster. Sketch your roller coaster in three dimensions. Graph the coaster in two dimensions (don't worry about right or left turns). Your roller coaster must have a minimum of 3 climbs and drops. Task 3: Determine the equations of all sections of your roller coaster. Design partners will check each other's work. Task 4: Design a poster, brochure or powerpoint to attract visitors to the park at which your roller coaster resides. Obviously, use your roller coaster as the featured attraction! **8th Grade Math: Ride the Wild Side** **RUBRIC** The elements of performance required by this task are: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (□,□) values. Interpret the rate of change and initial value of a linear function in terms of the situation it models. Describe qualitatively the functional relationship between two quantities by analyzing a graph. Possible Responses Point distribution 1. Total possible points - 5

a) Writes down at least three components of a good roller coaster. - 1 point

b) Finds data about a roller coaster on the internet. - 1 point

c) Superimposes a picture of the roller coaster on a graph - 1 pt

d) Analyzes the slopes of its descents and ascents - 2 pt

Partial credit

Computational errors for part (d) - deduct 1

2. Total possible points - 3

a) Sketch your roller coaster in three dimensions. - 1 pt

b) Graph the coaster in two dimensions. -2

3. Total possible points - 8

a) Determine the equations of all sections of your roller coaster. - 6 (2 each)

1 point for each correct slope, 1 point for each correct y-intercept

b) No errors on design partner's work. - 2

Partial credit

Computational error(s) in part 3(a) - deduct 1

4. Total possible points - 2

Poster, brochure or powerpoint receives 60% or more thumbs up from review board (classmates) - 2 points

Partial credit

40 % to 59% rating - deduct - 1

TOTAL POSSiBLE POiNTS = 18

8th Grade Math: Ride the Wild Side

Rubric

Performance Level Descriptions and Cut Scores

Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0–4 points)

The student's response shows few of the elements of performance that the task demands as defined by the CCSS. The student's work shows a minimal attempt and lack of coherence. The student fails to use appropriate tools strategically. The student is unable to make sense of the problem and apply mathematical concepts in this modeling situation.

Level 2: Performance Below Standard (5–10 points)

The student's response shows some of the elements of performance that the task demands as defined by the CCSS. The student might ignore or fail to address some of the constraints of the problem. The student may occasionally make sense of quantities or relationships in the problem. The student attempts to use some appropriate tools with limited success. The student may have trouble generalizing or applying mathematical methods in this modeling situation.

Level 3: Performance at Standard (11–15 points)
For most of the task, the student's response shows the main elements of performance that the tasks demand as defined by the CCSS with few minor errors or omissions. The student explains the problem and identifies constraints. The student makes sense of quantities and their relationships in the modeling situation. The student uses appropriate tools. The student might discern patterns or structures and make connections between representations. The student is able to make sense of the problem and apply geometric concepts to this modeling situation.

Level 4: Achieves Standards at a High Level (16–18 points)
The student's response meets the demands of nearly all of the tasks as defined by the CCSS and is organized in a coherent way. The communication is clear and precise. The body of work looks at the overall situation of the problem and process, while attending to the details. The student routinely interprets the mathematical results, applies concepts in the context of the situation, reflects on whether the results make sense and uses all appropriate tools strategically.

Engaging Learning Experiences

Engaging learning experiences are to be created with teacher input throughout the year.

A sample of an engaging scenario is included below. The scenario can be used for the performance task listed above.

"Ride The Wild Side"

93 mph! 130 feet in the air! Screams fill the air! Hands in the air! Where are you?

Students watch www.youtube.com/watch?v=bZTsRU8B8s4

What would make a roller coaster fun for you? In this activity, you will design the roller coaster of your dreams. It will be designed according to your preferences. Begin to think about what would make your roller coaster exciting to everyone. At the end of the activity your classmates will rate your creation so make it special!

Instructional Resources
Suggested Tools and Representations Graphing calculator Scatter plot Two-way tables ixl.com (Math) flippedmath.com (Grade 8 Math) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/grade-8-mathematics (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition	The modules that make up Grade 8 Mathematics propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement.

Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>**

	Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are “just right” for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day’s sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond. Assess by multiple means, including “show and tell” rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use

	models), not their accuracy in language. **Provide Multiple Means of Engagement** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities. Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**

New or Recently Introduced Terms

Association - a relationship between two variables. The tendency for two variables to vary together in a predictable way.

Column relative frequency - In a two-way table, a column relative frequency is a cell frequency divided by the column total for that cell.

Row relative frequency - In a two-way table, a row relative frequency is a cell frequency divided by the row total for that cell.

Two-way table - a table used to summarize data on two categorical variables. The rows of the table correspond to the possible categories for one of the variables, and the columns of the table correspond to the possible categories for the other variable. Entries in the cells of the table indicate the number of times that a particular category combination occurs in the data set or the frequency for that combination.

Familiar Terms and Symbols

- Categorical variable
- Intercept
- Initial value
- Numerical variable
- Scatter plot
- Slope

<u>Provide Multiple Means of Representation</u>

Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.

Guide students as they select and practice using their own graphic organizers and models to solve.

Use direct instruction for vocabulary with visual or concrete representations.

Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do."

Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.

Scaffold complex concepts and provide leveled problems for multiple entry points.

<u>Provide Multiple Means of Action and Expression</u>

First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.

Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..."

Incorporate written reflection, evaluation, and synthesis.

Allow creativity in expression and modeling solutions.

<u>Provide Multiple Means of Action and Expression</u>

Encourage students to explain their reasoning both orally and in writing.

Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.

Offer choices of independent or group assignments for early finishers.

	answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should	Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip- counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two- step problem). Adjust difficulty level by enhancing the operation (e. g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>**

	complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	Push student comprehension into higher levels of Bloom’s Taxonomy with questions such as: “What would happen if...?” “Can you propose an alternative...?” “How would you evaluate...?” “What choice would you have made...?” Ask “Why?” and “What if?” questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Introduction to Irrational Numbers Using Geometry

Overview

The module begins with work related to the Pythagorean Theorem and right triangles. Before the lessons of this module are presented to students, it is important that the lessons in Modules 2 and 3 related to the Pythagorean Theorem are taught (M2: Lessons 15 and 16, M3: Lessons 13 and 14). In Modules 2 and 3, students used the Pythagorean Theorem to determine the unknown length of a right triangle. In cases where the side length was an integer, students computed the length. When the side length was not an integer, students left the answer in the form of , where was not a perfect square number. Those solutions are revisited and are the motivation for learning about square roots and irrational numbers in general.

In Topic A, students learn the notation related to roots **(8.EE.A.2).** The definition for irrational numbers relies on students' understanding of rational numbers, that is, students know that rational numbers are points on a number line **(6.NS.C.6)** and that every quotient of integers (with a non-zero divisor) is a rational number **(7.NS.A.2)**. Then irrational numbers are numbers that can be placed in their approximate positions on a number line and not expressed as a quotient of integers. Though the term "irrational" is not introduced until Topic B, students learn that irrational numbers exist and are different from rational numbers. Students learn to find positive square roots and cube roots of expressions and know that there is only one such number **(8.EE.A.2)**. Topic A includes some extension work on simplifying perfect square factors of radicals in preparation for Algebra I.

In Topic B, students learn that to get the decimal expansion of a number **(8.NS.A.1)**, they must develop a deeper understanding of the long division algorithm learned in Grades 6 and 7 **(6.NS.B.2, 7.NS.A.2d)**. Students show that the decimal expansion for rational numbers repeats eventually, in some cases with zeros, and they can convert the decimal form of a number into a fraction **(8.NS.A.2)**. Students learn a procedure to get the approximate decimal expansion of numbers like and and compare the size of these irrational numbers using their rational approximations. At this point, students learn that the definition of an irrational number is a number that is not equal to a rational number **(8.NS.A.1)**. In the past, irrational numbers may have been described as numbers that are infinite decimals that cannot be expressed as a fraction, like the number . This may have led to confusion about irrational numbers because until now, students did not know how to write repeating decimals as fractions and further, students frequently approximated using leading to more confusion about the definition of irrational numbers. Defining irrational numbers as those that are not equal to rational numbers provides an important guidepost for students' knowledge of numbers. Students learn that an irrational number is something quite different than other numbers they have studied before. They are infinite decimals that can only be expressed by a decimal approximation. Now that students know that irrational numbers can be approximated, they extend their knowledge of the number line gained in Grade 6 **(6.NS.C.6)** to include being able to position irrational numbers on a line diagram in their approximate locations **(8.NS.A.2).**

Topic C revisits the Pythagorean Theorem and its applications, now in a context that includes the use of square roots and irrational numbers. Students learn another proof of the Pythagorean Theorem involving areas of squares off of each side of a right triangle **(8.G.B.6).** Another proof of the converse of the Pythagorean Theorem is presented to students, which requires an understanding of congruent triangles **(8.G.B.6).** With the concept of square roots firmly in place, students apply the Pythagorean Theorem to solve real-world and mathematical problems to determine an unknown side length of a right triangle and the distance between two points on the coordinate plane **(8.G.B.7, 8.G.B.8)**.

In Topic D, students learn that radical expressions naturally arise in geometry, such as the height of an isosceles triangle or the lateral length of a cone. The Pythagorean Theorem is applied to three-dimensional figures in Topic D as students learn some geometric applications of radicals and roots **(8.G.B.7).** In order for students to determine the volume of a cone or sphere, they must first apply the Pythagorean Theorem to determine the height of the cone or the radius of the sphere. Students learn that truncated cones are solids obtained by removing the top portion above a plane parallel to the base. Students know that to find the volume of a truncated cone they must access and apply their knowledge of similar figures learned in Module 3. Their work with truncated cones is an exploration of solids that is not formally assessed. In general, students solve real-world and mathematical problems in three dimensions in Topic D **(8.G.C.9)**. For example, now that students can compute with cube roots and understand the concept of rate of change, students compute the average rate of change in the height of the water level when water is poured into a conical container at a constant rate. Students also use what they learned about the volume of cylinders, cones, and spheres to compare volumes of composite solids.

It is recommended that students have access to a calculator to complete the End-of-Module Assessment but that they complete the Mid-Module Assessment without one.

The discussion of infinite decimals and the conversion of fractions to decimals in this module is taken from the following source: H. Wu, *Mathematics of the Secondary School Curriculum, Volume III* (to appear in 2015).

Introduction to Irrational Numbers Using Geometry

Unit 7

Subject: Mathematics

Grade/Course: Grade 8

Pacing: 35 days

Unit of Study: Unit 7: Introduction to Irrational Numbers Using Geometry

Priority Standards:

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions

Work with radicals and integer exponents.

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that is the square root of 2 is irrational.

Understand and apply the Pythagorean Theorem.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.C.9 Know the volumes of cones, cylinders, and spheres and use them to solve real world and mathematical problems.

Foundational Standards

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.B.2 Fluently divide multi-digit numbers using the standard algorithm.

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., and that is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal and vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If and are integers, then Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational numbers terminates in 0s or eventually repeats.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.A.2 Draw (freehand, with rule and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Focus Standards for Mathematical Practice

MP.6 Attend to precision.
Students begin attending to precision by recognizing and identifying numbers as rational or irrational. Students know the definition of an irrational number and can represent the number in different ways, e.g., as a root, non-repeating decimal block, or symbol such as Students will attend to precision when clarifying the difference between an exact value of an irrational number compared to the decimal approximation of the irrational number. Students use appropriate symbols and definitions when they work through proofs of the Pythagorean Theorem and its converse. Students know and apply formulas related to volume of cones and truncated cones.

MP.7 Look for and make use of structure.
Students learn that a radicand can be rewritten as a product and that sometimes one or more of the factors of the product can be simplified to a rational number. Students look for structure in repeating decimals, recognize repeating blocks, and know that every fraction is equal to a repeating decimal. Additionally, students learn to see composite solids as made up of simpler solids. Students interpret numerical expressions as representations of volumes of complex figures.

MP.8 Look for and express regularity in repeated reasoning.
While using the long division algorithm to convert fractions to decimals, students recognize that when a sequence of remainders repeats that the decimal form of the number will contain a repeat block. Students recognize that when the decimal expansion of a number does not repeat nor terminate, the number is irrational and can be represented with a method of rational approximation using a sequence of rational numbers to get closer and closer to the given number.

“Unwrapped” Standards

8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that the square root of 2 is irrational.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

8.G.C.9 Know the volumes of cones, cylinders, and spheres and use them to solve real world and mathematical problems.

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
Numbers that are not rational are called irrational.	know (L1)
Every number has a decimal expansion.	understand (L2)
The decimal expansion repeats eventually for rational numbers.	show (L1)
A decimal expansion which repeats into a rational number.	convert (L4)
Irrational numbers.	compare (L2) - the size locate (L2) - on a number line estimate (L2) - the value
Square root and cube root symbols.	use (L1) - to represent solutions to equations
Square roots of small perfect squares and cube roots of small perfect cubes.	evaluate (L4)
The square root of 2 is irrational.	know (L1)
A proof of the Pythagorean Theorem and its converse.	explain (L2)
The Pythagorean Theorem.	apply (L2) - to determine unknown side lengths in right triangles apply (L2) - to find the distance between two points in a coordinate system
The volumes of cones, cylinders, and spheres.	know (L1)
Real world and mathematical problems.	solve (L2) - using formulas

Essential Questions	**Big ideas**
What problem solving strategy will work best for a real world problem? How does explaining my process help me to understand a problem's solution better? How do I use algebraic expressions to analyze or solve problems? How is thinking algebraically different from thinking arithmetically? What kinds of experiences help develop number sense?	There can be different strategies to solve a problem, but some are more effective and efficient than others are. A problem solver understands what has been done, knows why the process was appropriate, and can support it with reasons and evidence. Algebraic expressions and equations generalize relationships from specific cases. Real world situations can be represented symbolically and graphically. Number sense develops through experience.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary New Terms Perfect Square Square Root Cube Root Irrational Number Infinite Decimals Rational Approximation Truncated Cone Familiar Terms and Symbols Volume Rate of Change Number Line Rational Number Finite Decimals Decimal Expansion Rate of Change	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences IXL Math	Type: Mid-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addressed: 8.NS.A.1, 8.NS.A.2, 8.EE.A.2 Type: End-of-Module Assessment Task Administered: After Topic D Format: Constructed response with rubric Standards Addresses: 8.NS.A.1, 8.NS.A.2, 8.EE.A.2, 8.G.B.6, 8.G.B.7, 8.G.B.8, 8.G.C.9

Conduct opening exercise Use exit ticket as pre-assessment and post where applicable		

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task detailed in Unit 6.
Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 6.

Instructional Resources
Suggested Tools and Representations Scientific Calculator 3D models (truncated cone, pyramid) ixl.com (Math) flippedmath.com (Grade 8 Math) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.)

https://www.engageny.org/resource/grade-8-mathematics (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Grade 8 Mathematics propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as

	models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are "just right" for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day's sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond.

	Assess by multiple means, including “show and tell” rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently.

	Follow predictable routines to allow students to focus on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities. Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Perfect Square** - the square of an integer. **Square Root** **Cube Root** **Irrational Number** - numbers that are not rational. **Infinite Decimals** - decimals that do not repeat nor terminate. **Rational Approximation** - the method for determining the approximated rational form of an irrational number. **Truncated Cone** - a solid obtained from a cone by removing the top portion above a plane parallel to the base. **Familiar Terms and Symbols** Volume Rate of Change Number Line Rational Number Finite Decimals Decimal Expansion	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action</u>**

Rate of Change	**<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know	**<u>and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by

	the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	enhancing the operation (e. g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?" "Can you propose an alternative...?" "How would you evaluate...?" "What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Appendix A: Lesson Plan Sample
Module 4 Lesson 4

The following is a sample lesson plan from EngageNY. The lesson in its entirety can be found at **https://www.engageny.org/resource/algebra-i-module-4-topic-lesson-4**
In addition to the lesson plan, printable worksheets, sample student answers are available online. EngageNY can be used as a resource for all modules.

Lesson 4: Solving a Linear Equation

Student Outcomes

Students extend the use of the properties of equality to solve linear equations having rational coefficients.

Classwork

Concept Development (13 minutes)

To solve an equation means to find all of the numbers x, if they exist, so that the given equation is true.

In some cases, some simple guess work can lead us to a solution. For example, consider the following equation:

$$4x + 1 = 13.$$

What number x would make this equation true? That is, what value of x would make the left side equal to the right side? (Give students a moment to guess a solution.)

When $x = 3$, we get a true statement. The left side of the equal sign is equal to 13 and so is the right side of the equal sign.

In other cases, guessing the correct answer is not so easy. Consider the following equation:

$$3(4x - 9) + 10 = 15x + 2 + 7x.$$

Can you guess a number for x that would make this equation true? (Give students a minute to guess.)

Guessing is not always an efficient strategy for solving equations. In the last example, there are several terms in each of the linear expressions comprising the equation. This makes it more difficult to easily guess a solution. For this reason, we want to use what we know about the properties of equality to transform equations into equations with fewer terms.

The ultimate goal of solving any equation is to get it into the form of x (or whatever symbol is being used in the equation) equal to a constant.

Complete the activity described below to remind students of the properties of equality, then proceed with the discussion that follows.

Give students the equation: $4 + 1 = 7 - 2$, and ask them the following questions.

1. Is this equation true?

2. Perform each of the following operations, and state whether or not the equation is still true:

a. Add three to both sides of the equal sign.

b. Add three to the left side of the equal sign, and add two to the right side of the equal sign.

c. Subtract six from both sides of the equal sign.

d. Subtract three from one side of the equal sign, and subtract three from the other side of the equal sign.

e. Multiply both sides of the equal sign by ten.

f. Multiply the left side of the equation by ten and the right side of the equation by four.

g. Divide both sides of the equation by two.

h. Divide the left side of the equation by two and the right side of the equation by five.

3. What do you notice? Describe any patterns you see.

There are four properties of equality that will allow us to transform an equation into the form we want. If □, □, and □ are any rational numbers, then

If □ = □, then □ + □ = □ + □.

If □ = □, then □ − □ = □ − □.

If □ = □, then □ · □ = □ · □.

If □ = □, then □/□ = □/□ , where □ is not equal to zero.

All four of the properties require us to start off with □ = □. That is, we have to assume that a given equation has an expression on the left side that is equal to the expression on the right side. Working under that assumption, each time we use one of the properties of equality, we are transforming the equation into another equation that is also true, i.e., left side equals right side.

Example 1 (3 minutes)

Solve the linear equation 2□ −3 = 4□ for the number □.

Examine the properties of equality. Choose "something" to add, subtract, multiply, or divide on both sides of the equation.

Validate the use of the properties of equality by having students share their thoughts. Then, discuss the "best" choice for the first step in solving the equation with the points below. Be sure to remind students throughout this and the other examples that our goal is to get □ equal to a constant; therefore, the "best" choice is one that gets us to that goal most efficiently.

First, we must assume that there is a number □ that makes the equation true. Working under that assumption, when we use the property, if □ = □, then □ − □ = □ − □, we get an equation that is also true.

2□ −3 = 4□

2□ − 2□ −3 = 4□ −2□

Now, using the distributive property, we get another set of equations that is also true.

$$(2-2)\square - 3 = (4-2)\square$$
$$0\square - 3 = 2\square$$
$$-3 = 2\square$$

Using another property, if $\square = \square$, then $\square/\square = \square/\square$, we get another equation that is also true.

$$-3/2 = 2\square/2$$

After simplifying the fraction 2/2 , we have

$$-3/2 = \square,$$

which is also true.

The last step is to check to see if $\square = -3/2$ satisfies the equation $2\square - 3 = 4\square$.
The left side of the equation is equal to $2 \cdot (-3/2) - 3 = -3 - 3 = -6$.
The right side of the equation is equal to $4 \cdot (-3/2) = 2 \cdot (-3) = -6$.
Since the left side equals the right side, we know we have found the number $\square$ that solves the equation $2\square - 3 = 4\square$.

Example 2 (4 minutes)

Solve the linear equation $\frac{3}{5}\square - 21 = 15$. Keep in mind that our goal is to transform the equation so that it is in the form of $\square$ equal to a constant. If we assume that the equation is true for some number $\square$, which property should we use to help us reach our goal, and how should we use it?

Again, provide students time to decide which property is "best" to use first.

We should use the property if $\square = \square$, then $\square + \square = \square + \square$, where the number $\square$ is 21.

Note that if students suggest that we subtract 15 from both sides (i.e., where $\square$ is -15), remind them that we want the form of $\square$ equal to a constant. Subtracting 15 from both sides of the equal sign puts the $\square$ and all of the constants on the same side of the equal sign. There is nothing mathematically incorrect about subtracting 15, but it does not get us any closer to reaching our goal.

If we use $\square + \square = \square + \square$, then we have the true statement:

$$\frac{3}{5}\square - 21 + 21 = 15 + 21$$

and

⅗ ▯ = 36.

Which property should we use to reach our goal, and how should we use it?

We should use the property if ▯ = ▯, then ▯ · ▯ = ▯ · ▯, where ▯ is 5/3 .

If we use ▯ · ▯ = ▯ · ▯, then we have the true statement:

3/5▯ · 5/3 = 36 · 5/3 ,

and by the commutative property and the cancellation law, we have

▯ = 12 · 5 = 60.

Does ▯ = 60 satisfy the equation 3 /5 ▯ − 21 = 15?

Yes, because the left side of the equation is equal to 180/5 − 21 = 36 − 21 = 15. Since the right side is also 15, then we know that 60 is a solution to 3/5 ▯ −21 = 15.

Example 3 (5 minutes)

The properties of equality are not the only properties we can use with equations. What other properties do we know that could make solving an equation more efficient?

We know the distributive property, which allows us to expand and simplify expressions.

We know the commutative and associative properties, which allow us to rearrange and group terms within expressions.

Now we will solve the linear equation ⅕ ▯ + 13 + ▯ = 1 − 9▯ + 22. Is there anything we can do to the linear expression on the left side to transform it into an expression with fewer terms?

Yes, we can use the commutative and distributive properties:

1/5 ▯ + 13 + ▯ = 1/5 ▯ +▯ + 13
= 6/5 ▯ +13.

Is there anything we can do to the linear expression on the right side to transform it into an expression with fewer terms?

Yes, we can use the commutative property:

1 −9▯ + 22 = 1+ 22 − 9▯
= 23− 9▯.

Now we have the equation 6/5 ▯ + 13 = 23 −9▯. What should we do now to solve the equation?

Students should come up with the following four responses as to what should be done next. A case can be made for each of them being the "best" move. In this case, each move gets us one step closer to our goal of having the solution in the form of x equal to a constant. Select one option and move forward with solving the equation (the notes that follow align to the first choice, subtracting 13 from both sides of the equal sign).

We should subtract 13 from both sides of the equal sign.
We should subtract 23 from both sides of the equal sign.
We should add 9x to both sides of the equal sign.
We should subtract 6/5 x from both sides of the equal sign.

Let's choose to subtract 13 from both sides of the equal sign. Though all options were generally equal with respect to being the "best" first step, I chose this one because when I subtract 13 on both sides, the value of the constant on the left side is positive. I prefer to work with positive numbers. Then we have

$$6/5\, x + 13 - 13 = 23 - 13 - 9x$$
$$6/5\, x = 10 - 9x.$$

What should we do next? Why?

We should add 9x to both sides of the equal sign. We want our solution in the form of x equal to a constant, and this move puts all terms with an x on the same side of the equal sign.

Adding 9x to both sides of the equal sign, we have

$$6/5\, x + 9x = 10 - 9x + 9x$$
$$51/5\, x = 10.$$

What do we need to do now?

We should multiply 5/51 on both sides of the equal sign.

Then we have

$$51/5\, x \cdot 5/51 = 10 \cdot 5/51.$$

By the commutative property and the fact that $5/51 \times 51/5 = 1$, we have

$$x = 50/51.$$

All of the work we did is only valid if our assumption that $1/5\, x + 13 + x = 1 - 9x + 22$ is a true statement. Therefore, check to see if 50/51 makes the original equation a true statement.

$$1/5\, x + 13 + x = 1 - 9x + 22$$
$$1/5\,(50/51) + 13 + 50/51 = 1 - 9\,(50/51) + 22$$
$$6/5\,(50/51) + 13 = 23 - 9\,(50/51)$$

300/255 + 13 = 23 − 450/51
3615/255 = 723/51
723/51 = 723/51

Since both sides of our equation equal 723/51 , then we know that 50/51 is a solution of the equation.

Exercises 1–5 (10 minutes)

Students work on Exercises 1–5 independently.

Exercises
For each problem, show your work and check that your solution is correct.

1. Solve the linear equation □ + □ + □ + □ + □ + □ + □ = −□□. State the property that justifies your first step and why you chose it.

The left side of the equation can be transformed from □ + □ + □ + □ + □ + □ + □ to □□ + □□ using the commutative and distributive properties. Using these properties decreases the number of terms of the equation. Now we have the equation:

□□ + □□ = −□□
□□ + □□ − □□ = −□□ − □□
□□ = −□□
□/□ · □□ = −□□ · □/□
□ = −□□.

The left side of the equation is equal to (−□□) + (−□□) + □ + (−□□) + □ + (−□□) + □, which is −□□. Since the left side is equal to the right side, then □ = −□□ is the solution to the equation.

Note: Students could use the division property in the last step to get the answer.

2. Solve the linear equation □(□□ + □) = □□ − □ + □. State the property that justifies your first step and why you chose it.

Both sides of equation can be rewritten using the distributive property. I have to use it on the left side to expand the expression. I have to use it on the right side to collect like terms.
The left side is

□(□□ + □) = □□ + □.

The right side is

□□ − □ + □ = □□ + □ − □
= □□ − □.

The equation is

□□ + □ = □□ − □

□□ + □ − □ = □□ − □ − □

□□ = □□ − □

□□ − □□ = □□ − □□ − □

(□ − □)□ = (□ − □)□ − □

□ = −□

□/□ · □□ = □/□ · (−□)

□ = − □/□ .

The left side of the equation is equal to □(−□ + □) = □(−□) = −□. The right side of the equation is equal to −□ − □ = −□. Since both sides are equal to −□, then □ = − □/□ is a solution to □(□□ + □) = □□ − □ + □.

Note: Students could use the division property in the last step to get the answer.

3. Solve the linear equation □ − □ = □ □ □. State the property that justifies your first step and why you chose it.

I chose to use the subtraction property of equality to get all terms with an □ on one side of the equal sign.

□ − □ = □/□ □

□ − □ − □ = □/□ □ − □

(□ − □)□ − □ = (□/□ − □) □

−□ = − □/□ □

− □/□ · (−□) = − □/□ · − □/□ □

□□/□ = □

The left side of the equation is □□ /□ − □□/□ = □□/□ . The right side is □/□ · □□/□ = □/□ · □/□ = □□/□ . Since both sides are equal to the same number, then □ = □□/□ is a solution to □ − □ = □/□ □.

4. Solve the linear equation □□ − □□ = □□ + □. State the property that justifies your first step and why you chose it.

I chose to use the addition property of equality to get all terms with an □ on one side of the equal sign.

□□ − □□ = □□ + □

□□ − □□ + □□ = □□ + □□ + □

□□ = □□ + □

□□ − □ = □□ + □ − □

□□ = □□

□/□ · □□ = □/□ · □□

□ = □

The left side of the equal sign is □□ − □(□) = □□ − □ = □□. The right side is equal to □(□) + □ = □□ + □ = □□. Since both sides are equal, □ = □ is a solution to □□ − □□ = □□ + □.

Note: Students could use the division property in the last step to get the answer.

5. Solve the linear equation □/□ □ − □ + □□□ = □. State the property that justifies your first step and why you chose it.

I chose to combine the constants −□ and □□□. Then, I used the subtraction property of equality to get all terms with an □ on one side of the equal sign.

□/□ □ − □ + □□□ = □

□/□ □ + □□□ = □

□/□ □ − □/□ □ + □□□ = □ − □/□ □

□□□ = □/□ □

□□□ · □/□ = □/□ · □/□ □

□□ × □ = □

□□□ = □

The left side of the equation is □/□ · □□□ − □ + □□□ = □□ − □ + □□□ = □□ + □□□ = □□□, which is exactly equal to the right side. Therefore, □ = □□□ is a solution to □/□ □ − □ + □□□ = □.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

We know that properties of equality, when used to transform equations, make equations with fewer terms that are simpler to solve.

When solving an equation, we want the answer to be in the form of the symbol □ equal to a constant.

Lesson Summary

The properties of equality, shown below, are used to transform equations into simpler forms. If □, □, □ are rational numbers, then

If □ = □, then □ + □ = □ + □.	Addition property of equality
If □ = □, then □ − □ = □ − □.	Subtraction property of equality
If □ = □, then □ · □ = □ · □.	Multiplication property of equality
If □ = □, then □ □ = □ □ , where □ is not equal to zero.	Division property of equality

To solve an equation, transform the equation until you get to the form of □ equal to a constant (□ = □, for example).

Exit Ticket (5 minutes)