Thomaston Public Schools

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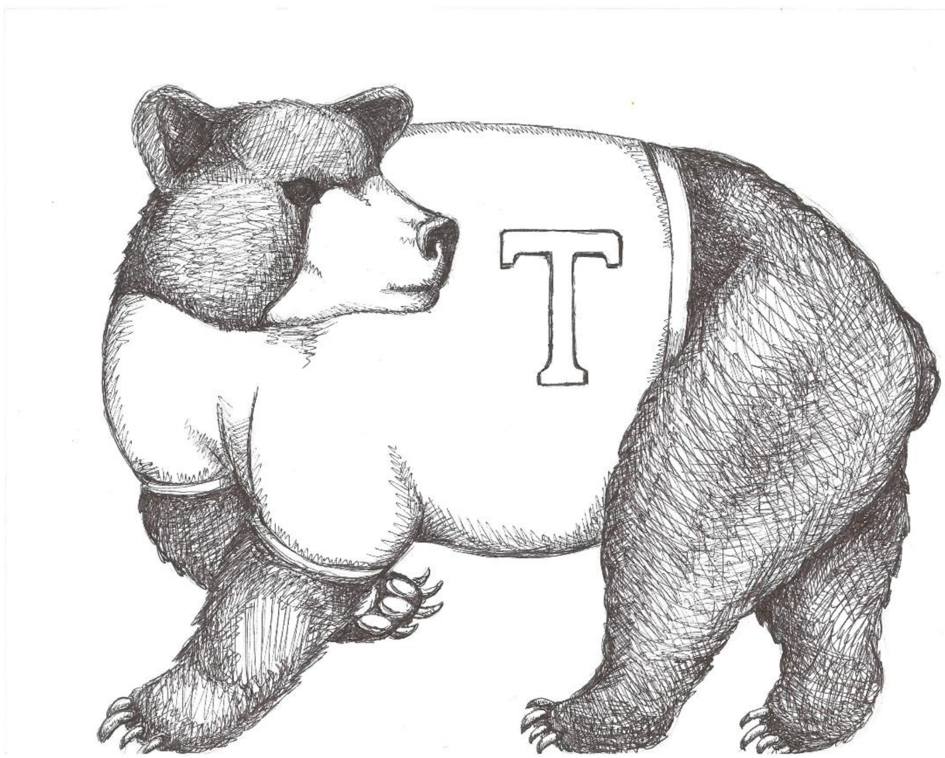

Thomaston Public Schools Curriculum
Thomaston High School
Grade(s): 7 Mathematics
2015

Learn to Live….Live to Learn

Acknowledgements

Curriculum Writer(s):

Alisha DiCorpo

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

Alisha DiCorpo

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Director of Curriculum and Professional Development

Date of Presentation to the Board of Education: _ August 2015 ___________

(Math Curriculum Grade 7)

[Grade 7 Mathematics]

Board of Education Mission Statement:

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:
The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main Resource used when writing this curriculum:
NYS COMMON CORE MATHEMATICS CURRICULUM A Story of Units Curriculum. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. A Story of Ratios: A Curriculum Overview for Grades 6-8 Date: 7/31/13 5 © 2013 Common Core, Inc. Some rights reserved. commoncore.org

Course Description:
Sequence of Grade 7 Modules (Units) Aligned with the Standards

Unit 1: Ratios and Proportional Relationships
Unit 2: Rational Numbers
Unit 3: Expressions and Equations
Unit 4: Percent and Proportional Relationships
Unit 5: Statistics and Probability
Unit 6: Geometry

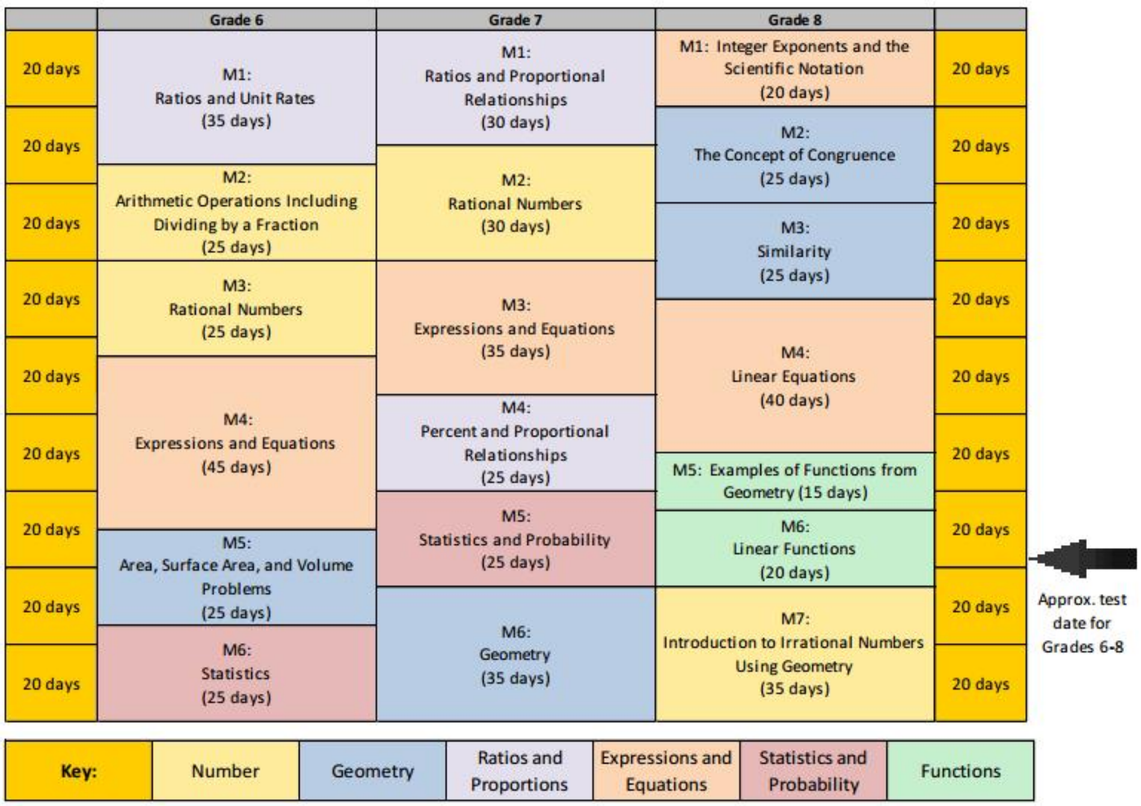
Summary of the Year
Seventh grade mathematics is about (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two-and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

Key Areas of Focus for Grade 7: Ratios and proportional reasoning; arithmetic of rational numbers

Rationale for Module Sequence in Grade 7

In Unit 1, students build on their Grade 6 experiences with ratios, unit rates, and fraction division to analyze proportional relationships. They decide whether two quantities are in a proportional relationship, identify constants of proportionality, and represent the relationship by equations. These skills are then applied to real-world problems including scale drawings.

Students continue to build an understanding of the number line in Module (Unit) 2 from their work in Grade 6. They learn to add, subtract, multiply, and divide rational numbers. Module (Unit) 2 includes rational numbers as they appear in expressions and equations-work that is continued in Module (Unit) 3.



Ratios and Proportional Relationships

OVERVIEW

In Unit 1, students build upon their Grade 6 reasoning about ratios, rates, and unit rates (**6.RP.A.1**, **6.RP.A.2**, **6.RP.3**) to formally define proportional relationships and the constant of proportionality (**7.RP.A.2**). In Topic A, students examine situations carefully to determine if they are describing a proportional relationship. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions (**7.RP.A.2a**).

In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the *constant of proportionality* and can be used to represent proportional relationships with equations of the form $y = kx$, where k is the constant of proportionality (**7.RP.A.2b**, **7.RP.A.2c**, **7.EE.B.4a**). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (**7.RP.A.2d**).

In Topic C, students extend their reasoning about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers, such as a speed of $\frac{1}{2}$ mile per $\frac{1}{4}$ hour (**7.RP.A.1**). Students apply their experience in the first two topics and their new understanding of unit rates for ratios and rates involving fractions to solve multistep ratio word problems (**7.RP.A.3**, **7.EE.B.4a**).

In the final topic of this unit, students bring the sum of their experience with proportional relationships to the context of scale drawings (**7.RP.A.2b**, **7.G.A.1**). Given a scale drawing, students rely on their background in working with side lengths and areas of polygons (**6.G.A.1**, **6.G.A.3**) as they identify the scale factor as the constant of proportionality, calculate the actual lengths and areas of objects in the drawing, and create their own scale drawings of a two-dimensional view of a room or building. The topic culminates with a two-day experience of students creating a new scale drawing by changing the scale of an existing drawing.

Later in the year, in Unit 4, students will extend the concepts of this module to percent problems.

The unit is comprised of 22 lessons; 8 days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Mathematics/Grade 7/Unit1

Rigorous Curriculum Design Template

Unit: 1 Ratios and Proportional Relationships

Subject: Mathematics

Grade/Course: 7 Mathematics

Pacing: 22 Days

Unit of Study: Ratios and Proportional Relationships

Priority Standards:

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost, t, is proportional to the number, n, of items purchased at a constant price, p, the relationship between the total cost and the number of items can be expressed as t = pn.*

d. Explain what a point *(x,y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points *(0,0)* and *(1,r)*, where *r* is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4[1] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[2]

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units

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[1] In this module, the equations are derived from ratio problems. 7.EE.B.4a is returned to in Modules 2 and 3.

[2] Expectations for unit rates in this grade are limited to non-complex fractions.

appropriately when multiplying or dividing quantities.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of and solve multistep ratio problems, including cases with pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane, and equations, and relate these representations to each other and to the context of the problem. Students depict the meaning of constant proportionality in proportional relationships, the importance of $(0,0)$ and $(1,$ □$)$ on graphs and the implications of how scale factors magnify or shrink actual lengths of figures on a scale drawing.

MP.2 **Reason abstractly and quantitatively.** Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Use of concrete numbers will be analyzed to create and implement equations, including □ = □□, where □is the constant of proportionality. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, □□□□□□□□ = □□□□ × □□□□. In scale drawings, scale factors will be changed to create additional scale drawings of a given picture.

"Unwrapped" Standards

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

e. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

f. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

g. Represent proportional relationships by equations. *For example, if total cost, t, is proportional to the number, n, of items purchased at a constant price, p, the relationship between the total cost and the number of items can be expressed as t = pn.*

h. Explain what a point *(x,y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points *(0,0)* and *(1,r)*, where *r* is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent*

increase and decrease, percent error. **7.EE.B.4**[3] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Levels**

[3] In this module, the equations are derived from ratio problems. 7.EE.B.4a is returned to in Modules 2 and 3.

7.RP.A.1 Unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	Compute (L2)
7.RP.A.2 Proportional relationships between quantities.	
Two quantities are in a proportional relationship,	Recognize (L1)
Constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	Represent (L2) Decide (L3)
Proportional relationships by equations. *For example, if total cost, t, is proportional to the number, n, of items purchased at a constant price, p, the relationship between the total cost and the number of items can be expressed as t = pn.*	Identify (L1)
i. A point *(x,y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points *(0,0)* and *(1,r)*, where *r* is the unit rate.	Represent (L3) Explain (L1)
7.RP.A.3 Proportional relationships to solve multistep ratio and percent problems.	Use (L1)
7.EE.B.4[4] Variables quantities in a real-world or mathematical problem, and simple equations and inequalities, problems by quantities.	Use (L1)
a. Word problems leading to equations of the form *px + q = r* and *p(x + q) = r*, where *p*, *q*, and *r* are specific rational numbers, equations of these forms fluently, algebraic solution to an arithmetic solution, the sequence of the operations used in each approach. *For example, the perimeter of a*	Represent (L3) Construct (L3) Solve (L2)

[4] In this module, the equations are derived from ratio problems. 7.EE.B.4a is returned to in Modules 2 and 3.

rectangle is 54 cm. Its length is 6 cm. What is its width? **7.G.A.1** Problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	Reasoning(L3) Solve (L2) Compare (L3) Identify the sequence (L1) Solve (L2)

Essential Question	**Big ideas**
How are ratios and their relationships used to solve real world problems? How can a complex fraction be simplified? What is the difference between a unit rate and a ratio? What is a proportion? Why are multiplicative relationships proportional?	Reasoning with ratios involves attending to and coordinating two quantities. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of

	interest. A proportion can be represented with ratios, tables or ordered pairs to help illustrate the relationships between quantities. A number of mathematical connections link ratios and fractions: o Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning. o Ratios are often used to make "part-part" comparisons, but fractions are not. o Ratios and fractions can be thought of as overlapping sets. o Ratios can often be meaningfully reinterpreted as fractions. Ratios can be meaningfully reinterpreted as quotients. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change. Proportional reasoning is complex and involves understanding that - o Equivalent ratios can be created by iterating and/or partitioning a composed unit; o If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and o The two types of ratios - composed units and multiplicative comparisons - are related. A rate is set of infinitely many equivalent ratios. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.

	Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities. Geometric images provide the content in relation to which properties can be noticed, definitions can be made, and invariances can be discerned. Geometric awareness develops through practice in visualizing, diagramming, and constructing.

Assessments

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.RP.A.2
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.4a, 7.G.A.1

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Exit Tickets for Pre-Assessment of each lesson.	**Application problems** **Student Debriefs** **Problem Set Data**	**(See chart above)** **Exit tickets as post assessments for each lesson**

Performance Task
Leaky Faucet Performance Task http://schools.nyc.gov/NR/rdonlyres/FFF98A0A-16D1-4E10-BF2C-01A83F011B67/140497/NYCDOE_G7_Math_LeakyFaucets_FINAL.pdf

Engaging Learning Experiences

Task Description:
While restoring an old house, it was noticed that there was a leaky faucet. At first it was the kitchen faucet, then extended to the bathroom. The task requires students to determine at what rate the f aucet is leaking and create a table to display observations. The task continues by having students create an equation to represent the situation. Students are asked to compare the unit rates of both faucets and identify the constant of proportionality. On a coordinate plane, students are asked to sketch the graph of both faucets and use evidence from the graph to justify a proportional relation.

Instructional Resources

Suggested Tools and Representations

- Ratio Table (See example below)
- Coordinate Plane (See

example below)

- Equations of the form
  □ = □□
- IXL Math

Ratio Table

Sugar	*Flour*

Coordinate Plane

Instructional Strategies	Meeting the Needs of All Students
Marzano's Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning.

21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **Scaffolds for Students with Disabilities** Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. ● Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model

and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…”

- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “_______ is _____ hundreds, _____ tens, and _____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.”
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are “just right” for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day’s sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including “show and tell” rather than written.

	- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” - Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” - Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** - Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. - Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. - Teach in small chunks so students get a lot of practice with one step at a time. - Know, use, and make the most of Deaf culture and sign language. - Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” - Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. - Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently.

	- Follow predictable routines to allow students to focus on content rather than behavior. - Allow “everyday” and first language to express math understanding. - Re-teach the same concept with a variety of fluency games. - Allow students to lead group and pair-share activities. - Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms ▪ **Proportional To** (Measures of one type of quantity are *proportional to* measures of a second type of quantity if there is a number □ > 0 so that for every measure □ of a quantity of the first type the corresponding measure □ of a quantity of the second type is given by □□, i.e., □ $=$ □□.) ▪ **Proportional Relationship** (A one-to-one matching between two types of quantities such that the measures of quantities of the first type are proportional to the measures of quantities of the second type.) ▪ **Constant of Proportionality** (If a proportional relationship is described by the set of ordered pairs that satisfies the equation □ $=$ □□, where □ is a positive constant, then □ is called the *constant of proportionality*. For example, if the ratio of □ to □ is 2 to 3, then the constant of proportionality is $\frac{2}{3}$ and □ $= \frac{2}{3}$□.)	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or

▪ **One-to- One Correspondence** (Two figures in the plane, □ and □′, are said to be in one-to-one correspondence if there is a pairing between the points in □ and □′, so that each point □ of □ is paired with one and only one point □′ in □′, and likewise, each point □′ in □′ is paired with one and only one point □ in □.) ▪ **Scale Drawing and Scale Factor**[5] (For two figures in the plane, □ and □′, □′ is said to be a *scale drawing* of □ with *scale factor* □ if there exists a one-to-one correspondence between □ and □′ so that under the pairing of this one-to-one correspondence, the distance |□□| between any two points □ and □ of □ is related to the distance |□′□′| between corresponding points □′ and □′ of □′ by |□′□′| = □|□□|.) **Familiar Terms and Symbols**[6]	objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving.	group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip- counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing

[5] These terms will be formally defined in Grade 8. A description is provided in Grade 7.

[6]

▪ Ratio ▪ Rate ▪ Unit Rate ▪ Equivalent Ratio ▪ Ratio Table	Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds.	numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games.

	Set goals with the students regarding next steps and what to focus on next.	Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Unit 2 OVERVIEW

In Grade 6, students formed a conceptual understanding of integers through the use of the number line, absolute value, and opposites and extended their understanding to include the ordering and comparing of rational numbers (**6.NS.C.5**, **6.NS.C.6**, **6.NS.C.7**). This unit uses the Integer Game: a card game that creates a conceptual understanding of integer operations and serves as a powerful mental model students can rely on during the module. Students build on their understanding of rational numbers to add, subtract, multiply, and divide signed numbers. Previous work in computing the sums, differences, products, and quotients of fractions and decimals serves as a significant foundation as well.

In Topic A, students return to the number line to model the addition and subtraction of integers (**7.NS.A.1**). They use the number line and the Integer Game to demonstrate that an integer added to its opposite equals zero, representing the additive inverse (**7.NS.A.1a**, **7.NS.A.1b**). Their findings are formalized as students develop rules for adding and subtracting integers, and they recognize that subtracting a number is the same as adding its opposite (**7.NS.A.1c**). Real-life situations are represented by the sums and differences of signed numbers. Students extend integer rules to include the rational numbers and use properties of operations to perform rational number calculations without the use of a calculator (**7.NS.A.1d**).

Students develop the rules for multiplying and dividing signed numbers in Topic B. They use the properties of operations and their previous understanding of multiplication as repeated addition to represent the multiplication of a negative number as repeated subtraction (**7.NS.A.2a**). Students make analogies to the Integer Game to understand that the product of two negative numbers is a positive number. From earlier grades, they recognize division as the inverse process of multiplication. Thus, signed number rules for division are consistent with those for multiplication, provided a divisor is not zero (**7.NS.A.2b**). Students represent the division of two integers as a fraction, extending product and quotient rules to all rational numbers. They realize that any rational number in fractional form can be represented as a decimal that either terminates in 0s or repeats (**7.NS.A.2d**). Students recognize that the context of a situation often determines the most appropriate form of a rational number, and they use long division, place value, and equivalent fractions to fluently convert between these fractions and decimal forms. Topic B concludes with students multiplying and dividing rational numbers using the properties of operations (**7.NS.A.2c**).

In Topic C, students problem-solve with rational numbers and draw upon their work from Grade 6 with expressions and equations (**6.EE.A.2**, **6.EE.A.3**, **6.EE.A.4**, **6.EE.B.5**, **6.EE.B.6**, **6.EE.B.7**). They perform operations with rational numbers (**7.NS.A.3**), incorporating them into algebraic expressions and equations. They represent and evaluate expressions in multiple forms, demonstrating how quantities are related (**7.EE.A.2**). The Integer Game is revisited as students discover "if-then" statements, relating changes in player's hands (who have the same card-value totals) to changes in both sides of a number sentence. Students translate word problems into algebraic equations and become proficient at solving equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r, are specific rational numbers (**7.EE.B.4a**). As they become fluent in generating algebraic solutions, students identify the operations, inverse operations, and order of steps, comparing these to an arithmetic solution. Use of algebra to represent contextual problems continues in Module 3.

This unit is comprised of 23 lessons; 7 days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic C.

Mathematics Unit -2 Grade 7

Rigorous Curriculum Design Template

Unit 2: Rational Numbers

Subject: Mathematics

Grade/Course: Grade 7

Pacing: 23 Days

Unit of Study: Unit 2: Rational Numbers

Priority Standards:

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[7]

Use properties of operations to generate equivalent expressions.

7.EE.A.2[8] Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4[9] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$,

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[7] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

[8] In this module, this standard is applied to expressions with rational numbers in them.

[9] In this module, the equations include negative rational numbers.

where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Foundational Standards

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars*.

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.2 **Write, read, and evaluate expressions in which letters stand for numbers.**

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\,s^2$ *to find the volume and surface area of a cube with sides of length s = 1/2.*

6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce* *the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.B.7 Solve real- world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them**. When problem-solving, students use a variety of techniques to make sense of a situation involving rational numbers. For example, they may draw a number line and use arrows to model and make sense of an integer addition or subtraction problem. Or when converting between forms of rational numbers, students persevere in carrying out the long division algorithm to determine a decimal's repeat pattern. A tape diagram may be constructed as an entry point to make sense of a working-backwards problem. As students fluently solve word problems using algebraic equations and inverse operations, they consider their steps and determine whether or not they make sense in relationship to the arithmetic reasoning that served as their foundation in earlier grades.

MP.2 **Reason abstractly and quantitatively**. Students make sense of integer addition and subtraction through the use of an integer card game and diagramming the distances and directions on the number line. They use different properties of operations to add, subtract, multiply, and divide rational numbers, applying the properties to generate equivalent expressions or explain a rule. Students use integer subtraction and absolute value to justify the distance between two numbers on the number line. Algebraic expressions and equations are created to represent relationships. Students know how to use the properties of operations to solve equations. They make "zeros and ones" when solving an algebraic equation, thereby demonstrating an understanding of how the use of inverse operations ultimately leads to the value of the variable.

MP.4 **Model with mathematics**. Through the use of number lines, tape diagrams, expressions, and equations, students model relationships between rational numbers. Students relate operations involving integers to contextual examples. For instance, an overdraft fee of $\$25$ that is applied to an account balance of $-\$73.06$, is represented by the expression $-73.06 - 25$ or $-73.06 + (-25)$ using the additive inverse. Students compare their answers and thought processes in the Integer Game and use number line diagrams to ensure accurate reasoning. They deconstruct a difficult word problem by writing an equation, drawing a number line, or drawing a tape diagram to represent quantities. To find a change in elevation, students may draw a picture representing the objects and label their heights to aid in their understanding of the mathematical operation(s) that must be performed.

MP.6 **Attend to precision**. In performing operations with rational numbers, students understand that the decimal representation reflects the specific place value of each digit. When converting fractions to decimals, they carry out their calculations to specific place values, indicating a terminating or repeating pattern. In stating answers to problems involving signed numbers, students use integer rules and properties of operations to verify that the sign of their answer is correct. For instance, when finding an average temperature for temperatures whose sum is a negative number, students realize that the quotient must be a negative number since the divisor is positive and the dividend is negative.

MP.7 **Look for and make use of structure.** Students formulate rules for operations with signed numbers by observing patterns. For instance, they notice that adding -7 to a number is the same as subtracting 7 from the number, and thus, they develop a rule for subtraction that relates to adding the inverse of the subtrahend. Students use the concept of absolute value and subtraction to represent the distance between two rational numbers on a number line. They use patterns related to the properties of operations to justify the rules for multiplying and dividing signed numbers. The order of operations provides the structure by which students evaluate and generate equivalent expressions.

"Unwrapped" Standards

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

e. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*.

f. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

g. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

h. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to

multiply and divide rational numbers. e. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. f. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. g. Apply properties of operations as strategies to multiply and divide rational numbers. h. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers.[10] **7.EE.A.2**[11] Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."* **7.EE.B.4**[12] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. b. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Levels**

[10] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

[11] In this module, this standard is applied to expressions with rational numbers in them.

[12] In this module, the equations include negative rational numbers.

7.NS.A.1 Previous understandings of addition and subtraction rational numbers; addition and subtraction on a horizontal or vertical number line diagram. i. Situations in which opposite quantities combine to make 0. j. *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. A number and its opposite have a sum of 0 (are additive inverses). Sums of rational numbers by real-world contexts. k. Subtraction of rational numbers as adding the additive inverse, *p* – *q* = *p* + (–*q*). distance between two rational numbers on the number line is the absolute value of their difference, and this principle in real-world contexts. l. Properties of operations as strategies to add and subtract rational numbers. **7.NS.A.2** Previous understandings of multiplication and division and of fractions rational numbers. i. Multiplication extended from fractions to rational numbers operations properties of operations, the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Products of rational numbers, real-world contexts. j. Integers divisor is not zero, every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then –(*p*/*q*) = (–*p*)/*q* = *p*/(–*q*). quotients of rational numbers by describing real-world contexts.	Apply (L1) Extend (L3) Represent (L2) Describe (L1) Understand (L1) Show (L2) Describe (L1) Interpret (L2) Understand (L1) Show (L2) Apply (L4) Apply (L4) Add (L1) Subtract (L1) Apply (L4) Extend (L3) Multiply (L2) Divide (L2) Understand (L1) Interpret (L2)

k. Properties of operations as strategies to multiply and divide rational numbers. l. Rational number a decimal using long division; the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.A.3** Real-world and mathematical problems involving the four operations with rational numbers.[13] **7.EE.A.2**[14] Expression in different forms in a problem context can shed light on the problem and how the quantities in it are related **7.EE.B.4**[15] Variables to quantities in a real-world or mathematical problem, and simple equations and inequalities, problems by reasoning about the quantities. c. Word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Algebraic solution to an arithmetic solution, the sequence of the operations used in each approach. d. Equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. equations of these forms fluently. an algebraic solution to an arithmetic solution, the sequence of the operations used in each approach.	Describe (L2) Understand (L1) Interpret (L2) Apply (L4) Convert (L3) Know (L1) Solve (L2) Understand (L1) Use (L1) Represent (L2) Construct (L3) Solve (L2) Compare (L3) Identify (L1) Solve (L2) Compare (L3) Identify (L1)

[13] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

[14] In this module, this standard is applied to expressions with rational numbers in them.

[15] In this module, the equations include negative rational numbers.

Essential Questions	Big ideas
When are negative numbers used and why are they important? How do I use positive and negative numbers in real life? Where are positive and negative numbers located on the number line? How does the coordinate plane and number line help me to determine the distance between points. What do algebraic inequalities represent? How are inequalities represented on a number line?	Negative numbers are used to represent quantities that are less than zero. Absolute value is useful in ordering and graphing positive and negative numbers. Rational numbers are points on a number line. Numbers in ordered pairs indicate locations in quadrants of the coordinate plan

Assessments

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.NS.A.1, 7.NS.A.2
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	7.NS.A.1, 7.NS.A.2, 7.NS.A.3, 7.EE.A.2, 7.EE.B.4a

Common Formative Pre-	Progress Monitoring Checks –	Common Formative Mid and or Post-

Assessments	“Dipsticks”	Assessments Resources
Exit Tickets for Pre-Assessment of each lesson.	**Checks for understanding** **Short and Extended Response questions used throughout the unit** **Reflections** **Formative Assessment Tasks (Embedded in lessons)**	**Performance tasks, unit tests** **Exit tickets as post assessments for each lesson**

Performance Task
To be created by teachers during year.

Engaging Learning Experiences
To be created by teachers during year.

Instructional Resources

Suggested Tools and Representations

- Equations
- Expressions
- Integer Game (See explanation on page 11)
- Number Line
- Tape Diagram
- IXL Math

Instructional Strategies	Meeting the Needs of All Students

Marzano's Effective Teaching Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **21st Century Skills** Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **Provide Multiple Means of Representation** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a

student searching to define “multiplication” may model groups of 6 with drawings or concrete objects and write the number sentence to match.

- Teach students how to ask questions (such as “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…”
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.”
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are “just right” for learners.
- Model each step of the algorithm before students begin.

	● Give students a chance to practice the next day's sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including "show and tell" rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" ● Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your

	legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

New or Recently Introduced Terms

- **Additive Identity** (The additive identity is 0.)
- **Additive Inverse** (The *additive inverse* of a real number is the opposite of that number on the real number line. For example, the opposite of -3 is 3. A number and its additive inverse have a sum of 0.)
- **Break-Even Point** (The *break-even point* is the point at which there is neither a profit nor loss.)
- **Distance Formula** (If ▯ and ▯ are rational numbers on a number line, then the distance between ▯ and ▯ is $|▯ - ▯|$.)
- **Loss** (A decrease in amount, as when the money earned is less than the money spent.)
- **Multiplicative Identity** (The *multiplicative identity* is 1.)
- **Profit** (A gain, as in the positive amount represented by the difference between the money earned and spent)
- **Repeating Decimal** (The decimal form of a rational number, for example, $\frac{1}{3} = 0.\underline{3}$.)
- **Terminating Decimal** (A decimal is called terminating if its

<u>Provide Multiple Means of Representation</u>

Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.

Guide students as they select and practice using their own graphic organizers and models to solve.

Use direct instruction for vocabulary with visual or concrete representations.

Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do."

Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.

Scaffold complex concepts and provide leveled problems for multiple entry points.

<u>Provide Multiple Means of Action and Expression</u>

First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.

Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?'

Encourage students to explain their thinking and strategy for the solution.

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?"
"I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..."

Incorporate written reflection, evaluation, and synthesis.

Allow creativity in expression and modeling solutions.

<u>Provide Multiple Means of Action and Expression</u>

Encourage students to explain their reasoning both orally and in writing.

Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.

Offer choices of independent or group assignments for early finishers.

Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling).

repeating digit is 0.) **Familiar Terms and Symbols**[16] ▪ Absolute Value ▪ Associative Property (of Multiplication and Addition) ▪ Commutative Property (of Multiplication and Addition) ▪ Credit ▪ Debit ▪ Deposit ▪ Distributive Property (of Multiplication Over Addition) ▪ Expression ▪ Equation ▪ Integer ▪ Inverse ▪ Multiplicative Inverse ▪ Opposites ▪ Overdraft ▪ Positives ▪ Negatives ▪ Rational Numbers ▪ Withdraw	Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **Provide Multiple Means of Engagement** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next.	Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **Provide Multiple Means of Engagement** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.

[16] These are terms and symbols students have seen previously.

		Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

OVERVIEW-Unit 3 Expressions and Equations

In Grade 6, students interpreted expressions and equations as they reasoned about one-variable equations (**6.EE.A.2**). This unit consolidates and expands upon students' understanding of equivalent expressions as they apply the properties of operations (associative, commutative, and distributive) to write expressions in both standard form (by expanding products into sums) and in factored form (by expanding sums into products). They use linear equations to solve unknown angle problems and other problems presented within context to understand that solving algebraic equations is all about the numbers. It is assumed that a number already exists to satisfy the equation and context; we just need to discover it. A number sentence is an equation that is said to be true if both numerical expressions evaluate to the same number; it is said to be false otherwise. Students use the number line to understand the properties of inequality and recognize when to *preserve the inequality* and when to *reverse the inequality* when solving problems leading to inequalities. They interpret solutions within the context of problems. Students extend their sixth-grade study of geometric figures and the relationships between them as they apply their work with expressions and equations to solve problems involving area of a circle and composite area in the plane, as well as volume and surface area of right prisms. In this module, students discover the most famous ratio of all, π, and begin to appreciate why it has been chosen as the symbol to represent the Grades 6–8 mathematics curriculum, *A Story of Ratios*.

To begin this module, students will generate equivalent expressions using the fact that addition and multiplication can be done in any order with any grouping and will extend this understanding to subtraction (adding the inverse) and division (multiplying by the multiplicative inverse, also known as the reciprocal) (**7.EE.A.1**). They extend the properties of operations with numbers (learned in earlier grades) and recognize how the same properties hold true for letters that represent numbers. Knowledge of rational number operations from Module 2 is demonstrated as students collect like terms containing both positive and negative integers.

An area model is used as a tool for students to rewrite products as sums and sums as products and to provide a visual representation leading students to recognize the repeated use of the distributive property in factoring and expanding linear expressions (**7.EE.A.1**). Students examine situations where more than one form of an expression may be used to represent the same context, and they see how looking at each form can bring a new perspective (and thus deeper understanding) to the problem. Students recognize and use the identity properties and the existence of additive inverses to efficiently write equivalent expressions in standard form, for example, $2□ + (-2□) + 3 = 0 + 3 = 3$ (**7.EE.A.2**). By the end of the topic, students have the opportunity to practice knowledge of operations with rational numbers gained in Module 2 (**7.NS.A.1, 7.NS.A.2)** as they collect like terms with rational number coefficients (**7.EE.A.1**).

In Topic B, students use linear equations and inequalities to solve problems (**7.EE.B.4**). They continue to use tape diagrams from earlier grades where they see fit, but will quickly discover that some problems would more reasonably be solved algebraically (as in the case of large numbers). Guiding students to arrive at this realization on their own develops the need for algebra. This algebraic approach builds upon work in Grade 6 with equations (**6.EE.B.6, 6.EE.B.7)** to now include multi-step equations and inequalities containing rational numbers (**7.EE.B.3**, **7.EE.B.4**). Students solve problems involving consecutive numbers; total cost; age comparisons; distance, rate, and time; area and perimeter; and missing angle measures. Solving equations with a variable is all about numbers, and students are challenged with the goal of finding the number that makes the equation true. When given in context, students recognize that a value exists, and it is simply their job to discover what that value is. Even the angles in each diagram have a precise value, which can be checked with a protractor to ensure students that the value they find does indeed create a true number sentence.

In Topic C, students continue work with geometry as they use equations and expressions to study area, perimeter, surface area, and volume. This final topic begins by modeling a circle with a bicycle tire and comparing its perimeter (one rotation of the tire) to the length across (measured with a string) to allow students to discover the most famous ratio of all, pi. Activities in comparing circumference to diameter are staged precisely for students to recognize that this symbol has a distinct value and can be approximated by $\frac{22}{7}$, or 3.14, to give students an intuitive sense of the relationship that exists. In addition to representing this value with the □ symbol, the fraction and decimal approximations allow for students to continue to practice their work with rational number operations. All problems are crafted in such a way as to allow students to practice skills in reducing within a problem, such as using $\frac{22}{7}$ for finding circumference with a given diameter length of 14 □□, and recognize what value would be best to approximate a solution. This understanding allows students to accurately assess work for reasonableness of answers. After discovering and understanding the value of this special ratio, students will continue to use pi as they solve problems of area and circumference (**7.G.B.4)**.

In this topic, students derive the formula for area of a circle by dividing a circle of radius □ into pieces of pi and rearranging the pieces so that they are lined up, alternating direction, and form a shape that resembles a rectangle. This "rectangle" has a length that is $\frac{1}{2}$ the circumference and a width of □. Students determine that the area of this rectangle (reconfigured from a circle of the same area) is the product of its length and its width: $\frac{1}{2}□ \cdot □ = \frac{1}{2}2□□ \cdot □ = □□^2$ (**7.G.B.4**). The precise definitions for diameter, circumference, pi, and circular region or disk will be developed during this topic with significant time being devoted to students' understanding of each term.

Students build upon their work in Grade 6 with surface area and nets to understand that surface area is simply the sum of the area of the lateral faces and the base(s) **(6.G.A.4)**. In Grade 7, they continue to solve real-life and mathematical problems involving area of two-dimensional shapes and surface area and volume of prisms, e.g., rectangular, triangular, focusing on problems that involve fractional values for length (**7.G.B.6**). Additional work (examples) with surface area will occur in Module 6 after a formal definition of rectangular pyramid is established.

This module is comprised of 26 lessons; 9 days are reserved for administering the Mid-Module and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic C.

Math Unit -3 Grade 7

Rigorous Curriculum Design Template

Unit 3: Expressions and Equations

Subject: Mathematics

Grade/Course: Grade 7

Pacing: 26 Days

Unit of Study: Unit 3: Expressions and Equations

Priority Standards: Focus Standards

Use properties of operations to generate equivalent expressions.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation*.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Foundational Standards

Understand and apply properties of operations and the relationship between addition and subtraction

1.OA.B.3 Apply properties of operations as strategies to add and subtract.[17] *Examples: If* $\boldsymbol{8+3=11}$ *is known, then* $\boldsymbol{3+8=11}$ *is also known. (Commutative property of addition.) To add* $\boldsymbol{2+6+4}$*, the second two numbers can be added to make a ten, so* $\boldsymbol{2+6+4=2+10=12}$*. (Associative property of addition.)*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.B.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If* $\boldsymbol{6\times 4=24}$ *is known, then* $\boldsymbol{4\times 6=24}$ *is also known. (Commutative property of multiplication.)*
$\boldsymbol{3\times 5\times 2}$ *can be found by* $\boldsymbol{3\times 5=15}$*, then* $\boldsymbol{15\times 2=30}$*, or by* $\boldsymbol{5\times 2=10}$*, then* $\boldsymbol{3\times 10=30}$*. (Associative property of multiplication.) Knowing that* $\boldsymbol{8\times 5=40}$ *and* $\boldsymbol{8\times 2=16}$*, one can find* $\boldsymbol{8\times 7}$ *as* $\boldsymbol{8\times(5+2)=(8\times 5)+(8\times 2)=40+16=56}$*. (Distributive property.)*

Geometric measurement: understand concepts of angle and measure angles.

4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\boldsymbol{1/360}$ of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through □ one-degree angles is said to have an angle measure of □ degrees.

4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

[17] Students need not use formal terms for these properties.

4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.*

6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations in the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.

6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- **a.** Describe situations in which opposite quantities combine to make $\boldsymbol{0}$. *For example, a hydrogen atom has* $\boldsymbol{0}$ *charge because its two constituents are oppositely charged.*
- b. Understand $\square + \square$ as the number located a distance $|\square|$ from $\square$, in the positive or negative direction depending on whether $\square$ is positive or negative. Show that a number and its opposite have a sum of $\boldsymbol{0}$ (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- c. Understand subtraction of rational numbers as adding the additive inverse, $\square - \square = \square + (-\square)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-\boldsymbol{1})(-\boldsymbol{1}) = \boldsymbol{1}$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $\square$ and $\square$ are integers, then $-(\square/\square) = (-\square)/\square = \square/(-\square)$. Interpret quotients of rational numbers by describing real-world contexts.
- c. Apply properties of operations as strategies to multiply and divide rational numbers.
- d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in $\boldsymbol{0}$s or eventually repeats.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively**. Students make sense of how quantities are related within a given context and formulate algebraic equations to represent this relationship. They use the properties of operations to manipulate the symbols that are used in place of numbers, in particular, pi. In doing so, students reflect upon each step in solving and recognize that these properties hold true since the variable is really just holding the place for a number. Students analyze solutions and connect back to ensure reasonableness within context.

MP.4 **Model with mathematics**. Throughout the module, students use equations and inequalities as models to solve mathematical and real-world problems. In discovering the relationship between circumference and diameter in a circle, they will use real objects to analyze the relationship and draw conclusions. Students test conclusions with a variety of objects to see if the results hold true, possibly improving the model if it has not served its purpose.

MP.6 **Attend to precision**. Students are precise in defining variables. They understand that a variable represents one number. They use appropriate vocabulary and terminology when communicating about expressions, equations, and inequalities. They use the definition of equation from Grade 6 to understand how to use the equal sign consistently and appropriately. Circles and related notions about circles are precisely defined in this module.

MP.7 **Look for and make use of structure.** Students recognize the repeated use of the distributive property as they write equivalent expressions. Students recognize how equations leading to the form $px + q = r$ and $p(x + q) = r$ are useful in solving a variety of problems. They see patterns in the way that these equations are solved. Students apply this structure as they understand the similarities and differences in how an inequality of the type $px + q > r$ or $px + q < r$ is solved.

MP.8 **Look for and express regularity in repeated reasoning**. Students use area models to write products as sums and sums as products and recognize how this model is a way to organize results from repeated use of the distributive property. As students work to solve problems, they maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of solutions as they are represented in contexts that allow for students to know that they found the intended value for a given variable. As they solve problems involving pi, they notice how a problem may be reduced by using a given estimate for pi to make calculations more efficient.

“Unwrapped” Standards

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Concepts (What Students Need to Know)	Skills (What Students Need to Be Able to Do) Depth of Knowledge Level
7.EE.A.1 Properties of operations as strategies linear expressions with rational coefficients. **7.EE.A.2** Expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. **7.EE.B.3** Multi-step real-life and mathematical problems positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Properties of operations numbers in any form; forms as appropriate; and the reasonableness of answers, mental computation and estimation strategies. **7.EE.B.4** Variables to represent quantities in a real-world or mathematical problem, and simple equations and inequalities problems the quantities. a. Word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Equations of these forms. Algebraic solution to an arithmetic solution, sequence of the operations. b. Word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers.	Apply (L4) Add (L2) Subtract (L2) Factor (L2) Expand (L3) Understand (L1) Rewriting (L2) Solve (L3) Apply (L4) Use (L2) Construct (L3) Solve (L2) Compare (L3) Solve (L2) Graph (L2)

The solution set of the inequality and in the context of the problem. **7.G.B.4** Formulas for the area and circumference of a circle and solve problems; informal derivation of the relationship between the circumference and area of a circle. **7.G.B.5** Supplementary, complementary, vertical, and adjacent angles in a multi-step problem to simple equations for an unknown angle in a figure. **7.G.B.6** Real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Know (L1) Use (L1) Give (L1) Use (L1) Write (L1) Solve (L2) Solve (L2)

Essential Questions	**Big idea**

What real life situations/careers require rational expressions?	The study of rational expressions is foundational to further levels of mathematics. Rational expressions, which are fractions involving variables, have applications in fields like physics, chemistry, biochemistry, circuitry, economics and calculus.

Assessments

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.EE.A.1, 7.EE.A.2, 7.EE.B.3, 7.EE.B.4, 7.G.B.5
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	7.EE.A.1, 7.EE.A.2, 7.G.B.4, 7.G.B.5, 7.G.B.6

Suggested Tools and Resources

- Area Model
- Coordinate Plane
- Equations and Inequalities
- Expressions
- Geometric Figures
- Nets for Three-Dimensional Figures
- Number Line
- Protractor
- Tape Diagram
- IXL Math

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest any new vocabulary **Conduct opening exercise**	**Post-test the vocabulary** **Opening Exercise (give again and**	**Mid-Module and End of Module Assessments (given as prescribed**

Use exit ticket as pre-assessment and post where applicable	**reflect on results from first administration prior to the unit)** **Exploratory Challenge** **Exit Ticket** **Student Conferences**	**in the pacing guide)**

Performance Task
To be created by teachers during year.
Engaging Learning Experiences To be created by teachers during year.

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **Provide Multiple Means of Representation** ● Teach from simple to complex, moving

from concrete to representation to abstract at the student's pace.

- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than

	a snap or “show”) to elicit responses from deaf/hard of hearing students. ● Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones. ● Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?”

	● Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-

	share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms ▪ **An Expression in Expanded Form** (description) (An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form.) ▪ **An Expression in Factored Form** (middle school description) (An expression that is a product of two or more expressions is said to be in *factored form*.) ▪ **An Expression in Standard Form** (description) (An expression that is in expanded form where all like	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...”

terms have been collected is said to be in *standard form.)* ▪ **Coefficient of the Term** (The number found by multiplying just the numbers in a term together is called the *coefficient of the term*.) ▪ **Circle** (Given a point ⍰ in the plane and a number ⍰ > 0, the *circle* with center ⍰ and radius ⍰ is the set of all points in the plane that are distance ⍰ from the point ⍰.) ▪ **Diameter of a Circle** (*The diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If ⍰ is the radius of a circle, then the diameter is 2⍰.) ▪ **Circumference** (The *circumference* is the length around a circle.)[18] ▪ **Pi** (The number *pi*, denoted ⍰, is the value of the ratio given by the circumference to the diameter in a circle; that is, ⍰ $=$ (circumference)/(diameter).) ▪ **Circular Region or Disk** (Given a point ⍰ in the plane and a number ⍰ > 0, the *circular region* (or disk) with center ⍰ and radius ⍰ is the set of all points in the plane whose distance from the point ⍰ is less than or equal to ⍰. The interior of a circle with center ⍰ and radius ⍰ is the set of all points in the plane whose distance from	be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).	Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather

[18] "Distance around a circular arc" is taken as an undefined term in G-CO.A.1.

the point ? is less than ?.) **Familiar Terms and Symbols**[19] ▪ Adjacent Angles ▪ Cube ▪ Distribute ▪ Equation ▪ Equivalent Expressions ▪ Expression (middle school description) ▪ Factor ▪ Figure ▪ Identity ▪ Inequality ▪ Length of a Segment ▪ Linear Expression ▪ Measure of an Angle ▪ Number Sentence ▪ Numerical Expression (middle school description) ▪ Properties of Operations (distributive, commutative, associative) ▪ Right Rectangular Prism ▪ Segment ▪ Square ▪ Surface of a Prism ▪ Term ▪ Triangle ▪ True or False Number Sentence ▪ Truth Values of a Number Sentence ▪ Value of a Numerical Expression	Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next.	than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two- step problem). Adjust difficulty level by enhancing the operation (e. g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to

[19] These are terms and symbols students have seen previously.

▪ Variable (middle school description) ▪ Vertical Angles		extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

OVERVIEW-Unit 4- Percent and Proportional Relationships

In Module 4, students deepen their understanding of ratios and proportional relationships from Module 1 (**7.RP.A.1**, **7.RP.A.2**, **7.RP.A.3**, **7.EE.B.4**, **7.G.A.1**) by solving a variety of percent problems. They convert between fractions, decimals, and percents to further develop a conceptual understanding of percent (introduced in Grade 6, Module 1) and use algebraic expressions and equations to represent and solve multi-step percent scenarios (**7.EE.B.3**). An initial focus on relating 100% to "the whole" serves as a foundation for students. Students begin the module by solving problems without the use of a calculator to develop a greater fluency and deeper reasoning behind calculations with percent. Material in early lessons is designed to reinforce students' understanding by having them use mental math and basic computational skills. To develop a conceptual understanding, students will use visual models and equations, building on earlier work with these strategies. As the lessons and topics progress and more complex calculations are required to solve multi-step percent problems, teachers may let students use calculators so that their computational fluency does not interfere with the primary concept(s) being addressed. This will also be noted in the teacher's lesson materials.

Topic A builds on students' conceptual understanding of percent from Grade 6 (**6.RP.A.3c**) and relates 100% to "the whole." Students represent percents as decimals and fractions and extend their understanding from Grade 6 to include percents greater than 100%, such as 225%, and percents less than 1%, such as $\frac{1}{2}\%$ or 0.5%. They understand that, for instance, 225% means $\frac{225}{100}$, which ultimately simplifies to the equivalent decimal value of 2.25 (**7.RP.A.1**). Students use complex fractions to represent non-whole number percents (e.g., $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8} = 0.125$).

Module 3's focus on algebra prepares students to move from the visual models used for percents in Grade 6 to algebraic equations in Grade 7. They write equations to solve multi-step percent problems and relate their conceptual understanding to the representation: $\text{Quantity} = \text{Percent} \times \text{Whole}$ (**7.RP.A.2c**). Students solve percent increase and decrease problems with and without equations (**7.RP.A.3**). For instance, given a multi-step word problem where there is an increase of 20% and "the whole" equals $\$200$, students recognize that $\$200$ can be multiplied by 120%, or 1.2, to get an answer of $\$240$. They use visual models such as a double number line diagram to justify their answers. In this case, 100% aligns to $\$200$ in the diagram, and intervals of fifths are used (since $20\% = \frac{1}{5}$) to partition both number line segments to create a scale indicating that 120% aligns to $\$240$. Topic A concludes with students representing 1% of a quantity using a ratio and then using that ratio to find the amounts of other percents. While representing 1% of a quantity and using it to find the amount of other percents is a strategy that will always work when solving a problem, students recognize that when the percent is a factor of 100, they can use mental math and proportional reasoning to find the amount of other percents in a more efficient way.

In Topic B, students create algebraic representations and apply their understanding of percent from Topic A to interpret and solve multi-step word problems related to markups or markdowns, simple interest, sales tax, commissions, fees, and percent error (**7.RP.A.3**, **7.EE.B.3**). They apply their understanding of proportional relationships from Module 1, creating an equation, graph, or table to model a tax or commission rate that is represented as a percent (**7.RP.A.1**, **7.RP.A.2**). Students solve problems related to changing percents and use their understanding of percent and proportional relationships to solve scenarios such as the following: A soccer league has 300 players, 60% of whom are boys. If some of the boys switch to baseball, leaving only 52% of the soccer players as boys, how many players remain in the soccer league? Students first determine that $100\% - 60\% = 40\%$ of the players are girls, and 40% of 300 equals 120. Then, after some boys switched to baseball, $100\% - 52\% = 48\%$ of the soccer players are girls; so, $0.48\square = 120$, or $\square = \frac{120}{0.48}$. Therefore, there are now 250 players in the soccer league.

In Topic B, students also apply their understanding of absolute value from Module 2 (**7.NS.A.1b**) when solving percent error problems. To determine the percent error for an estimated concert attendance of $5{,}000$ people, when actually $6{,}372$ people attended, students calculate the percent error as $\frac{|5000-6372|}{|6372|} \cdot 100\%$, which is about 21.5%.

Students revisit scale drawings in Topic C to solve problems in which the scale factor is represented by a percent (**7.RP.A.2b**, **7.G.A.1**). They understand from their work in Module 1, for example, that if they have two drawings, and if Drawing 2 is a scale model of Drawing 1 under a scale factor of 80%, then Drawing 1 is also a scale model of Drawing 2, and that scale factor is determined using inverse operations. Since $80\% = \frac{4}{5}$, the scale factor is found by taking the complex fraction $\frac{1}{\frac{4}{5}}$, or $\frac{5}{4}$, and multiplying it by 100%, resulting in a scale factor of 125%. As in Module 1, students construct scale drawings, finding scale lengths and areas given the actual quantities and the scale factor (and vice versa); however, in this module the scale factor is represented as a percent. Students are encouraged to develop multiple methods for making scale drawings. Students may find the multiplicative relationship between figures; they may also find a multiplicative relationship among lengths within the same figure.

The problem-solving materials in Topic D provide students with further applications of percent and exposure to problems involving population, mixtures, and counting in preparation for later topics in middle school and high school mathematics and science. Students will apply their understanding of percent (**7.RP.A.2c**, **7.RP.A.3**, **7.EE.B.3**) to solve complex word problems by identifying a set that meets a certain percentage criterion. Additionally, students will explore problems involving mixtures of ingredients and determine the percentage of a mixture that already exists, or on the contrary, the amount of ingredient needed in order to meet a certain percentage criterion.

This module spans 25 days and includes 18 lessons. Seven days are reserved for administering the assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic D.

Math Unit -4 Percent and Proportional Relationships

Rigorous Curriculum Design Template

Unit: 4 Percent and Proportional Relationships

Subject: Mathematics

Grade/Course: Grade 7

Pacing: 25 Days

Unit of Study: Unit 4: Percent and Proportional Relationships

Priority Standards: Focus Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

j. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

k. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

l. Represent proportional relationships by equations. *For example, if total cost ▯ is proportional to the number ▯ of items purchased at a constant price ▯, the relationship between the total cost and the number of items can be expressed as ▯ = ▯▯.*

m. Explain what a point (▯, ▯) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1,$ ▯$)$, where ▯ is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Solve real-life and mathematical problems using numerical and algebraic expressions

and equations.[20]

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional* $\frac{1}{10}$ *of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar* $9\frac{3}{4}$ *inches long in the center of a door that is* $27\frac{1}{2}$ *inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1[21] Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was* $2:1$*, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*

6.RP.A.2 Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”*[22]

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns*

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[20] 7.EE.B.3 is introduced in Module 3. The majority of this cluster was taught in the first three modules.

[21] 7.G.A.1 is introduced in Module 1. The balance of this cluster is taught in Module 6.

[22] Expectations for unit rates in this grade are limited to non-complex fractions.

could be mowed in 35 hours? At what rate were lawns being mowed?

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

b. Understand $\square + \square$ as the number located a distance $|\square|$ from $\square$, in the positive or negative direction depending on whether $\square$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[23]

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $\square\square + \square = \square$ and $\square(\square + \square) = \square$, where $\square$, $\square$, and $\square$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 $\square\square$. Its length is 6 $\square\square$. What is its width?*

Focus Standards for Mathematical Practice

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[23] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of percent problems by modeling the proportional relationship using an equation, a table, a graph, a double number line diagram, mental math, and factors of 100. When solving a multi-step percent word problem, students use estimation and number sense to determine if their steps and logic lead to a reasonable answer. Students know they can always find 1% of a quantity by dividing it by 100 or multiplying it by $\frac{1}{100}$, and they also know that finding 1% first allows them to then find other percents easily. For instance, if students are trying to find the amount of money after 4 years in a savings account with an annual interest rate of $\frac{1}{2}\%$ on an account balance of $\$300$, they use the fact that 1% of 300 equals $\frac{300}{100}$, or $\$3$; thus, $\frac{1}{2}\%$ of 300 equals $\frac{1}{2}$ of $\$3$, or $\$1.50$. $\$1.50$ multiplied by 4 is $\$6$ interest, and adding $\$6$ to $\$300$ makes the total balance, including interest, equal to $\$306$.

MP.2 **Reason abstractly and quantitatively.** Students use proportional reasoning to recognize that when they find a certain percent of a given quantity, the answer must be greater than the given quantity if they found more than 100% of it and less than the given quantity if they found less than 100% of it. Double number line models are used to visually represent proportional reasoning related to percents in problems such as the following: If a father has 70% more money in his savings account than his 25-year-old daughter has in her savings account, and the daughter has $\$4{,}500$, how much is in the father's account? Students represent this information with a visual model by equating $4{,}500$ to 100% and the father's unknown savings amount to 170% of $4{,}500$. Students represent the amount of money in the father's savings account by writing the expression $\frac{170}{100} \times 4{,}500$, or $1.7(4{,}500)$. When working with scale drawings, given an original two-dimensional picture and a scale factor as a percent, students generate a scale drawing so that each corresponding measurement increases or decreases by a certain percentage of measurements of the original figure. Students work backward to create a new scale factor and scale drawing when given a scale factor represented as a percent greater or less than 100%. For instance, given a scale drawing with a scale factor of 25%, students create a new scale drawing with a scale factor of 10%. They relate working backward in their visual model to the following steps: (1) multiplying all lengths in the original scale drawing by $\frac{1}{0.25}$ (or dividing by 25%) to get back to their original lengths, and then (2) multiplying each original length by 10% to get the new scale drawing.

MP.5 **Use appropriate tools strategically.** Students solve word problems involving percents using a variety of tools, including equations and double number line models. They choose their model strategically. For instance, given that 75% of a class of learners is represented by 21 students, they recognize that since 75 is $\frac{3}{4}$ of 100, and 75 and 21 are both divisible by 3, a double number line diagram can be used to establish intervals of 25's and 7's to show that 100% would correspond to $21 + 7$, which equals 28. For percent problems that do not involve benchmark fractions, decimals, or percents, students use math sense and estimation to assess the reasonableness of their answers and computational work. For instance, if a problem indicates that a bicycle is marked up 18% and it is sold at a retail price of $\$599$, students are able to estimate by using rounded values such as 120% and $\$600$ to determine that the solution that will represent the wholesale price of the bicycle must be in the realm of $600 \div 1.2$, or $6{,}000 \div 12$, to arrive at an estimate of $\$500$.

MP.6 **Attend to precision.** Students pay close attention to the context of the situation when working with percent problems involving a percent markup, markdown, increase, or decrease. They construct models based on the language of a word problem. For instance, a markdown of 15% on an $\$88$ item is represented by $0.85(88)$; however, a markup of 15% is represented by $1.15(88)$. Students attend to precision when writing the answer to a percent problem. If they are finding a percent, they use the $\%$ symbol in the answer or write the answer as a fraction with 100 as the denominator (or in an equivalent form). Double number line diagrams display correct segment lengths, and if a line in the diagram represents percents, it is either labeled as such or the percent sign is shown after each number. When stating the area of a scale drawing or actual drawing, students include the square units along with the numerical part of the answer.

MP.7 **Look for and make use of structure**. Students understand percent to be a rate per 100 and express □ percent as $\frac{□}{100}$. They know that, for instance, 5% means 5 for every 100, 1% means 1 for every 100, and 225% means 225 for every 100. They use their number sense to find benchmark percents. Since 100% is one whole, then 25% is one-fourth, 50% is one-half, and 75% is three-fourths. So, to find 75% of 24, they find $\frac{1}{4}$ of 24, which is 6, and multiply it by 3 to arrive at 18. They use factors of 100 and mental math to solve problems involving other benchmark percents as well. Students know that 1% of a quantity represents $\frac{1}{100}$ of it and use place value and the structure of the base-ten number system to find 1% or $\frac{1}{100}$ of a quantity. They use "finding 1%" as a method to solve percent problems. For instance, to find 14% of 245, students first find 1% of 245 by dividing 245 by 100, which equals 2.45. Since 1% of 245 equals 2.45, 14% of 245 would equal $2.45 \times 14 = 34.3$. Students observe the steps involved in finding a discount price or price including sales tax and use the properties of operations to efficiently find the answer. To find the discounted price of a $\$73$ item that is on sale for 15% off, students realize that the distributive property allows them to arrive at an answer in one step, by multiplying $\$73$ by 0.85, since
$73(100\%) - 73(15\%) = 73(1) - 73(0.15) = 73(0.85)$.

"Unwrapped" Standards"

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks* $\frac{1}{2}$ *mile in each* $\frac{1}{4}$ *hour, compute the unit rate as the complex fraction* $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

n. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

o. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

p. Represent proportional relationships by equations. *For example, if total cost □ is proportional to the number □ of items purchased at a constant price □, the relationship between the total cost and the number of items can be expressed as* □ = □□.

q. Explain what a point (□, □) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1,$ □$)$, where □ is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.[24]

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional* $\frac{1}{10}$ *of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar* $9\frac{3}{4}$ *inches long in the center of a door that is* $27\frac{1}{2}$ *inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

Draw, construct, and describe geometrical figures and describe the relationships between

[24] 7.EE.B.3 is introduced in Module 3. The majority of this cluster was taught in the first three modules.

them.

7.G.A.1[25]	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Concepts (What Students Need to Know)		Skills (What Students Need to Be Able to Do)
		Depth of Knowledge Level
7.RP.A.1	unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *equivalently 2 miles per hour.*	Compute unit rates
7.RP.A.2	proportional relationships between quantities.	Recognize and represent
7.RP.A.3	multistep ratio and percent problems.	Use proportional relationships to solve

25 7.G.A.1 is introduced in Module 1. The balance of this cluster is taught in Module 6.

7.EE.B.3 posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically with numbers in any form; as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies **7.G.A.1**[26] involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	Solve multi-step real-life and mathematical problems Apply properties of operations to calculate Convert between forms Solve problems

Essential Questions	**Big ideas**

[26] 7.G.A.1 is introduced in Module 1. The balance of this cluster is taught in Module 6.

What information do I get when I compare two numbers using a ratio? What kinds of problems can I solve by using ratios? How are ratios and their relationships used to solve real world problems? What conditions help to recognize and represent proportional relationships between quantities?	Fractions, decimals and percents can be used interchangeably. Ratios use division to represent relationships between two quantities. The constant of proportionality is also considered to be the unit rate.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.3, 7.G.A.1

Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources

Exit slips as pre-assessment	**Application problems** **Student Debriefs** **Problem Set Data**	**Exit tickets as post assessments for each lesson** **See chart above for mid and end of module assessment.**

Performance Task
To be created by teachers during year.
Engaging Learning Experiences
To be created by teachers during year.

Instructional Resources

Suggested Tools and Representations

- Calculator
- Coordinate Plane
- Double Number Line Diagrams
- Equations
- Expressions
- Geometric Figures
- Ratio Tables
- Tape Diagrams
- IXL Math

Instructional Strategies	Meeting the Needs of All Students
Marzano's Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **21st Century Skills** Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **Provide Multiple Means of Representation** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice.

- Partner key words with visuals (e.g., photo of “ticket”) and gestures (e.g., for “paid”). Connect language (such as ‘tens’) with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with “math-they-can-see,” such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define “multiplication” may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…”
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in

"Happy Counting."

- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

<u>Provide Multiple Means of Engagement</u>

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones

	up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**

New or Recently Introduced Terms

- **Absolute Error** (Given the exact value of a quantity and an approximate value of it, the absolute error is $| - |$.)
- **Percent Error** (The percent error is the percent the absolute error is of the exact value $(\frac{| - |}{||})(100\%)$, where is the exact value of the quantity, and is an approximate value of the quantity.)

Familiar Terms and Symbols[27]

- Area
- Circumference
- Coefficient of the Term
- Complex Fraction
- Constant of Proportionality
- Discount Price
- Equation
- Equivalent Ratios
- Expression
- Fee
- Fraction
- Greatest Common Factor
- Length of a Segment
- One-to-One

<u>Provide Multiple Means of Representation</u>

Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.

Guide students as they select and practice using their own graphic organizers and models to solve.

Use direct instruction for vocabulary with visual or concrete representations.

Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do."

Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.

Scaffold complex concepts and provide leveled problems for multiple entry points.

<u>Provide Multiple Means of Action and Expression</u>

First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.

Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?'

Encourage students to explain their

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?"
"I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..."

Incorporate written reflection, evaluation, and synthesis.

Allow creativity in expression and modeling solutions.

<u>Provide Multiple Means of Action and Expression</u>

Encourage students to explain their reasoning both orally and in writing.

Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.

Offer choices of independent or group assignments for early finishers.

Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g.,

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[27] These are terms and symbols students have seen previously.

Correspondence ▪ Original Price ▪ Percent ▪ Perimeter ▪ Pi ▪ Proportional Relationship ▪ Proportional To ▪ Rate ▪ Ratio ▪ Rational Number ▪ Sales Price ▪ Scale Drawing ▪ Scale Factor ▪ Unit Rate	thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next.	journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.

		Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

OVERVIEW-Unit 5 Statistics and Probability

In this module, students begin their study of probability, learning how to interpret probabilities and how to compute probabilities in simple settings. They also learn how to estimate probabilities empirically. The concept of probability provides a foundation for the thinking required to make inferential reasoning that will be developed in the second half of this module. Additionally, students build on their knowledge of data distributions that they studied in Grade 6, compare data distributions of two or more populations, and are introduced to the idea of drawing informal inferences based on data collected from random samples.

In Topics A and B, students learn to interpret the probability of an event as the proportion of the time that the event will occur when a chance experiment is repeated many times (**7.SP.C.5**). They learn to compute or estimate probabilities using a variety of methods, including collecting data, using tree diagrams, and using simulations. In Topic B, students move to comparing probabilities from simulations to computed probabilities that are based on theoretical models (**7.SP.C.6**, **7.SP.C.7**). They calculate probabilities of compound events using lists, tables, tree diagrams, and simulations (**7.SP.C.8**). They learn to use probabilities to make decisions and to determine whether or not a given probability model is plausible (**7.SP.C.7**). The Mid-Module Assessment follows Topic B.

In Topics C and D, students focus on using random sampling to draw informal inferences about a population (**7.SP.A.1**, **7.SP.A.2**). In Topic C, they investigate sampling from a population (**7.SP.A.2**). They learn to estimate a population mean using numerical data from a random sample (**7.SP.A.2**). They also learn how to estimate a population proportion using categorical data from a random sample. In Topic D, students learn to compare two populations with similar variability. They learn to consider sampling variability when deciding if there is evidence that the means or the proportions of two populations are actually different (**7.SP.B.3**, **7.SP.B.4**). The End-of-Module Assessment follows Topic D.

Rigorous Curriculum Design Template

Unit 5: Statistics and Probability

Subject: Mathematics

Grade/Course: Grade 7

Pacing: 20 Days

Unit of Study: Unit 5: Statistics and Probability

Priority Standards: Focus Standards

Using random sampling to draw inferences about a population.

7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 �� greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.C.5 Understand that the probability of a chance event is a number between *0* and *1* that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $1/2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Foundational Standards

Summarize and describe distributions.

6.SP.B.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.2 Recognize and represent proportional relationships between quantities.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students reason quantitatively by posing statistical questions about variables and the relationship between variables. Students reason abstractly about chance experiments by analyzing possible outcomes and designing simulations to estimate probabilities.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students construct viable arguments by using sample data to explore conjectures about a population. Students critique the reasoning of other students as part of poster or similar presentations.

MP.4 **Model with mathematics.** Students use probability models to describe outcomes of chance experiments. They evaluate probability models by calculating the theoretical probabilities of chance events, and by comparing these probabilities to observed relative frequencies.

MP.5 **Use appropriate tools strategically.** Students use simulation to approximate probabilities. Students use appropriate technology to calculate measures of center and variability. Students use graphical displays to visually represent distributions.

MP.6 **Attend to precision.** Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students make appropriate use of statistical terminology.

“Unwrapped” Standards

7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $1/2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

c. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

d. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

d. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

e. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

f. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
7.SP.A.1 Statistics can be used to gain information about a population sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Random sampling tends to produce representative samples and support valid inferences. **7.SP.A.2** Data from a random sample population with an unknown characteristic of interest, Multiple samples (or simulated samples) of the same size, variation in estimates or predictions. **7.SP.C.5** Probability of a chance event is a number between *0* and *1* that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near *0* indicates an unlikely event, a probability around *1/2* indicates an event that is neither unlikely nor likely, and a probability near *1* indicates a likely event. **7.SP.C.6** Probability of a chance event, data on the chance process that produces it, long-run relative frequency, and predict the approximate relative frequency given the probability. **7.SP.C.7** Probability model and use it to find probabilities of events. Probabilities	Understand (L2) Examine (L2) Understand (L2) Use (L2) Draw inferences (L2) Generate (L3) Understand (L2) Approximate (L2)

from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **7.SP.C.8** Probabilities of compound events, organized lists, tables, tree diagrams, and simulation.	Find (L1)

Essential Questions	**Big ideas**
1. How can we gather, organize and display data to communicate and justify results in the real world? 2. How can we analyze data to make inferences and/or predictions, based on surveys, experiments, probability and observational studies	A generalization about a population is only valid if the sample is representative of that population. It is difficult to gather statistics on an entire population, instead a random sample can be representative of the total population.

Assessments

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.8
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	7.SP.A.1, 7.SP.A.2, 7.SP.B.3, 7.SP.B.4, 7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.8

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Exit Tickets for Pre-Assessment of each lesson.	**Application problems** **Student Debriefs** **Problem Set Data**	**Exit tickets as post assessments for each lesson**

Performance Task
To be created by teachers during year.

Engaging Learning Experiences
To be created by teachers during year.

Instructional Resources

Sugges[illegible]s and Representations

- Gr[illegible]lator (See example below.)
- Do[illegible] example below.)
- Histograms (See example below.)
- IXL Math

Instructional Strategies	Meeting the Needs of All Students

<u>Marzano's Strategies</u> Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **<u>21st Century Skills</u>** Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.

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- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones.
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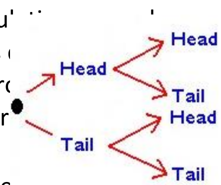
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are “just right” for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day’s sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the

	information before giving the signal to respond. ● Assess by multiple means, including "show and tell" rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" ● Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games,

	such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow “everyday” and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding.

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**
New or Recently Introduced Terms ▪ **Compound Event** (A *compound event* is an event consisting of more than one outcome from the sample space of a chance experiment.) ▪ **Inference** (*Inference* is the act of drawing conclusions about a population using data from a sample.) ▪ **Long-Run Relative Frequency** (The proportion of the time some outcome occurs in a very long sequence of observations is called a *long-run relative frequency*.) ▪ **Probability** (Probability is a number between *0* and *1* that represents the likelihood that an outcome will occur.)	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>**

▪ **Probability Model** (A *probability model* for a chance experiment specifies the set of possible outcomes of the experiment—the sample space—and the probability associated with each outcome.) ▪ **Random Sample** (A *random sample* is a sample selected in a way that gives every different possible sample of the same size an equal chance of being selected.) ▪ **Simulation** (A *simulation* is the process of generating “artificial” data that are consistent with a given probability model or with sampling from a known population.) ▪ **Tree Diagram** (A *tree diagram* consists of a sequence of nodes and branches. Tree diagrams are sometimes used as a way of representing the outcomes of a chance experiment that consists of a sequence of steps, such as rolling two number cubes, viewed as first rolling one number cube and then rolling the second.) ▪ **Uniform Probability Model** (A *uniform probability model* is a probability model in which all outcomes in the sample space of a chance experiment are equally	multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipu objects (such as make transfer fr pictorial to abstr  Have students re learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are “just right” for learners but teach the same concepts. Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip- counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two- step problem). Adjust difficulty level by enhancing the operation (e. g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.

likely.) **Familiar Terms and Symbols**[28] ▪ Mean Absolute Deviation (MAD) ▪ Measures of Center ▪ Measures of Variability ▪ Shape	calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	Let students write word problems to show mastery and/or extension of the content. **Provide Multiple Means of Engagement** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games.

[28] These are terms and symbols students have seen previously.

		Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

OVERVIEW-Unit 6 Geometry

In Module 6, students delve further into several geometry topics they have been developing over the years. Grade 7 presents some of these topics (e.g., angles, area, surface area, and volume) in the most challenging form students have experienced yet. Module 6 assumes students understand the basics; the goal is to build a fluency in these difficult problems. The remaining topics (i.e., working on constructing triangles and taking slices (or cross sections) of three-dimensional figures) are new to students.

In Topic A, students solve for unknown angles. The supporting work for unknown angles began in Grade 4, Module 4 (**4.MD.C.5–7**), where all of the key terms in this Topic were first defined, including: adjacent, vertical, complementary, and supplementary angles, angles on a line, and angles at a point. In Grade 4, students used those definitions as a basis to solve for unknown angles by using a combination of reasoning (through simple number sentences and equations), and measurement (using a protractor). For example, students learned to solve for a missing angle in a pair of supplementary angles where one angle measurement is known.

In Grade 7, Module 3, students studied how expressions and equations are an efficient way to solve problems. Two lessons were dedicated to applying the properties of equality to isolate the variable in the context of missing angle problems. The diagrams in those lessons were drawn to scale to help students more easily make the connection between the variable and what it actually represents. Now in Module 6, the most challenging examples of unknown angle problems (both diagram-based and verbal) require students to use a synthesis of angle relationships and algebra. The problems are multi-step, requiring students to identify several layers of angle relationships and to fit them with an appropriate equation to solve. Unknown angle problems show students how to look for, and make use of, structure (MP.7). In this case, they use angle relationships to find the measurement of an angle.

Next, in Topic B, students work extensively with a ruler, compass, and protractor to construct geometric shapes, mainly triangles (**7.G.A.2**). The use of a compass is new (e.g., how to hold it, and to how to create equal segment lengths). Students use the tools to build triangles, provided given conditions, such side length and the measurement of the included angle (MP.5). Students also explore how changes in arrangement and measurement affect a triangle, culminating in a list of conditions that determine a unique triangle. Students understand two new concepts about unique triangles. They learn that under a condition that determines a unique triangle: (1) a triangle can be drawn and (2) any two triangles drawn under the condition will be identical. It is important to note that there is no mention of congruence in the CCSS until Grade 8, after a study of rigid motions. Rather, the focus of Topic B is developing students' intuitive understanding of the structure of a triangle. This includes students noticing the conditions that determine a unique triangle, more than one triangle, or no triangle (**7.G.A.2**). Understanding what makes triangles unique requires understanding what makes them identical.

Topic C introduces the idea of a slice (or cross section) of a three-dimensional figure. Students explore the two-dimensional figures that result from taking slices of right rectangular prisms and right rectangular pyramids parallel to the base, parallel to a lateral face, and slices that are not parallel to the base nor lateral face, but are skewed slices (**7.G.A.3**). The goal of the first three lessons is to get students to consider three-dimensional figures from a new perspective. One way students do this is by experimenting with an interactive website which requires students to choose how to position a three-dimensional figure so that a slice yields a particular result (e.g., how a cube should be sliced to get a pentagonal cross section).

Similar to Topic A, the subjects of area, surface area, and volume in Topics D and E are not new to students, but provide opportunities for students to expand their knowledge by working with challenging applications. In Grade 6, students verified that the volume of a right rectangular prism is the same whether it is found by packing it with unit cubes or by multiplying the edge lengths of the prism (**6.G.A.2**). In Grade 7, the volume formula $V = Bh$, where B represents the area of the base, will be tested on a set of three-dimensional figures that extends beyond right rectangular prisms to right prisms in general. In Grade 6, students practiced composing and decomposing two-dimensional shapes into shapes they could work with to determine area (**6.G.A.1**). Now, they learn to apply this skill to volume as well. The most challenging problems in these topics are not pure area or pure volume questions, but problems that incorporate a broader mathematical knowledge such as rates, ratios, and unit conversion. It is this use of multiple skills and contexts that distinguishes real-world problems from purely mathematical ones (**7.G.B.6**).

Math Unit -7 Geometry

Rigorous Curriculum Design Template

Unit 7: Geometry

Subject: Geometry

Grade/Course: Grade 7

Pacing: 27 Days

Unit of Study: Unit 7: Geometry Unit

Priority Standards: Focus Standards

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.B.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Foundational Standards

Geometric measurement: understand concepts of angle and measure angles.

4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas
$V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Solve real-life and mathematical problems involving area, surface area, and volume.

7.G.B.4 Know the formulas for area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them**. This mathematical practice is particularly applicable for this module, as students tackle multi-step problems that require them to tie together knowledge about their current and former topics of study (i.e., a real-life composite area question that also requires proportions and unit conversion). In many cases, students will have to make sense of new and different contexts and engage in significant struggle to solve problems.

MP.3 **Construct viable arguments and critique the reasoning of others**. In Topic B, students examine the conditions that determine a unique triangle, more than one triangle, or no triangle. They will have the opportunity to defend and critique the reasoning of their own arguments as well as the arguments of others. In Topic C, students will predict what a given slice through a three-dimensional figure will yield (or how to slice a three-dimensional figure for a given cross section) and must provide a basis for their predictions.

MP.5 **Use appropriate tools strategically**. In Topic B, students will learn how to strategically use a protractor, ruler, and compass to build triangles according to provided conditions. An example of this is when students are asked to build a triangle provided three side lengths. Proper use of the tools will help them understand the conditions by which three side lengths will determine one triangle or no triangle. Students will have opportunities to reflect on the appropriateness of a tool for a particular task.

MP.7 **Look for and make use of structure**. Students must examine combinations of angle facts within a given diagram in Topic A to create an equation that correctly models the angle relationships. If the unknown angle problem is a verbal problem, such as an example that asks for the measurements of three angles on a line where the values of the measurements are consecutive numbers, students will have to create an equation without a visual aid and rely on the inherent structure of the angle fact. In Topics D and E, students will find area, surface area, and volume of composite figures based on the structure of two- and three-dimensional figures.

“Unwrapped” Standards	
7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7.G.A.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
7.G.B.6	Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Concepts (What Students Need to Know)		**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
7.G.A.2	geometric shapes with given conditions. triangles from three measures of angles or sides, conditions determine a unique triangle, more than one triangle, or no triangle.	Draw (L2) Focus (L2) Describe (L2) Use (L1) Write (L2) Solve (L2)
7.G.A.3	two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	
7.G.B.5	facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem, simple equations for an unknown angle in a figure.	
7.G.B.6	real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	

Essential Questions	**Big ideas**
Essential Questions: Why are geometry and geometric figures relevant and important? How can geometric ideas be communicated using a variety of representations? (i.e. maps, grids, charts, spreadsheets) How can geometry be used to solve problems about real world situations, spatial relationships, and logical reasoning	Decomposing and rearranging provide a geometric way of both *seeing that* a measurement formula is the right one and *seeing why* it is the right one. Geometric images provide the content in relation to which properties can be noticed, definitions can be made, and invariances can be discerned. Symmetry provides a powerful way of working geometrically. Tools provide new sources of imagery as well as specific ways of thinking about geometric objects and processes. Geometric thinking turns tools into objects, and in geometry the process of turning an action undertaken with a tool into an object happens over and over again.

Assessments

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.G.B.2, 7.G.A.5
End-of-Module Assessment Task	After Topic E	Constructed response with rubric	7.G.A.2, 7.G.A.3, 7.G.B.5, 7.G.B.6

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Exit Tickets for Pre-Assessment of each lesson.	**Application problems** **Student Debriefs** **Problem Set Data**	**See Chart Above** **Exit tickets as post assessments for each lesson**

Performance Task
To be created by teachers during year.
Engaging Learning Experiences
To be created by teachers during year.

Instructional Resources
Suggested Tools and Representations ▪ Familiar objects and pictures to begin discussions around cross sections, such as an apple, a car, a couch, a cup, a guitar, etc. ▪ A site on Annenberg Learner that illustrates cross sections: http://www.learner.org/courses/learningmath/geometry/session9/part_c/

- IXL Math

Instructional Strategies	Meeting the Needs of All Students

Marzano's Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **21st Century Skills** Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **Provide Multiple Means of Representation** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as

money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.

- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such

	as hand pointed downward means count backwards in "Happy Counting." ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are "just right" for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day's sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including "show and tell" rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" ● Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify

	directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow “everyday” and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary **New or Recently Introduced Terms** ▪ **Correspondence** (A *correspondence* between two triangles is a pairing of each vertex of one triangle with one and only one vertex of the other triangle. A triangle correspondence also induces a correspondence between the angles of the triangles and the sides of the triangles.)	**Students Achieving Below Standard** **<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual	**Students Achieving Above Standard** The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of</u>**

▪ **Identical Triangles** (Two triangles are said to be identical if there is a triangle correspondence that pairs angles with angles of equal measure and sides with sides of equal length.) ▪ **Unique Triangle** (A set of conditions for two triangles is said to determine a *unique* triangle if whenever the conditions are satisfied, the triangles are identical.) ▪ **Three sides condition** (Two triangles satisfy the *three sides condition* if there is a triangle correspondence that pairs all three sides of one triangle with sides of equal length. The three sides condition determines a unique triangle.) ▪ **Two angles and the included side condition** (Two triangles satisfy the *two angles and the included side condition* if there is a triangle correspondence that pairs two angles and the included side of one triangle with angles of equal measure and a side of equal length. This condition determines a unique triangle.) ▪ **Two angles and the side opposite a given angle condition** (Two triangles satisfy the t*wo angles and the side opposite a given angle condition* if there is a triangle correspondence that pairs two angles and a side opposite one of the angles with angles of equal measure and a side of equal length. The two angles and the side opposite a given angle condition determines a unique triangle.) ▪ **Two sides and the included angle condition** (Two triangles satisfy the *two sides and the included angle condition* if there is a triangle correspondence that pairs two sides and the included angle with sides of equal length and an angle of equal measure. The two sides and the included angle condition determines	displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?'	**<u>Representation</u>** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.

a unique triangle.) ▪ **Two sides and a non-included angle condition** (Two triangles satisfy the *two sides and a non-included angle condition* if there is a triangle correspondence that pairs two sides and a non-included angle with sides of equal length and an angle of equal measure. The *two sides and a non-included angle condition* determines a unique triangle if the non-included angle measures $90°$ or greater. If the non-included angle is acute, the triangles are identical with one of two non-identical triangles.) ▪ **Right rectangular pyramid** (Given a rectangular region □ in a plane □, and a point □ not in □, the rectangular pyramid with base □ and vertex □ is the union of all segments □□ for any point □ in □. It can be shown that the planar region defined by a side of the base □ and the vertex □ is a triangular region, called a lateral face. If the vertex lies on the line perpendicular to the base at its center (the intersection of the rectangle's diagonals), the pyramid is called a right rectangular pyramid.) ▪ **Surface of a pyramid** (The *surface of a pyramid* is the union of its base region and its lateral faces.) **Familiar Terms and Symbols[29]** ▪ Vertical angles ▪ Adjacent angles ▪ Complementary Angles ▪ Supplementary Angles ▪ Angles on a line	Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions.	Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

[29] These are terms and symbols students have seen previously.

▪ Angles at a Point ▪ Right rectangular prism	Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	

Additional Lesson Plans

In 2014, the LearnZillion Dream Team created hundreds of task-based lesson plans. Digital, interactive, and ready to use, our math lesson plans engage students in the productive struggle that leads to higher engagement and deeper understanding of the Common Core standards.

To help with Focus, one of the three major shifts in the standards, we have flagged the "major work" of each grade.

Math Lesson Plans: 7th Grade

This is the complete collection of lesson plans for 7th grade.

Appendix A: 3 Lesson Plan Examples Module 1 Lesson 1 Module 2 Lesson 1 and Module 3 Lesson 1

Lesson 1: An Experience in Relationships as Measuring Rate

Student Outcomes

- Students compute unit rates associated with ratios of quantities measured in different units. Students use the context of the problem to recall the meaning of value of a ratio, equivalent ratios, rate, and unit rate, relating them to the context of the experience.

Classwork

Example 1 (15 minutes): How fast is our class?

To start this first class of the school year, conduct an exercise in how to pass out papers. The purpose of the task is not only to establish a routine at the start of the school year but also to provide a context to discuss ratio and rate.

Determine how papers will be passed out in class depending upon seating arrangement. For this task, it is best to divide the original stack so that one student (in each row or group) has a portion of the original stack. Based upon this determination, explain the system to students. A brief demonstration may help to provide a visual.

For example: If the room is arranged in rows, pass *across* the rows. Have students start on command and perhaps require that only the current paper-passing student may be out of his or her seat. If the room is arranged in groups or at tables, have the students pass papers to their left, on command, until everyone has a paper. *Note: this procedure is highly customizable for use in any classroom structure.*

Begin the task by handing a stack of papers to a starting person. Secretly start a stopwatch as the start command is given. Once every student has a paper, report the paper-passing time out loud. For example, "Twelve seconds. Not bad, but let's see if we can get these papers passed out in eleven seconds next time."

Tell students to begin returning papers back in to the original stack and then report the time upon completion.

- "Excellent job. Now pass them back out in ten seconds. Excellent. Now pass them back in eight seconds."

Pose the following questions to the students as a whole group, one question at a time.

Questions to discuss:

- How will we measure our rate of passing out papers?
    - *Using a stopwatch or similar tool to measure the number of seconds taken to pass out papers*
- What quantities will we use to describe our rate?
    - *The number of papers passed out and the time that it took to pass them out*

Complete the 2nd and 3rd columns (number of papers and time) on the table as a class.

- Describe the quantities you want to measure by talking about what units we use to measure each quantity.
    - *One quantity measures the number of papers, and the other measures the number of seconds.*

Review the Key Terms box defining ratio, rate, and unit rate in the student materials. Focus on reviewing the concept of ratio first, perhaps using a few quick examples.

Key Terms from Grade 6 Ratios and Unit Rates:

A ratio is an ordered pair of non-negative numbers, which are not both zero. The ratio is denoted to indicate the order of the numbers: the number is first and the number is second.

Two ratios and are equivalent ratios if there is a positive number, , such that and .

A ratio of two quantities, such as miles per hours, can be written as another quantity called a rate.

The numerical part of the rate is called the unit rate and is simply the value of the ratio, in this case . This means that in hour the car travels miles. The unit for the rate is miles/hour, read miles per hour.

Guide students to complete the ratio column in the table as shown below.

E x a m p l e 1 : H o w fast is our class?

Record the results from the paper-passing exercise in the table below.

Trial	Number of Papers Passed	Time (in seconds)	Ratio of Number of Papers Passed to Time	Rate	Unit Rate
1	*24*	*12*	*24 : 12, or 24 to 12, or equivalent ratio*		
2	*24*	*11*	*24 : 11, or 24 to 11, or equivalent ratio*		
3	*24*	*10*	*24 : 10, or 24 to 10, or equivalent ratio*		
4	*24*	*8*	*24 : 8, or 24 to 8, or equivalent ratio*		

- When we started passing papers, the ratio of the number of papers to the number of seconds was *24 to 12,* and by the end of the activity, the ratio of the number of papers to the number of seconds was *24 to 8*. Are these two ratios equivalent? Explain why or why not.

Guide students in a discussion about the fact that the number of papers was constant, and the time decreased with each successive trial. See if students can relate this to rate and ultimately determine which rate is greatest.

  - *The ratios are not equivalent since we passed the same number of papers in a shorter time. We passed 2 papers per second at the beginning and 3 papers per second by the end. Equivalent ratios must have the same value*.

The following questioning is meant to guide students into the realization that unit rate helps us to make comparisons between a variety of ratios and compare different data points.

- In another class period, students were able to pass 28 papers in 15 seconds, then 28 papers in 12 seconds. A third class period passed 18 papers in 10 seconds. How do these compare to our class?

Use sample data here or use real data collected from other classes prepared in advance.

  - *We could find how many papers were passed per second to make these comparisons. Answers on how they compare would vary depending on class results in the table.*

Review the meaning of rate and unit rate in the Key Terms box, and complete the last two columns of the table, modeling how to find both rate and unit rate. The associated unit rate is the numerical value $\frac{\square}{\square}$ when there are $\square$ units of one quantity for every $\square$ units of another quantity.

Trial	Number of Papers Passed	Time (in seconds)	Ratio of Number of Papers Passed to Time	Rate	Unit Rate
1	24	12	*$24:12$, or 24 to 12, or equivalent ratio*	*2 papers per second*	*2*
2	24	11	*$24:11$, or 24 to 11, or equivalent ratio*	*Approximately 2.2 papers per second*	*2.2*
3	24	10	*$24:10$, or 24 to 10, or equivalent ratio*	*2.4 papers per second*	*2.4*
4	24	8	*$24:8$, or 24 to 8, or equivalent ratio*	*3 papers per second*	*3*

Example 2 (15 minutes): Our Class by Gender

Let's make a comparison of two quantities that are measured in the same units by comparing the ratio of the number of boys to the number of girls in this class to the ratio for different classes (and the whole grade).

- Sample discussion: In this class, we have 14 boys and 12 girls. In another class, there are 7 boys and 6 girls. *Note: Any class may be used for comparison; the ratios do not need to be equivalent.*

Guide students to complete the table accordingly, pausing to pose the questions below.

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Our Class by Gender

	Number of Boys	Number of Girls	Ratio of Boys to Girls
Class 1	*14*	*12*	*7 to 6*
Class 2	*7*	*6*	*7 to 6*
Whole 7th Grade	*Answers vary*	*Answers vary*	

Create a pair of equivalent ratios by making a comparison of quantities discussed in this example.

- Are the ratios of boys to girls in the two classes equivalent?
- What could these ratios tell us?
- What does the ratio of the number of boys to the number of girls in Class 1 to the ratio of the number of boys to the ratio of the number of girls in the entire *7th* grade class tell us?

You will need this information in advance.

- Are they equivalent?
- If there is a larger ratio of boys to girls in one class than in the grade as a whole, what must be true about the boy to girl ratio in other classes? (It may be necessary to modify this question based upon real results or provide additional examples where this is true.)

Provide ratios from four classes and the total number of students in 7th grade. Using these provided ratios, challenge students to determine the ratio of the 5th class and derive a conclusion. (See detailed explanation in chart below.)

- *Sample solution: If the total number of students is 55 boys and 65 girls, or 120 students, then the missing number of boys for Class 5 is $55 - 47 = 8$, and the missing number of girls (for Class 5) is $65 - 49 = 16$, resulting in a boy to girl ratio, $8:16 = 1:2$, that is smaller than the whole grade ratio.*

This extension would also allow for students to see the usefulness of using unit rate when making comparisons.

- How do we compare ratios when we have varying sizes of quantities?
  - *Finding the unit rate may help. In the data given here, the unit rate for both Classes 1 and 2 is approximately 1.16, and the unit rate for the whole grade is approximately 0.85. The unit rate for Class 4 is approximately 0.53, and the unit rate for Class 5 is 0.5.*

	Number of Boys	**Number of Girls**	**Ratio of Boys to Girls**
Class 1	*14*	*12*	*7* to *6*

Class 2	7	6	7 to 6
Class 3	16	12	8 to 6 or 4 to 3
Class 4	10	19	10 to 19
Class 5	$? = 8$	$? = 16$	1 to 2
Whole 7th Grade	55	65	11 to 13

Larger than whole grade ratio

Smaller than whole grade ratio

he numbers for Class 5.

Review the Key Terms box focusing on the meaning of equivalent ratios, and give students 2 minutes to write equivalent ratios comparing boys to girls or a similar comparison from their class. Discuss responses as a whole class.

Exercise 1 (8 minutes): Which is the Better Buy?

Read the problem as a class, and then allow time for students to solve independently. Ask students to share responses regarding how to determine if the ratios are equivalent. Reinforce key vocabulary from Grade 6.

Exercise 1: Which is the Better Buy?

Value-Mart is advertising a Back-to-School sale on pencils. A pack of 30 sells for $\$7.97$, whereas a 12-pack of the same brand costs $\$\,4.77$. Which is the better buy? How do you know?

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The better buy is the pack of 30. The pack of 30 has a smaller unit rate, approximately 0.27, as compared to the pack of 12 with a unit price of 0.40. You would pay $\$0.27$ per pencil in the pack of 30, whereas you would pay $\$0.40$ per pencil in the pack of 12.

	Pack of 30	*Pack of 12*
Relationship:	*$7.97 for every 30 pencils* *$15.94 for every 60 pencils*	*$4.77 for every 12 pencils* *$23.85 for every 60 pencils*
Ratio:	$7.97: 30$; $15.94: 60$	$4.77: 12$; $23.85: 60$
Rate:	$7.97/30$, *or approx.* 0.27 *dollars per pencil*	$4.77/12$, *or approx.* 0.40 *dollars per pencil*

Unit Rate:	0.27	0.40
Unit of Measure:	*dollars per pencil*	*dollars per pencil*

Students may instead choose to compare the costs for every 60 pencils or every 360 pencils, etc. Facilitate a discussion of the different methods students may have used to arrive at their decisions.

Closing (2 minutes)

- How is finding an associated rate or unit rate helpful when making comparisons between quantities?
  - *The unit rate tells the number of units of one quantity per one unit of a second quantity. For example, a unit price of 0.4 means 1 juice box from a six-pack costs $\$0.40$.*

> **Lesson Summary**
>
> **Unit rate is often a useful means for comparing ratios and their associated rates when measured in different units. The unit rate allows us to compare varying sizes of quantities by examining the number of units of one quantity per one unit of the second quantity. This value of the ratio is the unit rate.**

Module 2 Lesson 1:

Opposite Quantities Combine to Make Zero

Student Outcomes

- Students add positive integers by counting up and negative integers by counting down (using curved arrows on the number line).
- Students play the Integer Game to combine integers, justifying that an integer plus its opposite add to zero.
- Students know the opposite of a number is called the additive inverse because the sum of the two numbers is zero.

Lesson Notes

There is some required preparation from teachers before the lesson begins. It is suggested that number lines are provided for students. However, it is best if students can reuse these number lines by having them laminated and using white board markers. Also, the Integer Game is used during this lesson, so the teacher should prepare the necessary cards for the game.

Classwork

Exercise 1 (3 minutes): Positive and Negative Numbers Review

In pairs, students will discuss "What I Know" about positive and negative integers to access prior knowledge. Have them record and organize their ideas in the graphic organizer in the student materials. At the end of discussion, the teacher will choose a few pairs to share out with the class.

Exercise 1: Positive and

Negative Numbers Review

With your partner, use the graphic organizer below to record what you know about positive and negative numbers. Add or remove statements during the whole class discussion.

Negative Numbers		Positive Numbers
They are to the left of on a number line and get smaller going to the left. *They can mean a loss, drop, decrease, or below sea level.* *They look like -.* *They are opposites of positive numbers.*	*Both are on a number line.*	*They are to the right of on a number line and get larger going to the right.* *They can mean a gain, increase, or above sea level.* *They don't have a sign.* *They are opposites of negative numbers.*

Scaffolding:

- Laminate (or place in sheet protectors) 1-page of number lines (vary blank and numbered) for individual use with white board markers.
- Create a number line on the floor using painters tape to model the "counting on" principle.
- Provide a wall model of the number line at the front of the room for visual reinforcement.

Example 1 (5 minutes): Introduction to the Integer Game

Read the Integer Game outline before the lesson. The teacher selects a group of 3 or 4 students to demonstrate to the whole class how to play the Integer Game.[30] The game will be played later in the lesson. The teacher should stress that the object of the game is to get a score of zero.

Example 2 (5 minutes): Counting Up and Counting Down on the Number Line

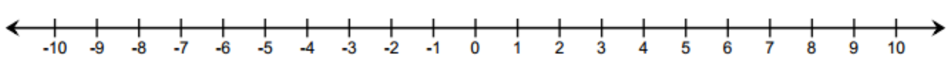
Model a few examples of counting with small curved arrows to locate numbers on the number line, where *counting* up corresponds to positive numbers and *counting down* corresponds to negative numbers.

[30] Refer to the Integer Game outline for player rules.

Example 2: Counting Up and Counting Down on the Number Line

Use the number line below to practice counting up and counting down.

- ***Counting up*** **corresponds to** ***_____positive_______*** **numbers.**
- ***Counting down*** **corresponds to** ***_____negative______*** **numbers.**



A negative is units to the left of .

A positive is units to the right of .

a. **Where do you begin when locating a number on the number line?**

Start at 0.

b. **What do you call the distance between a number and 0 on a number line?**

The absolute value.

c. What is the relationship between 7 and -7?

Answers will vary. 7 and -7 both have the same absolute values. They are both the same distance from zero, 0, but in opposite directions; therefore, 7 and -7 are opposites.

Example 3 (5 minutes): Using the Integer Game and the Number Line

The teacher leads the whole class using a number line to model the concept of counting on (addition) to calculate the value of a hand when playing the Integer Game. The hand's value is the sum of the card values.

First card:

Start at and end up at positive . This is the first card drawn, so the value of the hand is .

Second Card:

Start at , the value of the hand after the first card; move units to the left to end at .

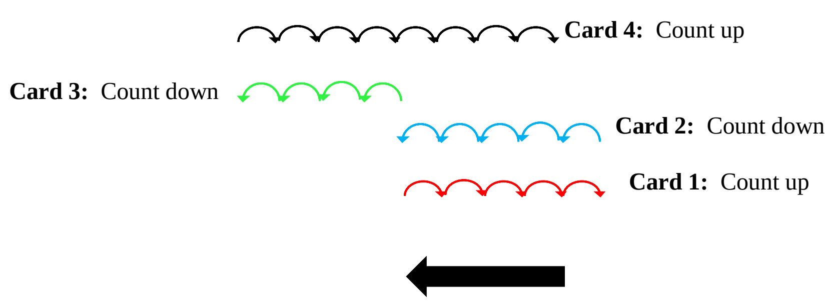
Third Card:

Start at , the value of the hand after the second card; move units to the left.

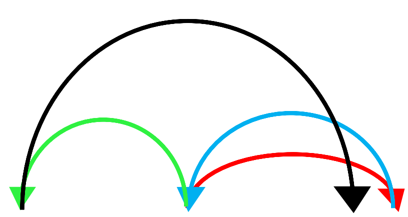
Fourth Card:

Start at , the value of the hand after the third card; move units to the right.

- What is the final position on the number line?
  - *The final position on the number line is 4.*



- What card or combination of cards would you need to get back to 0?
  - *In order to get a score of 0, I would need to count down 4 units. This means, I would need to draw a -4 card or a combination of cards whose sum is -4, such as -1 and -3.*



The final position is units to the right of .

We can use smaller, curved arrows to show the number of "hops" or "jumps" that correspond to each integer. Or, we can use larger, curved arrows to show the length of the "hop" or "jump" that corresponds to the distance between the tail and the tip of the arrow along the number line. Either way, the final position is 4 units to the right of zero. Playing the Integer Game will prepare students for integer addition using arrows (vectors) in Lesson 2.

Example 3: Using the Integer Game and the Number Line

What is the sum of the card values shown? Use the counting on method on the provided number line to justify your answer.

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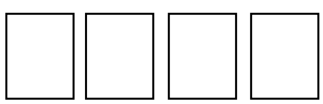
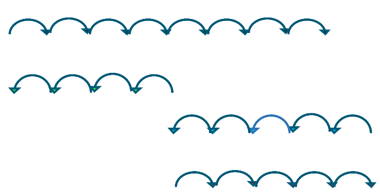




a. What is the final position on the number line? *4*

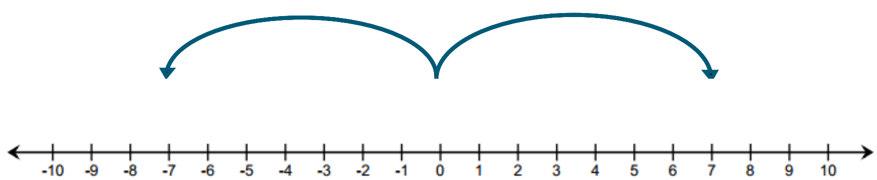
d. What card or combination of cards would you need to get back to 0? *-4 or -1 and -3*

Exercise 2 (5 minutes): The Additive Inverse

Before students begin, the teacher highlights part of the previous example where starting at zero and counting up five units and then back down five units brings us back to zero. This is because 5 and -5 are opposites. Students work independently to answer the questions. At the end of the exercise questions, formalize the definition of **additive inverse**.

Exercise 2: The Additive Inverse

Use the number line to answer each of the following questions.



a. **How far is 7 from 0 and in which direction?** *7 units to the right*

b. **What is the opposite of 7?** -7

c. **How far is -7 from 0 and in which direction?** *7 units to the left*

e. **Thinking back to our previous work, how would you use the counting on method to represent the following: While playing the Integer Game, the first card selected is 7, and the second card selected is -7.**

I would start at 0 and count up 7 by moving to the right. Then, I would start counting back down from 7 to 0.

f. **What does this tell us about the sum of 7 and its opposite, -7?**The sum of 7 and -7 equals 0.$7 + (-7) = 0$.

g. Look at the curved arrows you drew for 7 and -7. What relationship exists between these two arrows that would support your claim about the sum of 7 and -7?

The arrows are both the same distance from 0. They are just pointing in opposite directions.

h. Do you think this will hold true for the sum of any number and its opposite? Why?

I think this will be true for the sum of any number and its opposite because when you start at 0 on the number line and move in one direction, moving in the opposite direction the same number of times will always take you back to zero.

For all numbers there is a number , such that .

The additive inverse of a real number is the opposite of that number on the real number line. For example, the opposite of is . A number and its additive inverse have a sum of 0. The sum of any number and its opposite is equal to zero.

Example 4 (5 minutes): Modeling with Real-World Examples

The purpose of this example is to introduce real-world applications of opposite quantities to make zero. The teacher holds up an Integer Game card, for example -10, to the class and models how to write a story problem.

- How would the value of this card represent a temperature?
  - *-10 could mean 10 degrees below zero.*
- How would the temperature need to change in order to get back to 0 degrees?
  - *Temperature needs to rise 10 degrees.*
- With a partner, write a story problem using money that represents the expression $200 + (-200)$.
  - *Answers will vary. Timothy earned \$200 last week. He spent it on a new video game console. How much money does he have left over?*

Students share their responses to the last question with the class.

Exercise 3 (10 minutes): Playing the Integer Game

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cise 3: Playing the Integer Game

Play the Integer Game with your group. Use a number line to practice counting on.

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Students will play the Integer Game in groups. Students will practice counting using their number lines. Let students explore how they will model addition on the number line. Monitor student understanding by ensuring that the direction of the arrows appropriately represents positive or negative integers.

Module 3 Lesson 1 Lesson Plan:

Lesson 1: Generating Equivalent Expressions

Student Outcomes

- Students generate equivalent expressions using the fact that addition and multiplication can be done in *any order* (commutative property) and *any grouping* (associative property).
- Students recognize how *any order, any grouping* can be applied in a subtraction problem by using additive inverse relationships (adding the opposite) to form a sum and likewise with division problems by using the multiplicative inverse relationships (multiplying by the reciprocal) to form a product.
- Students recognize that *any order* does not apply to expressions mixing addition and multiplication, leading to the need to follow the order of operations.

Lesson Notes

The *any order, any grouping* property introduced in this lesson combines the commutative and associative properties of both addition and multiplication. The commutative and associative properties are regularly used in sequence to rearrange terms in an expression without necessarily making changes to the terms themselves. Therefore, students utilize the any order, any grouping property as a tool of efficiency for manipulating expressions. The any order, any grouping property is referenced in the Progressions for the Common Core State Standards in Mathematics: Grades 6–8, Expressions and Equations, and is introduced in Grade 6 Module 4, Expressions and Equations.

The definitions presented below, related to variables and expressions, form the foundation of the next few lessons in this topic. Please review these carefully in order to understand the structure of Topic A lessons.

VARIABLE: A *variable* is a symbol (such as a letter) that represents a number, i.e., it is a placeholder for a number.

A variable is actually quite a simple idea: it is a placeholder—a blank—in an expression or an equation where a number can be inserted. A variable holds a place for *a single number* throughout all calculations done with the variable—it does not vary. It is the *user of the variable* who has the ultimate power to change or vary what number is inserted, *as he/she desires*. The power to “vary” rests in the will of the student, not in the variable itself.

NUMERICAL EXPRESSION (IN MIDDLE SCHOOL): A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

Statements such as "$3 +$" or "$3 \div 0$" are not numerical expressions because neither represents a point on the number line.

VALUE OF A NUMERICAL EXPRESSION: The *value of a numerical expression* is the number found by evaluating the expression.

For example, $\frac{1}{3} \cdot (2 + 4) - 7$ is a numerical expression, and its value is -5. Note to teachers: Please do not stress words over meaning here; it is acceptable to use "number computed," "computation," "calculation," etc. to refer to the value as well.

EXPRESSION (IN MIDDLE SCHOOL): An *expression* is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.

There are two ways to build expressions:

- We can start out with a numerical expression, such as $\frac{1}{3} \cdot (2 + 4) - 7$, and replace some of the numbers with letters to get $\frac{1}{3} \cdot (\square + \square) - \square$.
- We can build such expressions from scratch, as in $\square + \square(\square - \square)$, and note that if numbers were placed in the expression for the variables $\square$, $\square$, and $\square$, the result would be a numerical expression.

The key is to strongly link expressions back to computations with numbers through building and evaluating them. Building an expression often occurs in the context of a word problem by thinking about examples of numerical expressions first, and then replacing some of the numbers with letters in a numerical expression. The act of evaluating an expression means to replace each of the variables with specific numbers to get a numerical expression, and then finding the value of that numerical expression.

The description of expression above is meant to work nicely with how students in 6^{th} and 7^{th} grades learn to manipulate expressions. In these grades, students spend a lot of time building and evaluating expressions for specific numbers substituted into the variables. Building and evaluating helps students see that expressions are really just a slight abstraction of arithmetic in elementary school.

EQUIVALENT EXPRESSIONS (IN MIDDLE SCHOOL): Two expressions are *equivalent* if both expressions evaluate to the same number for every substitution of numbers into all the letters in both expressions. This description becomes clearer through lots of examples and linking to the associative, commutative, and distributive properties.

AN EXPRESSION IN EXPANDED FORM (IN MIDDLE SCHOOL): An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in *expanded form*.

AN EXPRESSION IN STANDARD FORM (IN MIDDLE SCHOOL): An expression that is in expanded form where all like terms have been collected is said to be in *standard form*.

<u>Important</u>: An expression in *standard form* is the equivalent of what is traditionally referred to as a *simplified expression*. This curriculum does not utilize the term *simplify* when writing equivalent expressions, but rather asks students to "put an expression in standard form" or "expand the expression and combine like terms." However, students must know that the term *simplify* will be seen outside of this curriculum and that the term is directing them to write an expression in standard form.

Materials

Prepare a classroom set of manila envelopes (non-translucent). Cut and place four triangles and two quadrilaterals in each envelope (provided at the end of this lesson). These envelopes are used in the Opening Exercise of this lesson.

Classwork

Opening Exercise (15 minutes)

This exercise requires students to represent unknown quantities with variable symbols and reason mathematically with those symbols to represent another unknown value.

As students enter the classroom, provide each one with an envelope containing two quadrilaterals and four triangles; instruct students not to open their envelopes. Divide students into teams of two to complete parts (a) and (b).

> *Scaffolding:*
>
> To help students understand the given task, discuss a numerical expression, such as
>
> as an example

Opening Exercise

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let □ represent the number of triangles, and let □ represent the number of quadrilaterals.

MP.

i. **Write an expression using □ and □ that represents the total number of sides in your envelope. Explain what the terms in your expression represent.**

   $3\square + 4\square$. Triangles have 3 sides, so there will be 3 sides for each triangle in the envelope. This is represented by $3\square$. Quadrilaterals have 4 sides, so there will be 4 sides for each quadrilateral in the envelope. This is represented by $4\square$. The total number of sides will be the number of triangle sides and the number of quadrilateral sides together.

j. **You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.**

   $3\square + 4\square + 3\square + 4\square$; $2(3\square + 4\square)$; $6\square + 8\square$

Discuss the variations of the expressions in part (b) and whether those variations are equivalent. This discussion helps students understand what it means to combine like terms; some students have added their number of triangles together and number of quadrilaterals together, while others simply doubled their own number of triangles and quadrilaterals since the envelopes contain the same number. This discussion further shows how these different forms of the same expression relate to each other. Students then complete part (c).

MP.

k. **Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.**

Answer depends on the number of students in the classroom. For example, if there are 12 students in the classroom, the expression would be $12(3\square + 4\square)$, or an equivalent expression.

Next, discuss any variations (or possible variations) of the expression in part (c), and discuss whether those variations are equivalent. Are there as many variations in part (c), or did students use multiplication to consolidate the terms in their expressions? If the latter occurred, discuss the students' reasoning.

Choose one student to open an envelope and count the numbers of triangles and quadrilaterals. Record the values of □ and □ as reported by that student for all students to see. Next, students complete parts (d), (e), and (f).

l. **Use the given values of □ and □ and your expression from part (a) to determine the number of sides that should be found in your envelope.**

$3\square + 4\square 3(4) + 4(2) 12 + 8 20$

There should be 20 sides contained in my envelope.

m. **Use the same values for □ and □ and your expression from part (b) to determine the number of sides that should be contained in your envelope and your partner's envelope combined.**

Variation 1	*Variation 2*	*Variation 3*
$2(3\square + 4\square)$	$3\square + 4\square + 3\square + 4\square$	$6\square + 8\square$
$2(3(4) + 4(2))$	$3(4) + 4(2) + 3(4) + 4(2)$	$6(4) + 8(2) 2(12 +$
$8)$	$12 + 8 + 12 + 8$	$24 + 16$
$2(20)$	$20 + 12 + 8$	40
40	40	

My partner and I have a combined total of 40 sides.

n. **Use the same values for □ and □ and your expression from part (c) to determine the number of sides that should be contained in all of the envelopes combined.**

Answer will depend on the seat size of your classroom. Sample responses for a class size of 12:

Variation 1	*Variation 2*	*Variation 3*
$12(3\square + 4\square)$	$3\square + 4\square\square^{1} + 3\square + 4\square\square^{2} + \cdots + 3\square + 4\square\square^{12}$	$36\square + 48\square$
$12(3(4) + 4(2))$ $48(2)$	$3(4) + 4(2)\square^{1} + 3(4) + 4(2)\square^{2} + \cdots + 3(4) + 4(2)\square^{12}$	$36(4) +$
$12(12 + 8)$	$3(4) + 4(2)\square^{1} + 3(4) + 4(2)\square^{2} + \cdots + 3(4) + 4(2)\square^{12}$	$144 + 96$
$12(20)$	$12 + 8\square^{1} + 12 + 8\square^{2} + \cdots + 12 + 8\square^{12}$	240
240	$20\square^{1} + 20\square^{2} + \cdots + 20\square^{12}$	240

For a class size of 12 students, there should be 240 sides in all of the envelopes combined.

Have all students open their envelopes and confirm that the number of triangles and quadrilaterals matches the values of □ and □ recorded after part (c). Then, have students count the number of sides contained on the triangles and quadrilaterals from their own envelope and confirm with their answer to part (d). Next, have partners count how many sides they have combined and confirm that number with their answer to part (e). Finally, total the number of sides reported by each student in the classroom and confirm this number with the answer to part (f).

o. What do you notice about the various expressions in parts (e) and (f)?

The expressions in part (e) are all equivalent because they evaluate to the same number: 40. The expressions in part (f) are all equivalent because they evaluate to the same number: 240. The expressions themselves all involve the expression $3\square + 4\square$ in different ways. In part (e), $3\square + 3\square$ is equivalent to $6\square$, and $4\square + 4\square$ is equivalent to $8\square$. There appear to be several relationships among the representations involving the commutative, associative, and distributive properties.

When finished, have students return their triangles and quadrilaterals to their envelopes for use by other classes.

Example 1 (10 minutes): Any Order, Any Grouping Property with Addition

This example examines how and why we combine numbers and other like terms in expressions. An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include 324, $3\square$,
$5\square + 3 - 40$, $\square + 2\square + 3\square$, etc.

Each summand of an expression in expanded form is called a *term*, and the number found by multiplying just the numbers in a term together is called the *coefficient of the term.* After defining the word *term*, students can be shown what it means to "combine like terms" using the distributive property. Students saw in the Opening Exercise that terms sharing exactly the same letter could be combined by adding (or subtracting) the coefficients of the terms:

$$3\square + 3\square = (3+3)\square^{\square\square\square\square\square\square\square\square\square\square\square\square} \cdot \square = 6\square, \qquad \text{and} \qquad 4\square + 4\square = (4+4)\square^{\square\square\square\square\square\square\square\square\square\square\square\square} \cdot \square = 8\square.$$

An expression in expanded form with all its like terms collected is said to be in *standard form***.**

Scaffolding:

Refer students to the triangles and quadrilaterals from the opening exercise to understand why terms containing the

Example 1

d. Rewrite $5\square + 3\square$ and $5\square - 3\square$ by combining like terms.

Write the original expressions and expand each term using addition. What are the new expressions equivalent to?

$5\square + 3\square = \square + \square + \square + \square + \square\square^{5\square} + \square + \square + \square\square^{3\square}\square_{8\square} = 8\square$
$5\square - 3\square = \square + \square + \square + \square + \square\square_{3\square}\square^{5\square} = 2\square$

Scaffolding:

Note to the teacher: The distributive property was covered in Grade 6 (**6.EE.A.3**) and is reviewed here in preparation for further use in this module starting with Lesson 3.

- Because both terms have the common factor of x, we can use the distributive property to create an equivalent expression.

$5x + 3x = (5 + 3)x = 8x$ $\quad$ $5x - 3x = (5 - 3)x = 2x$

Ask students to try to find an example (a value for x) where $5x + 3x \neq 8x$ or where $5x - 3x \neq 2x$. Encourage them to use a variety of positive and negative rational numbers. Their failure to find a counterexample will help students realize what equivalence means.

In Example 1, part (b), students see that the commutative and associative properties of addition are regularly used in consecutive steps to reorder and regroup like terms so that they can be combined. Because the use of these properties does not change the value of an expression or any of the terms within the expression, the commutative and associative properties of addition can be used simultaneously. The simultaneous use of these properties is referred to as the *any order, any grouping property*.

Scaffolding:

Teacher may also want to show the expression as

in the same

MP.

p. Find the sum of $2x + 1$ and $5x$.

$(2x + 1) + 5x$	***Original expression***
$2x + (1 + 5x)$	***Associative property of addition***
$2x + (5x + 1)$	***Commutative property of addition***
$(2x + 5x) + 1$	***Associative property of addition***
$(2 + 5)x + 1$	***Combined like terms (the distributive property)***
$7x + 1$	***Equivalent expression to the given problem***

With a firm understanding of the commutative and associative properties of addition, students further understand that these steps can be combined.

- Why did we use the associative and commutative properties of addition?
  - *We reordered the terms in the expression to group together like terms so that they could be combined.*
- Did the use of these properties change the value of the expression? How do you know?
  - *The properties did not change the value of the expression because each equivalent expression includes the same terms as the original expression, just in a different order and grouping.*
- If a sequence of terms is being added, the *any order, any grouping* property allows us to add those terms

in any order by grouping them together in any way.

- How can we confirm that the expressions $(2⍰ + 1) + 5⍰$ and $7⍰ + 1$ are equivalent expressions?
  - *When a number is substituted for the ⍰ in both expressions, they both should yield equal results.*

The teacher and student should choose a number, such as 3, to substitute for the value of ⍰ and together check to see if both expressions evaluate to the same result.

Given Expression	***Equivalent Expression?***
$(2⍰ + 1) + 5⍰$ $(2 \cdot 3 + 1) + 5 \cdot 3$ $(6 + 1) + 15$ $(7) + 15$ 22	$7⍰ + 1$ $7 \cdot 3 + 1$ $21 + 1$ 22

The expressions both evaluate to 22; however, this is only one possible value of ⍰. Challenge students to find a value for ⍰ for which the expressions do not yield the same number. Students find that the expressions evaluate to equal results no matter what value is chosen for ⍰.

- What prevents us from using any order, any grouping in part (c), and what can we do about it?
  - *The second expression, $(5⍰ - 3)$, involves subtraction, which is not commutative or associative; however, subtracting a number ⍰ can be written as adding the opposite of that number. So, by changing subtraction to addition, we can use any order and any grouping.*

q. Find the sum of $-3⍰ + 2$ and $5⍰ - 3$.

$(-3⍰ + 2) + (5⍰ - 3)$	***Original expression***
$-3⍰ + 2 + 5⍰ + (-3)$	***Add the opposite (additive inverse)***
$-3⍰ + 5⍰ + 2 + (-3)$	***Any order, any grouping***
$2⍰ + (-1)$	***Combined like terms (Stress to students that the expression is not yet simplified.)***
$2⍰ - 1$	***Adding the inverse is subtracting***

- What was the only difference between this problem and those involving all addition?
  - *We first had to rewrite subtraction as addition; then, this problem was just like the others.*

Example 2 (3 minutes): Any Order, Any Grouping with Multiplication

Students relate a product to its expanded form and understand that the same result can be obtained using any order, any grouping since multiplication is also associative and commutative.

Example 2

Find the product of $2\square$ and 3.

$2\square \cdot 3 = 2\square + 2\square + 2\square = 6\square$

$2 \cdot (\square \cdot 3)$ ***Associative property of multiplication (any grouping)***

$2 \cdot (3 \cdot \square)$ ***Commutative property of multiplication (any order)***

$6\square$ ***Multiplication***

With a firm understanding of the commutative and associative properties of multiplication, students further understand that these steps can be combined.

MP.

- Why did we use the associative and commutative properties of multiplication?
  - *We reordered the factors to group together the numbers so that they could be multiplied.*
- Did the use of these properties change the value of the expression? How do you know?
  - *The properties did not change the value of the expression because each equivalent expression includes the same factors as the original expression, just in a different order or grouping.*
- If a product of factors is being multiplied, the *any order, any grouping* property allows us to multiply those factors in any order by grouping them together in any way.

Example 3 (9 minutes): Any Order, Any Grouping in Expressions with Addition and Multiplication

Students use any order, any grouping to rewrite products with a single coefficient first as terms only, then as terms within a sum, noticing that any order, any grouping cannot be used to mix multiplication with addition.

E
x
a
m
p
l
e

3

Use any order, any grouping to write equivalent expressions.

e. $3(2\square)$

$(3 \cdot 2)\square$

$6\square$

Ask students to try to find an example (a value for □) where $3(2\square) \neq 6\square$. Encourage them to use a variety of positive and negative rational numbers because in order for the expressions to be equivalent, the expressions must evaluate to equal numbers for *every* substitution of numbers into all the letters in both expressions. Again, the point is to help students recognize that they cannot find a value—that the two expressions are equivalent. Encourage students to follow the order of operations for the expression $3(2\square)$: multiply by 2 first, then by 3.

r. $4\square(5)$

$(4 \cdot 5)\square$
$20\square$

s. $4 \cdot$
$2 \cdot$
$\square$

$(4 \cdot 2)\square$
$8\square$

t. $3(2\square) +$
$4\square(5)$

$3(2\square) + 4\square(5) \quad = 2\square + 2\square + 2\square\square^{6\square} + 4\square + 4\square + 4\square + 4\square + 4\square\square^{20\square}$

$(3 \cdot 2)\square + (4 \cdot 5)\square$
$6\square + 20\square$

u. $3(2\square) + 4\square(5) + 4 \cdot 2 \cdot \square$

$3(2\square) + 4\square(5) + 4 \cdot 2 \cdot \square = \underbrace{2\square + 2\square + 2\square}_{6\square} + \underbrace{4\square + 4\square + 4\square + 4\square + 4\square}_{20\square} + \underbrace{\square + \square + \square + \square + \square + \square + \square + \square}_{8\square}$
$(3 \cdot 2)\square + (4 \cdot 5)\square + (4 \cdot 2)\square$
$6\square + 20\square + 8\square$

v. **Alexander says that $3\square + 4\square$ is equivalent to $(3)(4) + \square\square$ because of any order, any grouping. Is he correct? Why or why not?**

Encourage students to substitute a variety of positive and negative rational numbers for □ and □ because in order for the expressions to be equivalent, the expressions must evaluate to equal numbers for *every* substitution of numbers into all the letters in both expressions.

MP.

Alexander is incorrect; the expressions are not equivalent because if we, for example, let $\square = -2$ and let $\square = -3$, then we get the following:

$3\square + 4\square$	$(3)(4) + \square\square$
$3(-2) + 4(-3)$	$12 + (-2)(-3)$
$-6 + (-12)$	$12 + 6$
-18	18

$-18 \neq 18$*, so the expressions cannot be equivalent.*

- What can be concluded as a result of part (f)?
  - *Any order, any grouping cannot be used to mix multiplication with addition. Numbers and letters that are factors within a given term must remain factors within that term.*

Closing (3 minutes)

- We found that we can use any order, any grouping of terms in a sum, or of factors in a product. Why?
  - *Addition and multiplication are both associative and commutative, and these properties only change the order and grouping of terms in a sum or factors in a product without affecting the value of the expression.*
- Can we use any order, any grouping when subtracting expressions? Explain.
  - *We can use any order any grouping after rewriting subtraction as the sum of a number and the additive inverse of that number so that the expression becomes a sum.*
- Why can't we use any order, any grouping in addition and multiplication at the same time?
  - *Multiplication must be completed before addition. If you mix the operations, you change the value of the expression.*

Relevant Vocabulary

VARIABLE (DESCRIPTION): A *variable* is a symbol (such as a letter) that represents a number, i.e., it is a placeholder for a number.

NUMERICAL EXPRESSION (DESCRIPTION): A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

VALUE OF A NUMERICAL EXPRESSION: The *value of a numerical expression* is the number found by evaluating the expression.

EXPRESSION (DESCRIPTION): An *expression* is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.

EQUIVALENT EXPRESSIONS: Two expressions are *equivalent* if both expressions evaluate to the same number for every substitution of numbers into all the letters in both expressions.

AN EXPRESSION IN EXPANDED FORM: An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form.* A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include: 324, $3\square$, $5\square + 3 - 40$, $\square + 2\square + 3\square$, etc.

TERM (DESCRIPTION): Each summand of an expression in expanded form is called a *term.* For example, the expression $2\square + 3\square + 5$ consists of three terms: $2\square$, $3\square$, and 5.

COEFFICIENT OF THE TERM (DESCRIPTION): The number found by multiplying just the numbers in a term together. For example, given the product $2 \cdot \square \cdot 4$, its equivalent term is $8\square$. The number 8 is called the coefficient of the term $8\square$.

AN EXPRESSION IN STANDARD FORM: An expression in expanded form with all its like terms collected is said to be in *standard form.* For example, $2\square + 3\square + 5$ is an expression written in expanded form; however, to be written in standard form, the like terms $2\square$ and $3\square$ must be combined. The equivalent expression $5\square + 5$ is written in standard form

Closing (2 minutes)

> **Lesson Summary**
>
> **Terms that contain exactly the same variable symbol can be combined by addition or subtraction because the variable represents the same number. Any order, any grouping can be used where terms are added (or subtracted) in order to group together like terms. Changing the orders of the terms in a sum does not affect the value of the expression for given values of the variable(s).**

Students will discuss the following questions in their groups to summarize the lesson.

- How do you model addition using a number line?
  - *When adding a positive number on a number line, you count up by moving to the right. When adding a negative number on a number line, you count down by moving to the left.*
- Using a number line, how could you find the sum of $(-5) + 6$?
  - *Start at zero, then count down or move to the left five. From this point, count up or move to the right six.*
- Peter says he found the sum by thinking of it as $(-5) + 5 + 1$. Is this an appropriate strategy? Why do you think Peter did this?
  - *Peter did use an appropriate strategy to determine the sum of $(-5) + 6$. Peter did this because 5 and -5 are additive inverses, so they have a sum of zero. This made it easier to determine the sum to be one.*
- Why is the opposite of a number also called the additive inverse? What is the sum of a number and its opposite?
  - *The opposite of a number is called the additive inverse because the two numbers' sum is zero.*

> **Lesson Summary**
>
> **Add positive integers by counting up, and add negative integers by counting down.**
>
> **An integer plus its opposite sum to zero.**

Appendix B Example: Exit Tickets Unit 1 Lesson 1 and Unit 2 Lesson 1 and Mid Module Assessment Example

Name ____Date___________________

Module 1 Lesson 1: An Experience in Relationships as Measuring Rate Pre-Post Lesson Example

Exit Ticket

Watch the video clip of Tillman the English bulldog, the Guinness World Record holder for Fastest Dog on a Skateboard.

1. At the conclusion of the video, your classmate takes out his or her calculator and says, “Wow that was amazing! That means the dog went about 5 meters in 1 second!” Is your classmate correct, and how do you know?

2. After seeing this video, another dog owner trained his dog, Lightning, to try to break Tillman’s skateboarding record. Lightning’s fastest recorded time was on a 75-meter stretch where it took him 15.5 seconds. Based on this data,
   did Lightning break Tillman’s record for fastest dog on a skateboard? Explain how you know.

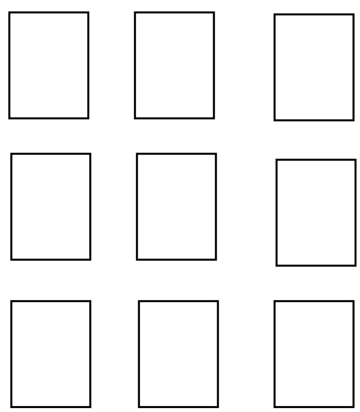
Name ____ Date ________________

Module 2 Lesson 1: Opposite Quantities Combine to Make Zero

Pre-Post Test Example

Exit Ticket

1. Your hand starts with the 7 card. Find three different pairs that would complete your hand and result in a value of zero.



2. Write an equation to model the sum of the situation below.

   A hydrogen atom has a zero charge because it has one negatively charged electron and one positively charged proton.

Write an equation for each diagram below. How are these equations alike? How are they different? What is it about the diagrams that lead to these similarities and differences?

3.

Diagram A:

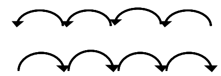
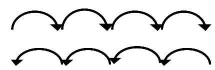


Diagram B:



Module 3 Lesson 1 Exit Ticket Pre-Post Test Example

Name ____ Date______________________

Lesson 1: Generating Equivalent Expressions

Exit Ticket

1. Write an equivalent expression to $2x + 3 + 5x + 6$ by combining like terms.

2. Find the sum of $(8a + 2b - 4)$ and $(3b - 5)$.

3. Write the expression in standard form: $4(2\square) + 7(-4\square) + (3 \cdot \square \cdot 5)$.

Name ____ Date ________

Mid Module Assessment Unit 1

1. It is a Saturday morning and Jeremy has discovered he has a leak coming from the water heater in his attic. Since plumbers charge extra to come out on weekends, Jeremy is planning to use buckets to catch the dripping water. He places a bucket under the drip and steps outside to walk the dog. In half an hour, the bucket is $\frac{1}{5}$ of the way full.

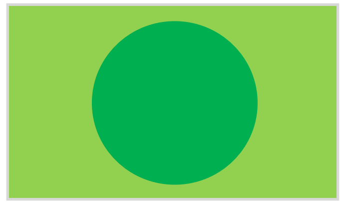
    a. What is the rate at which the water is leaking per hour?

    b. Write an equation that represents the relationship between the number of buckets filled, □, in □ hours.

    c. What is the longest that Jeremy can be away from the house before the bucket will overflow?

2. Farmers often plant crops in circular areas because one of the most efficient watering systems

for crops provides water in a circular area. Passengers in airplanes often notice the distinct circular patterns as they fly over land used for farming. A photographer takes an aerial photo of a field on which a circular crop area has been planted. He prints the photo out and notes that 2 centimeters of length in the photo corresponds to 100 meters in actual length.



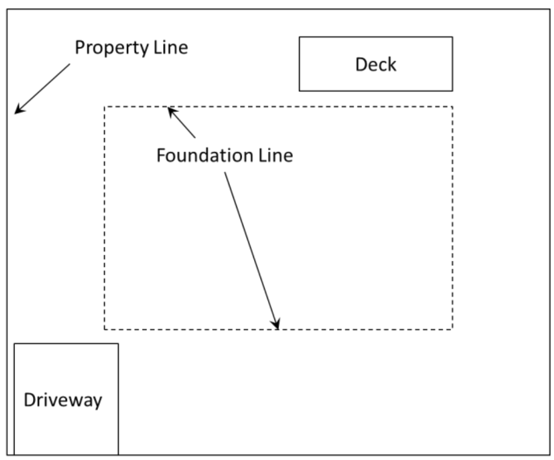
a. What is the scale factor of the actual farm to the photo?

b. If the dimensions of the entire photo are 25 cm by 20 cm, what are the actual dimensions of the rectangular land area in meters captured by the photo?

c. If the area of the rectangular photo is 5 cm^2, what is the actual area of the rectangular area in square meters?

3. A store is having a sale to celebrate President's Day. Every item in the store is advertised as one fifth off the original price. If an item is marked with a sale price of $\$140$, what was its original price? Show your work.

4. Over the break, your uncle and aunt ask you to help them cement the foundation of their newly purchased land and give you a top-view blueprint of the area and proposed layout. A small legend on the corner states that *4* inches of the length corresponds to an actual length of *52* feet.



a. What is the scale factor of the actual foundation to the blueprint?

b. If the dimensions of the foundation on the blueprint are *11* inches by *13* inches, what are the actual dimensions in feet?

c. You are asked to go buy bags of dry cement and know that one bag covers 350 square feet. How many bags do you need to buy to finish this project?

d. After the first 15 minutes of laying down the cement, you had used $\frac{1}{5}$ of the bag. What is the rate you are laying cement in bags per hour? What is the unit rate?

e. Write an equation that represents the relationship between the number of bags used, ▯, in ▯ hours.

f. Your uncle is able to work faster than you. He uses 3 bags for every 2 bags you use. Is the relationship proportional? Explain your reasoning using a graph on a coordinate plane.

g. What does ($0,0$) represent in terms of the situation being described by the graph created in part (f)?

h. Using a graph, show how many bags you would have used if your uncle used 18 bags.