Thomaston Public Schools

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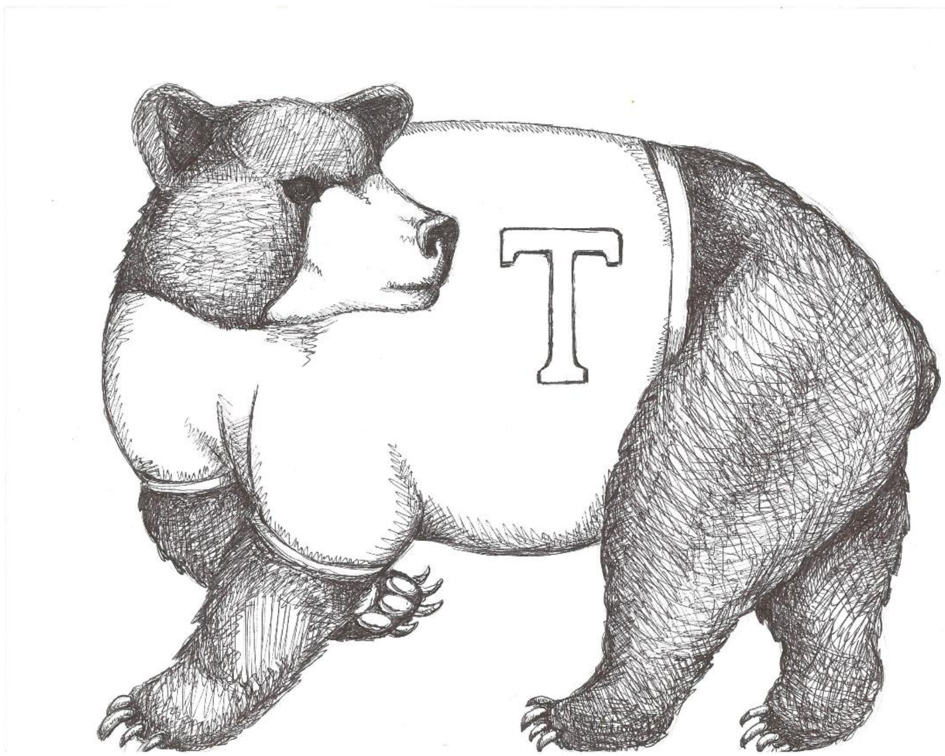

Thomaston Public Schools Curriculum
Thomaston Center School
Grade:6 Mathematics 2015

The Bridge to Adolescence

Acknowledgements

Curriculum Writer:

Gail Lascko

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

______*Alisha DiCorpo*________________________________
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Director of Curriculum and Professional Development

Date of Presentation to the Board of Education: August 2015

Math Curriculum Grade Six

Grade 6 Mathematics

Board of Education Mission Statement:

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:

The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main resource used when writing this curriculum:

NYS COMMON CORE MATHEMATICS CURRICULUM A Story of Ratios Curriculum. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. A Story of Ratios: A Story of Ratios Overview for Grades 6-8 Date: 7/31/13 5 © 2013 Common Core, Inc. Some rights reserved. commoncore.org

Course Description:

Sequence of Grade 6 Modules Aligned with the Standards

Module 1: Ratios and Unit Rates
Module 2: Arithmetic Operations Including Dividing by a Fraction
Module 3: Rational Numbers
Module 4: Expressions and Equations
Module 5: Area, Surface Area, and Volume Problems
Module 6: Statistics

Summary of Year

Sixth grade mathematics is about (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

Key Areas of Focus for Grade 6:	Ratios and proportional reasoning; early expressions and equations	
Required Fluency:	6.NS.2	Multi-digit division
	6.NS.3	Multi-digit decimal operations

Rationale for Module Sequence in Grade 6

In Module 1, students build on their prior work in measurement and in multiplication and division as they study the concepts and language of ratios and unit rates. They use proportional reasoning to solve problems. In particular, students solve ratio and rate using tape diagrams, tables of equivalent ratios, double number line diagrams, and equations. They plot pairs of values generated from a ratio or rate on the first quadrant of the coordinate plane.

Math Unit 1

Rigorous Curriculum Design Template

Unit One-Ratio and Unit Rates

Subject:Math

Grade/Course: Grade 6

Pacing: 35 days

Unit of Study: Ratio and Unit Rates

Priority Standards:

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Foundational Standards:

Use the four operations with whole numbers to solve problems.
4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.3 Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? Convert like measurement units within a given measurement system.
5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. Graph points on the coordinate plane to solve real-world and mathematical problems.
5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Math Practice Standards:

MP.1 Make sense of problems and persevere in solving them. Students make sense of and solve real-world and mathematical ratio, rate, and percent problems using representations, such as tape diagrams, ratio tables, the coordinate plane, and double number line diagrams. They identify and explain the correspondences between the verbal descriptions and their representations and articulate how the representation depicts the relationship of the quantities in the problem. Problems include ratio problems involving the comparison of three quantities, multistep changing ratio problems, using a given ratio to find associated ratios, and constant rate problems including two or more people or machines working together.

MP.2 Reason abstractly and quantitatively. Students solve problems by analyzing and comparing ratios and unit rates given in tables, equations, and graphs. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, distance = rate x time.

MP.5 Use appropriate tools strategically. Students become proficient using a variety of representations that are useful in reasoning with rate and ratio problems, such as tape diagrams, double line diagrams, ratio tables, a coordinate plane, and equations. They then use judgment in selecting appropriate tools as they solve ratio and rate problems.

MP.6 Attend to precision. Students define and distinguish between ratio, the value of a ratio, a unit rate, a rate unit, and a rate. Students use precise language and symbols to describe ratios and rates. Students learn and apply the precise definition of percent.

MP.7 Look for and make use of structure. Students recognize the structure of equivalent ratios in solving word problems using tape diagrams. Students identify the structure of a ratio table and use it to find missing values in the table. Students make use of the structure of division and ratios to model 5 miles/2 hours as a quantity 2.5 mph.

“Unwrapped” Standards	
● **6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.** ● **6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.** ● **6.RP.3Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.**	
Concepts -What Students Need to Know(Context)	**Skills (What Students Need to Be Able to Do) Depth of Knowledge Level(DOK)**
Concept of a ratio Ratio language(to describe a ratio relationship between two quantities) Concept of a unit rate a/b Rate language(context of a ration relationship) Ratio and rate reasoning(to solve real-world and mathematical problems)	Understand(2) Use(3) Understand(2) Use(3) Use(3)

Essential Questions	**Big ideas**
● What is the difference between part-to-whole ratios and part-to-part ratios? ● How are ratios and fractions alike and different? ● How are measurement conversions related to ratios? ● How are a ratio and rate related?	Mathematical relationships among numbers can be represented, compared and communication.

Assessments See Appendix B		
Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments

***Assessment Summary**

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.RP.A.1, 6.RP.A.3 (Stem Only), 6.RP.A.3a
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	6.RP.A.1, 6.RP.A.2, 6.RP.A.3

Performance Assessment (*To be completed by grade level team)
Overview:Sharing Costs Equitably: Traveling to School(See Appendix A) **Engaging Scenario: Students will be challenged by discovering if they can help their parents/guardians save money on gas cost, they will receive the savings in a reward to an amusement park of their choice.**
Engaging Learning Experiences
Task 1 :Calculate costs to worksheet S-1. Task 2:Partners will collorate by an analysis of sample responses on worksheet S-2, S-3, and S-4. Task 3:Small group of students will create a poster that proves their responses is the best. Task 4:Individually students will reflect and critique problem responses on worksheet S-5.

Instructional Resources

See Appendix C for Three Sample Lessons

Useful Websites:

Engage NY Curriculum overview and guiding documents:
https://www.engageny.org/

Engage NY Grade 6 Resources:
https://www.engageny.org/resource/grade-6-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 6th Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/6th.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Suggested Tools and Representations

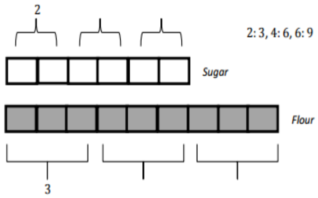
Tape Diagrams (See example below.)

Double Number Line Diagrams (See example below.)

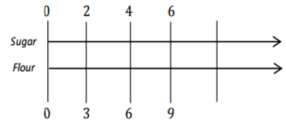
Ratio Tables (See example below.)

Coordinate Plane (See example below.) 4 These are terms and symbols students have seen previously. Representing Equivalent Ratios for a cake recipe that uses 2 cups of sugar for every 3 cups of flour Coordinate Plane 0 2 4 6 0 3 6 9

Tape Diagram



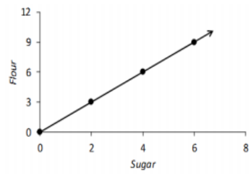
Double Number Line



Ratio Table

Sugar	Flour
2	3
4	6
6	9

Coordinate Plane



Instructional Strategies	Meeting the Needs of All Students
21st Century Skills ● Critical thinking and problem solving ● Collaboration and leadership ● Agility and adaptability ● Initiative and entrepreneurialism ● Effective oral and written communication ● Accessing and analyzing information ● Curiosity and imagination **Marzano's Nine Instructional Strategies for Effective Teaching and Learning** 1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way 2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words 3. Reinforcing Effort and Providing Recognition: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. 4. Homework and Practice: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. 5. Nonlinguistic Representations: has recently been proven to stimulate and increase brain activity. 6. Cooperative Learning: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.	**Meeting the Needs of All Students** **The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students.** **Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations.** **Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.** **It It is important to note that the scaffold/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.** **Scaffolds for Students with Disabilities** **Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text**

7. Setting Objectives and Providing Feedback: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied. 8. Generating and Testing Hypotheses: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. 9. Cues, Questions, and Advanced Organizers: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	**boxes at appropriate points, demonstrating how and when they might be used.** **Provide Multiple Means of Representation** · **Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.** · **Clarify, compare, and make connections to math words in discussion, particularly during and after practice.** · **Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.** · **Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"** · **Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:** · **Enlarge sprint print for visually impaired learners.** · **Use student boards to work on one calculation at a time.** · **Invest in or make math picture dictionaries or word walls.** **Provide Multiple Means of Action and Expression** · **Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.** · **Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.**

	· Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting." · Adjust wait time for interpreters of deaf and hard-of-hearing students. · Select numbers and tasks that are "just right" for learners. · Model each step of the algorithm before students begin. · Give students a chance to practice the next day's sprint beforehand. (At home, for example.) · Give students a few extra minutes to process the information before giving the signal to respond. · Assess by multiple means, including "show and tell" rather than written. · Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" · Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" · Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. Provide Multiple Means of Engagement · Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. · Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use nonverbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. · Teach in small chunks so students get a lot of practice with one step at a time. · Know, use, and make the most of Deaf culture and sign language. · Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up

	first/Make a bundle if you can!” · Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. · Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. · Follow predictable routines to allow students to focus on content rather than behavior. · Allow “everyday” and first language to express math understanding. · Re-teach the same concept with a variety of fluency games. · Allow students to lead group and pair-share activities. · Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
Ratio (A pair of nonnegative numbers, A:B, where both are not zero, and that are used to indicate that there is a relationship between two quantities such that when there are A units of one quantity, there are B units of the second quantity.) **Rate** (A rate indicates, for a proportional relationship between two quantities, how many units of one quantity there are for every 1 unit of the second quantity. For a ratio of A: B between two quantities, the rate is A/B units of the first quantity per unit of the second quantity.) **Unit Rate** (The numeric value of the rate, e.g., in the rate 2.5 mph, the unit rate is 2.5.) **Value of a Ratio** (For the ratio A:B: , the value of the ratio is the quotient A/B.) **Equivalent Ratios** (Ratios that have the same value.) Percent (Percent of a quantity is a rate per 100.) **Associated Ratios** (e.g., if a popular shade of purple is made by mixing 2 cups of blue paint for every 3 cups of red paint, not only can we say that the ratio of blue paint to red paint in the mixture is 2: 3, but we can discuss associated ratios such as the ratio of cups of red paint to cups of blue paint, the ratio of cups of blue paint to total cups of purple paint, the ratio of cups of red paint to total cups of purple paint, etc.) **Double Number Line** (See example under Suggested Tools and Representations.) Ratio Table (A table listing pairs of numbers that form equivalent ratios; see example under	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. “I do, we do, you do.” ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. ● Model and post conversation “starters,” such as: “I agree because...” “Can you ● explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships

Suggested Tools and Representations.)	diagram?' ● Encourage students to explain their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?" "Can you propose an alternative...?" "How would you evaluate...?" "What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds).

		● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Grade 6 • Module 2

Arithmetic Operations Including Division of Fractions

OVERVIEW

In Module 1, students used their existing understanding of multiplication and division as they began their study of ratios and rates. In Module 2, students complete their understanding of the four operations as they study division of whole numbers, division by a fraction, and operations on multi-digit decimals. This expanded understanding serves to complete their study of the four operations with positive rational numbers, thereby preparing students for understanding, locating, and ordering negative rational numbers (Module 3) and algebraic expressions (Module 4).

In Topic A, students extend their previous understanding of multiplication and division to divide fractions by fractions. They construct division stories and solve word problems involving division of fractions (**6.NS.A.1**). Through the context of word problems, students understand and use partitive division of fractions to determine how much is in each group. They explore real-life situations that require them to ask, "How much is one share?" and "What part of the unit is that share?" Students use measurement to determine quotients of fractions. They are presented conceptual problems where they determine that the quotient represents how many of the divisor is in the dividend. For example, students understand that $\frac{6\text{ cm}}{2\text{ cm}}$ derives a quotient of 3 because 2 divides into 6 three times. They apply this method to quotients of fractions, understanding $\frac{6}{7} \div \frac{2}{7} = \frac{6\text{ sevenths}}{2\text{ sevenths}} = 3$ because, again, 2 divides into 6 three times. Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. Using this relationship, students create equations and formulas to represent and solve problems. Later in the module, students learn the direct correlation of division of fractions to division of decimals along with the application of this concept.

Prior to division of decimals, students will revisit all decimal operations in Topic B. Students have had extensive experience with decimal operations to the hundredths and thousandths (**5.NBT.B.7**), which prepares them to easily compute with more decimal places. Students begin by relating the first lesson in this topic to the last lesson in Topic A, which focused on mixed numbers. They find that sums and differences of large mixed numbers can sometimes be more efficiently determined by first converting the number to a decimal and then applying the standard algorithms (**6.NS.B.3**). They use estimation to justify their answers.

Within decimal multiplication, students begin to practice the distributive property. Students use arrays and partial products to understand and apply the distributive property as they solve multiplication problems involving decimals. By gaining fluency in the distributive property throughout this module and the next, students will be proficient in applying the distributive property in Module 4 (**6.EE.A.3**). Estimation and place value enable students to determine the placement of the decimal point in products and recognize that the size of a product is relative to each factor. Students learn to use connections between fraction multiplication and decimal multiplication.

In Grades 4 and 5, students used concrete models, pictorial representations, and properties to divide whole numbers (**4.NBT.B.6**, **5.NBT.B.6**). They became efficient in applying the standard algorithm for long division. Students broke dividends apart into like base-ten units, applying the distributive property to find quotients place by place. In Topic C, students connect estimation to place value and determine that the standard algorithm is simply a tally system arranged in place value columns (**6.NS.B.2**). Students understand that when they "bring down" the next digit in the algorithm, they are essentially distributing, recording, and shifting to the next place value. They understand that the steps in the algorithm continually provide better approximations to the answer. Students further their understanding of division as they develop fluency in the use of the standard algorithm to divide multi-digit decimals (**6.NS.B.3**). They make connections to division of fractions and rely on mental math strategies to implement the division algorithm when finding the quotients of decimals.

In the final topic, students think logically about multiplicative arithmetic. In Topic D, students apply odd and even number properties and divisibility rules to find factors and multiples. They extend this application to consider common factors and multiples and find greatest common factors and least common multiples. Students explore and discover that Euclid's Algorithm is a more efficient way to find the greatest common factor of larger numbers and see that Euclid's Algorithm is based on long division.

The module comprises 19 lessons; six days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Rigorous Curriculum Design Template

Unit Two-Arithmetic Operations Including Dividing by a Fraction

Subject:Math

Grade/Course: Grade 6

Pacing:25 days

Unit of Study: Arithmetic Operations Including Dividing by a Fraction

Priority Standards:

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Foundational Standards:

Gain familiarity with factors and multiples.

4.OA.B.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. Understand the place value system. **5.NBT.A.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by fractions.2 a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.

Math Practice Standards:

MP.1 Make sense of problems and persevere in solving them. Students use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, “What is this problem asking me to find?” For instance, when determining the quotient of fractions, students ask themselves how many sets or groups of the divisor is in the dividend. That quantity is the quotient of the problem. They solve simpler problems to gain insight into the solution. They will confirm, for example, that 10 ÷ 2 can be found by determining how many groups of two are in ten. They will apply that strategy to the division of fractions. Students may use pictorial representations such as area models, array models, number lines, and drawings to conceptualize and solve problems.

MP.2 Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problems. They understand “how many” as it pertains to the divisor in a quotient of fractions problem. They understand and use connections between divisibility and the greatest common factor to apply the distributive property. Students consider units and labels for numbers in contextual problems and consistently refer to what the labels represent to make sense in the problem. Students rely on estimation and properties of operations to justify the reason for their answers when manipulating decimal numbers and their operations. Students reason abstractly when applying place value and fraction sense when determining the placement of a decimal point.

MP.6 Attend to Precision. Students use precise language and place value when adding, subtracting, multiplying, and dividing by multi-digit decimal numbers. Students read decimal numbers using place value. For example, 326.31 is read as three hundred twentysix and thirty-one hundredths. Students calculate sums, differences, products, and quotients of decimal numbers with a degree of precision appropriate to the problem context.

MP.7 Look for and make use of structure. Students find patterns and connections when multiplying and dividing multi-digit decimals. For instance, they use place value to recognize that the quotient of 22.5 ÷ 0.15 is the same as the quotient of 2250 ÷ 15. In the example 36 + 48 = 12(3 + 4), students recognize that when expressing the sum of two whole numbers using the distributive property, the number 12 represents the greatest common factor of 36 and 48 and that 36 and 48 are both multiples of 12. When dividing fractions, students recognize and make use of a related multiplication problem or create a number line and use skip counting to determine the number of times the divisor is added to obtain the dividend. Students use the familiar structure of long division to find the greatest common factor in another way, specifically the Euclidean Algorithm.

MP.8 Look for and express regularity in repeated reasoning. Students determine reasonable answers to problems involving operations with decimals. Estimation skills and compatible numbers are used. For instance, when 24.385 is divided by 3.91, students determine that the answer will be close to the quotient of 24 ÷ 4, which equals 6. Students discover, relate, and apply strategies when problem-solving, such as the use of the distributive property to solve a multiplication problem involving fractions and/or decimals (e.g., 350 × 1.8 = 350(1 + 0.8) = 350 + 280 = 630). When dividing fractions, students may use the following reasoning: Since 2/7 + 2/7 + 2/7 = 6/7, then 6/7 ÷ 2/7 = 3, so I can solve fraction division problems by first getting common denominators and then solving the division problem created by the numerators. Students understand the long-division algorithm and the continual breakdown of the dividend into different place value units. Further, students use those repeated calculations and reasoning to determine the greatest common factor of two numbers using the Euclidean Algorithm.

“Unwrapped” Standards

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.	
Concepts -What Students Need to Know(Context)	**Skills (What Students Need to Be Able to Do) Depth of Knowledge Level (DOK)**
quotients of fractions word problems involving division of fractions by fractions	Interpret (2) Compute(2) Solve(2)

Essential Questions	**Big ideas**
• How can fraction strips and bar models be used to make sense of multiplication and division of fractions problems? • How can you use estimation to help make sense of problems involving multiplication and division of fractions? • How do you know which fraction is the divisor and which is the dividend in a word problem?	Dividing fractions should be thought of as “fair sharing” just like dividing whole numbers.

Assessments		
Common Formative	Progress Monitoring Checks	Common Formative Mid and or Post-Assessments

Pre-Assessments	– “Dipsticks”	
Exit tickets for pre-assessment of each lesson.	• **Application problems** • **Exploratory Challenge** • **Problem set**	**Exit Ticket** **Mid-Module Assessment Task*** **End-of-Module Assessment Task*** ***See Table Below.**

***Assessment Summary**

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	**After Topic B**	**Constructed response with rubric**	**6.NS.A.1, 6.NS.B.3**
End-of-Module Assessment Task	**After Topic D**	**Constructed response with rubric**	**6.NS.A.1, 6.NS.B.2, 6.NS.B.3, 6.NS.B.4**

Performance Assessment (*To be completed by grade level team)
Overview:
Engaging Learning Experiences
Task 1: Task 2: Task 3: Task 4:

Instructional Resources

Useful Websites:
Engage NY Curriculum overview and guiding documents:
https://www.engageny.org/

Engage NY Grade 6 Resources:
https://www.engageny.org/resource/grade-6-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 6th Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/6th.pdf

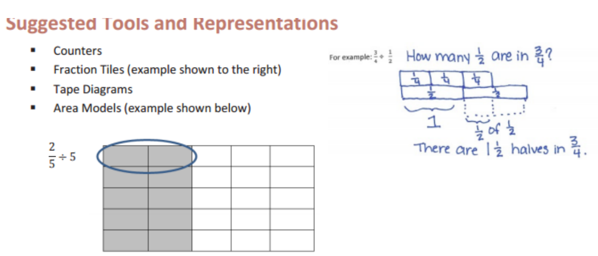
Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library



Instructional Strategies	Meeting the Needs of All Students

<u>21st Century Skills</u> ● Critical thinking and problem solving ● Collaboration and leadership ● Agility and adaptability ● Initiative and entrepreneurialism ● Effective oral and written communication ● Accessing and analyzing information ● Curiosity and imagination **<u>Marzano's Nine Instructional Strategies for Effective Teaching and Learning</u>** 1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way 2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words 3. Reinforcing Effort and Providing Recognition: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. 4. Homework and Practice: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. 5. Nonlinguistic Representations: has recently been proven to stimulate and increase brain activity. 6. Cooperative Learning: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner. 7. Setting Objectives and Providing Feedback: provide students with a direction. Objectives should not be too specific and should be adaptable to students’	**Meeting the Needs of All Students** **The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students.** **Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations.** **Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.** **It It is important to note that the scaffold/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.** **Scaffolds for Students with Disabilities** **Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.**

individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied. 8. Generating and Testing Hypotheses: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. 9. Cues, Questions, and Advanced Organizers: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	**Provide Multiple Means of Representation** **· Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.** **· Clarify, compare, and make connections to math words in discussion, particularly during and after practice.** **· Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.** **· Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"** **· Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:** **· Enlarge sprint print for visually impaired learners.** **· Use student boards to work on one calculation at a time.** **· Invest in or make math picture dictionaries or word walls.** **Provide Multiple Means of Action and Expression** **· Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.** **· Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.** **· Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the**

	sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting." **· Adjust wait time for interpreters of deaf and hard-of-hearing students.** **· Select numbers and tasks that are "just right" for learners.** **· Model each step of the algorithm before students begin.** **· Give students a chance to practice the next day's sprint beforehand. (At home, for example.)** **· Give students a few extra minutes to process the information before giving the signal to respond.** **· Assess by multiple means, including "show and tell" rather than written.** **· Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"** **· Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"** **· Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.** **Provide Multiple Means of Engagement** **· Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.** **· Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.** **· Teach in small chunks so students get a lot of practice with one step at a time.** **· Know, use, and make the most of Deaf culture and sign language.** **· Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"** **· Point to visuals and captions while speaking, using**

	your hands to clearly indicate the image that corresponds to your words. · Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. · Follow predictable routines to allow students to focus on content rather than behavior. · Allow “everyday” and first language to express math understanding. · Re-teach the same concept with a variety of fluency games. · Allow students to lead group and pair-share activities. · Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

Greatest Common Factor (The largest positive integer that divides into two or more integers without a remainder; the GCF of 24 and 36 is 12 because when all of the factors of 24 and 36 are listed, the largest factor they share is 12.) **Least Common Multiple** (The smallest positive integer that is divisible by two or more given integers without a remainder; the LCM of 4 and 6 is 12 because when the multiples of 4 and 6 are listed, the smallest or first multiple they share is 12.) **Multiplicative Inverses** (Two numbers whose product is 1 are multiplicative inverses of one another. For example, 3 4 and 4 3 are multiplicative inverses of one another because 3 4 × 4 3 = 4 3 × 3 4 = 1. Multiplicative inverses do not always have to be the reciprocal. For example 1 5 and 10 2 both have a product of 1, which makes them multiplicative inverses.)	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. "I do, we do, you do." ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' ● Encourage students to explain	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. ● Model and post conversation "starters," such as: "I agree because..." "Can you ● explain how you solved it?""I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and

	their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance

		and support

Grade 6 • Module 3

Rational Numbers

OVERVIEW

Students are familiar with the number line and determining the location of positive fractions, decimals, and whole numbers from previous grades. Students extend the number line (both horizontally and vertically) in Module 3 to include the opposites of whole numbers. The number line serves as a model to relate integers and other rational numbers to statements of order in real-world contexts. In this module's final topic, the number line model is extended to two-dimensions, as students use the coordinate plane to model and solve real-world problems involving rational numbers.

Topic A focuses on the development of the number line in the opposite direction (to the left or below zero). Students use positive integers to locate negative integers, understanding that a number and its opposite will be on opposite sides of zero and that both lie the same distance from zero. Students represent the opposite of a positive number as a negative number and vice-versa. Students realize that zero is its own opposite and that the opposite of the opposite of a number is actually the number itself (**6.NS.C.6a**). They use positive and negative numbers to represent real-world quantities, such as -50 to represent a $\$50$ debt or 50 to represent a $\$50$ deposit into a savings account (**6.NS.C.5**). Topic A concludes with students furthering their understanding of signed numbers to include the rational numbers. Students recognize that finding the opposite of any rational number is the same as finding an integer's opposite (**6.NS.C.6c**) and that two rational numbers that lie on the same side of zero will have the same sign, while those that lie on opposites sides of zero will have opposite signs.

In Topic B, students apply their understanding of a rational number's position on the number line (**6.NS.C.6c**) to order rational numbers. Students understand that when using a conventional horizontal number line, the numbers increase as you move along the line to the right and decrease as you move to the left. They recognize that if a and b are rational numbers and $a < b$, then it must be true that $-a > -b$. Students compare rational numbers using inequality symbols and words to state the relationship between two or more rational numbers. They describe the relationship between rational numbers in real-world situations and with respect to numbers' positions on the number line (**6.NS.C.7a**, **6.NS.C.7b**). For instance, students explain that $-10°\text{F}$ is warmer than $-11°\text{F}$ because -10 is to the right (or above) -11 on a number line and write

$-10°\text{F} > -11°\text{F}$. Students use the concept of absolute value and its notation to show a number's distance from zero on the number line and recognize that opposite numbers have the same absolute value (**6.NS.C.7c**). In a real-world scenario, students interpret absolute value as magnitude for a positive or negative quantity. They apply their understanding of order and absolute value to determine that, for instance, a checking account balance that is less than -25 dollars represents a debt of more than $\$25$ (**6.NS.C.7d**).

In Topic C, students extend their understanding of the ordering of rational numbers in one dimension (on a number line) to the two-dimensional space of the coordinate plane. They construct the plane's vertical and horizontal axes, discovering the relationship between the four quadrants and the signs of the coordinates of points that lie in each quadrant (**6.NS.C.6b**, **6.NS.C.6c**). Students build upon their foundational understanding

Math Unit 3

Rigorous Curriculum Design Template

Unit Three-Rational Numbers

Subject:Math

Grade/Course: Grade 6

Pacing: 25 days

Unit of Study: Rational Numbers

Priority Standards:

6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

6.NS.7 Understand ordering and absolute value of rational numbers.

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Foundational Standards:

Develop understanding of fractions as numbers.
3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.**a.** Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. Graph points on the coordinate plane to solve real-world and mathematical problems.
5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
5.G.A.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Math Practice Standards:

MP.2 Reason abstractly and quantitatively. Students read a word problem involving integers, draw a number line or coordinate plane model, and write about their conclusions. They understand the meaning of quantities as they relate to the real world. For instance, a loss of 14 yards in a football game can be represented by −14, and a distance of 25 feet below sea level is greater than a distance of 5 feet above sea level because $|-25| > |5|$. Students decontextualize word problems related to distance by creating number lines and coordinate plane models. In doing so, they count the number of units between endpoints and use the concept of absolute value to justify their answer. For instance, when given the coordinate (2, 6), students determine that the point (2, −6) would be the same distance from the x-axis but in the opposite direction because both points have the same x-coordinate and their y-coordinates (6 and −6) have the same absolute value.

MP.4 Model with mathematics. Students use vertical and horizontal number lines to visualize integers and better understand their connection to whole numbers. They divide number line intervals into sub-intervals of tenths to determine the correct placement of rational numbers. Students may represent a decimal as a fraction or a fraction as a decimal to better understand its relationship to other rational numbers to which it is being compared. To explain the meaning of a quantity in a real-life situation (involving elevation, temperature, or direction), students may draw a diagram and/or number line to illustrate the location of the quantity in relation to zero or an established level that represents zero in that situation.

MP.6 Attend to precision. In representing signed numbers on a number line or as a quantity, students pay close attention to the direction and sign of a number. They realize that a negative number must lie to the left of zero on a horizontal number line or below zero on a vertical number line. They recognize that the way they represent their answer depends on the phrasing of a question and context of a word problem. For instance, a question that asks a student: "How many feet below sea level is the diver?" would require the answer to be stated as a positive number. Whereas, a question that is phrased: "Which integer would represent 40 feet below sea level?" would require the answer to be written as −40.

MP.7 Look for and make use of structure. Students understand the placement of negative numbers on a number line by observing the patterns that exist between negative and positive numbers with respect to zero. They recognize that two numbers are opposites if they are the same distance from zero and that zero is its own opposite. Students extend their understanding of the number line's structure to the coordinate plane to determine a point's location. They recognize the relationship between the signs of a point's coordinates and the quadrant in which the point lies.

<table>
<tr><td colspan="2">"Unwrapped" Standards
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
6.NS.7 Understand ordering and absolute value of rational numbers.
6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with same first coordinate or the same second coordinate.</td></tr>
<tr><td>Concepts -What Students Need to Know(Context)</td><td>Skills -What Students Need to Be Able to Do
Depth of Knowledge Level (DOK)</td></tr>
</table>

Positive and negative numbers Numbers together to describe quantities (having directions or values) Positive and negative numbers to represent quantities(in real-world contexts, explaining the meaning of 0 in each situation.) Rational number(as a point on the number line.) Number line diagrams and coordinate axes familiar(focusing on points on the line and the plane with negative number coordinates.) Ordering and absolute value of rational numbers. By graphing points in all four quadrants(using real-world and mathematical problems. Use of coordinates and absolute value to find distances between points with same first coordinate or the same second coordinate.	Understand(3) Use(3) Use(3) Understand(3) Extend(3) Understand(3) Solve(1) Include(1)

Essential Questions	**Big ideas**

• What is absolute value? • What are the similarities and differences between positive and negative numbers? • How do you know if one integer is smaller or larger than another? • How are inequalities used to understand integers? • How do you add and subtract integers?	All numbers have an opposite. Inequalities are used to express the relationship between two rational numbers.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Exit tickets for pre-assessment of each lesson.	• **Application problems** • **Exploratory Challenge** • **Problem set**	**Exit Ticket** **Mid-Module Assessment Task*** **End-of-Module Assessment Task*** ***See Table Below.**

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.NS.C.5, 6.NS.C.6a, 6.NS.C.6c, 6.NS.C.7
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	6.NS.C.5, 6.NS.C.6a, 6.NS.C.6c, 6.NS.C.7, 6.NS.C.8

Performance Assessment (*To be completed by grade level team)

Overview:
Engaging Learning Experiences
Task 1: Task 2: Task 3: Task 4:

Instructional Resources
Useful Websites: Engage NY Curriculum overview and guiding documents: https://www.engageny.org/ Engage NY Grade 6 Resources: https://www.engageny.org/resource/grade-6-mathematics Eureka Math Module PDFs: http://greatminds.net/maps/math/module-pdfs North Carolina 6th Grade Standards Unpacked: http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/6th.pdf Illustrative Mathematics – problems and tasks by grade and standard https://www.illustrativemathematics.org/ NCTM Illuminations – problems, tasks and interactives by grade and standard http://illuminations.nctm.org/Default.aspx Inside Mathematics – Problems of the Month and Performance Assessment tasks http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks https://learnzillion.com/resources/17132 SBAC Digital Library **Suggested Tools and Representations** Horizontal and Vertical Number Lines Coordinate Plane

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills ● Critical thinking and problem solving ● Collaboration and leadership ● Agility and adaptability ● Initiative and entrepreneurialism ● Effective oral and written communication ● Accessing and analyzing information ● Curiosity and imagination **Marzano's Nine Instructional Strategies for Effective Teaching and Learning** 1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way 2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words 3. Reinforcing Effort and Providing Recognition:	**Meeting the Needs of All Students** **The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students.** **Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other**

showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. 4. Homework and Practice: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. 5. Nonlinguistic Representations: has recently been proven to stimulate and increase brain activity. 6. Cooperative Learning: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner. 7. Setting Objectives and Providing Feedback: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied. 8. Generating and Testing Hypotheses: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. 9. Cues, Questions, and Advanced Organizers: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	**students and overlapping populations.** **Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.** **It It is important to note that the scaffold/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.** **Scaffolds for Students with Disabilities** **Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.** **Provide Multiple Means of Representation** · **Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.** · **Clarify, compare, and make connections to math words in discussion, particularly during and after practice.** · **Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.** · **Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post**

	conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" · Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with: · Enlarge sprint print for visually impaired learners. · Use student boards to work on one calculation at a time. · Invest in or make math picture dictionaries or word walls. Provide Multiple Means of Action and Expression · Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. · Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones. · Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting." · Adjust wait time for interpreters of deaf and hard-of-hearing students. · Select numbers and tasks that are "just right" for learners. · Model each step of the algorithm before students begin. · Give students a chance to practice the next day's sprint beforehand. (At home, for example.) · Give students a few extra minutes to process the information before giving the signal to respond. · Assess by multiple means, including "show and tell" rather than written. · Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions

	for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" **· Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"** **· Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.** **Provide Multiple Means of Engagement** **· Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.** **· Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use nonverbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.** **· Teach in small chunks so students get a lot of practice with one step at a time.** **· Know, use, and make the most of Deaf culture and sign language.** **· Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"** **· Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.** **· Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.** **· Follow predictable routines to allow students to focus on content rather than behavior.** **· Allow "everyday" and first language to express math understanding.** **· Re-teach the same concept with a variety of fluency games.** **· Allow students to lead group and pair-share activities.** **· Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding**

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
Absolute Value (The absolute value of a number is the distance between the number and zero on the number line. For example, |3| = 3, | − 4| = 4, etc.) **Charge** (A charge is the amount of money a person must pay, as in a charge to an account, or a fee charged.) **Credit** (A credit is a decrease in an expense, as in money credited to an account. For instance, when a deposit is made into a checking account, the money is credited to the account. A credit is the opposite of a debit.) **Debit** (A debit is an increase in an expense or money paid out of an account. For instance, using a debit card to make a purchase will result in an expense, and money will be deducted from the related bank account.) **Deposit** (A deposit is the act of putting money into a bank account.) **Elevation** (Elevation is the height of a person, place, or thing above a certain reference level.) Integers (The numbers ... ,−3, −2, −1, 0, 1, 2, 3, … are	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. “I do, we do, you do.” ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action</u>**	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. ● Model and post conversation “starters,” such as: “I agree because...” “Can you ● explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.

integers on the number line.) **Magnitude** (The magnitude is the absolute value of a measurement, given the measurement of a positive or negative quantity.) **Negative Number** (A negative number is a number less than zero.) **Opposite** (In a position on the other side; for example, negative numbers are the opposite direction from zero as positive numbers.) **Positive Number** (A positive number is a number greater than zero.) **Quadrants** (The four sections of the coordinate plane formed by the intersection of the axes are called quadrants.) **Rational Number** (A rational number is a fraction or the opposite of a fraction on the number line.) **Withdraw** (To withdraw is to take away; for example, to take money out of a bank account.) **Withdrawal** (A withdrawal is the act of taking money out of a bank account.)	**and Expression** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' ● Encourage students to explain their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **Provide Multiple Means of Engagement** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to	● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **Provide Multiple Means of Engagement** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.

	focus on next.	● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Grade 6 • Module 4

Expressions and Equations

OVERVIEW

In Module 4, students extend their arithmetic work to include using letters to represent numbers. Students understand that letters are simply "stand-ins" for numbers and that arithmetic is carried out exactly as it is with numbers. Students explore operations in terms of verbal expressions and determine that arithmetic properties hold true with expressions because nothing has changed—they are still doing arithmetic with numbers. Students determine that letters are used to represent specific but unknown numbers and are used to make statements or identities that are true for all numbers or a range of numbers. Students understand the importance of specifying units when defining letters. Students say, "Let $K =$ Karolyn's weight in pounds" instead of "Let $K =$ Karolyn's weight" because weight cannot be a specific number until it is associated with a unit, such as pounds, ounces, grams, etc. They also determine that it is inaccurate to define K as Karolyn because Karolyn is not a number. Students conclude that in word problems, each letter (or variable) represents a number, and its meaning is clearly stated.

To begin this module, students are introduced to important identities that will be useful in solving equations and developing proficiency with solving problems algebraically. In Topic A, students understand the relationships of operations and use them to generate equivalent expressions (**6.EE.A.3**). By this time, students have had ample experience with the four operations since they have worked with them from kindergarten through Grade 5 (**1.OA.B.3**, **3.OA.B.5**). The topic opens with the opportunity to clarify those relationships, providing students with the knowledge to build and evaluate identities that are important for solving equations. In this topic, students discover and work with the following identities: $w - x + x = w$, $w + x - x = w$, $a \div b \cdot b = a$, $a \cdot b \div b = a$ (when $b \neq 0$), and $3x = x + x + x$. Students will also discover that if $12 \div x = 4$, then $12 - x - x - x - x = 0$.

In Topic B, students experience special notations of operations. They determine that $3x = x + x + x$ is not the same as x^3, which is $x \cdot x \cdot x$. Applying their prior knowledge from Grade 5, where whole number exponents were used to express powers of ten (**5.NBT.A.2**), students examine exponents and carry out the order of operations, including exponents. Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents (**6.EE.A.1**).

Students represent letters with numbers and numbers with letters in Topic C. In past grades, students discovered properties of operations through example (**1.OA.B.3**, **3.OA.B.5**). Now, they use letters to represent numbers in order to write the properties precisely. Students realize that nothing has changed because the properties still remain statements about numbers. They are not properties of letters, nor are they new rules introduced for the first time. Now, students can extend arithmetic properties from manipulating numbers to manipulating expressions. In particular, they develop the following identities: $a \cdot b = b \cdot a$, $a + b = b + a$, $g \cdot 1 = g$, $g + 0 = g$, $g \div 1 = g$, $g \div g = 1$, and $1 \div g = \frac{1}{g}$. Students understand that a letter in an expression represents a number. When that number replaces that letter, the expression can be evaluated to one number. Similarly, they understand that a letter in an expression can represent a number. When that number is replaced by a letter, an expression is stated (**6.EE.A.2**).

In Topic D, students become comfortable with new notations of multiplication and division and recognize their equivalence to the familiar notations of the prior grades. The expression $2 \times b$ is exactly the same as $2 \cdot b$, and both are exactly the same as $2b$. Similarly, $6 \div 2$ is exactly the same as $\frac{6}{2}$. These new conventions are practiced to automaticity, both with and without variables. Students extend their knowledge of GCF and the distributive property from Module 2 to expand, factor, and distribute expressions using new notation (**6.NS.B.4**). In particular, students are introduced to factoring and distributing as algebraic identities. These include: $a + a = 2 \cdot a = 2a$, $(a + b) + (a + b) = 2 \cdot (a + b) = 2(a + b) = 2a + 2b$, and $a \div b = \frac{a}{b}$.

In Topic E, students express operations in algebraic form. They read and write expressions in which letters stand for and represent numbers (**6.EE.A.2**). They also learn to use the correct terminology for operation symbols when reading expressions. For example, the expression $\frac{3}{2x-4}$ is read as "the quotient of three and the difference of twice a number and four." Similarly, students write algebraic expressions that record operations with numbers and letters that stand for numbers. Students determine that $3a + b$ can represent the story: "Martina tripled her money and added it to her sister's money" (**6.EE.A.2b**).

Students write and evaluate expressions and formulas in Topic F. They use variables to write expressions and evaluate those expressions when given the value of the variable (**6.EE.A.2**). From there, students create formulas by setting expressions equal to another variable. For example, if there are 4 bags containing c colored cubes in each bag with 3 additional cubes, students use this information to express the total number of cubes as $4c + 3$. From this expression, students develop the formula $t = 4c + 3$, where t is the total number of cubes. Once provided with a value for the amount of cubes in each bag ($c = 12$ cubes), students can evaluate the formula for t: $t = 4(12) + 3, t = 48 + 3, t = 51$. Students continue to evaluate given formulas such as the volume of a cube, $V = s^3$, given the side length, or the volume of a rectangular prism, $V = l \cdot w \cdot h$, given those dimensions (**6.EE.A.2c**).

Math Unit 4

Rigorous Curriculum Design Template

Unit 4-Expressions and Equations

Subject:Math

Grade/Course: Grade 6

Pacing:45 days

Unit of Study: Expressions and Equations

Priority Standards:

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

6.EE.3 Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

6.EE.5 Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.

6.EE.8 Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Foundational Standards:

Understand and apply properties of operations and the relationship between addition and subtraction

1.OA.B.3 Apply properties of operations as strategies to add and subtract.4 Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)Understand properties of multiplication and the relationship between multiplication and division.

3.OA.B.5 Apply properties of operations as strategies to multiply and divide.5 Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive property.) Gain familiarity with factors and multiples.

4.OA.B.4 Find all factors for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. Geometric measurement: understand concepts of angle and measure angles.

4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1 360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through □□ one-degree angles is said to have an angle measure of □□ degrees.

4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. Write and interpret numerical expressions

. **5.OA.A**.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product. Analyze patterns and relationships.

5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. Understand the place value system. **5.NBT.A.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. Graph points on the coordinate plane to solve real-world and mathematical problems

5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. Understand ratio concepts and use ratio reasoning to solve problems. 6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? Compute fluently with multi-digit numbers and find common factors and multiples. 6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).

Math Practice Standards:

MP.2 Reason abstractly and quantitatively. Students connect symbols to their numerical referents. They understand exponential notation as repeated multiplication of the base number. Students realize that 32 is represented as 3 × 3, with a product of 9, and explain how 32 differs from 3 × 2, where the product is 6. Students determine the meaning of a variable within a real-life context. They write equations and inequalities to represent mathematical situations. Students manipulate equations using the properties so that the meaning of the symbols and variables can be more closely related to the real-world context. For example, given the expression 12x represents how many beads are available to make necklaces, students rewrite 12x as 4x + 4x + 4x when trying to show the portion each person gets if there are three people, or rewrite 12x as 6x + 6x if there are two people sharing. Students recognize that these

expressions are equivalent. Students can also use equivalent expressions to express the area of rectangles and to calculate the dimensions of a rectangle when the area is given. Also, students make connections between a table of ordered pairs of numbers and the graph of those data.

MP.6 Attend to precision. Students are precise in defining variables. They understand that a variable represents one number. For example, students understand that in the equation a + 4 = 12, the variable a can only represent one number to make the equation true. That number is 8, so a = 8. When variables are represented in a real-world problem, students precisely define the variables. In the equation 2w = 18, students define the variable as weight in pounds (or some other unit) rather than just weight. Students are precise in using operation symbols and can connect between previously learned symbols and new symbols (3 × 2 can be represented with parentheses 3(2) or with the multiplication dot 3 · 2; similarly 3 ÷ 2 is also represented with the fraction bar 3 2). In addition, students use appropriate vocabulary and terminology when communicating about expressions, equations, and inequalities. For example, students write expressions, equations, and inequalities from verbal or written descriptions. "A number increased by 7 is equal to 11" can be written as x + 7 = 11. Students refer to 7y as a term or expression, whereas 7y = 56 is referred to as an equation.

MP.7 Look for and make use of structure. Students look for structure in expressions by deconstructing them into a sequence of operations. They make use of structure to interpret an expression's meaning in terms of the quantities represented by the variables. In addition, students make use of structure by creating equivalent expressions using properties. For example, students write 6x as x+ x + x + x + x + x, 4x + 2x, 3(2x), or other equivalent expressions. Students also make sense of algebraic solutions when solving an equation for the value of the variable through connections to bar diagrams and properties. For example, when there are two copies of a + b, this can be expressed as either (a + b) + (a + b), 2a + 2b, or 2(a + b). Students use tables and graphs to compare different expressions or equations to make decisions in real-world scenarios. These models also create structure as students gain knowledge in writing expressions and equations.

MP.8 Look for and express regularity in repeated reasoning. Students look for regularity in a repeated calculation and express it with a general formula. Students work with variable expressions while focusing more on the patterns that develop than the actual numbers that the variable represents. For example, students move from an expression such as 3 + 3 + 3 + 3 = 4 · 3 to the general form m + m + m + m = 4 · m, or 4m. Similarly, students move from expressions such as 5 · 5 · 5 · 5 = 54 to the general form m · m · m · m = m4. These are especially important when moving from the general form back to a specific value for the variable.

"Unwrapped" Standards

- **6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.**
- **6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.**
- **6.EE.3 Apply the properties of operations to generate equivalent expressions.**
- **6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).**
- **6.EE.5 Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.**
- **6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.**
- **6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.**
- **6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.**
- **6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.**

Concepts -What Students Need to Know(Context)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level (DOK)**
Numerical expressions involving whole-number exponents.	Write(1)Evaluate(4)
Expressions in which letters stand for numbers.	Write(1)Read(1)Evaluate(4)
The properties of operations to generate equivalent expressions.	Apply(4)
When two expressions are equivalent.	Identify(1)
Solving an equation or inequality as a process of answering a question(Which values from a specified set, if any, make the equation or inequality true?)	Understand(3)
Variables to represent numbers Expressions(when solving a real-world or mathematical problem A variable can represent an unknown number(any number in a specified set)	Use(1)Write(1)Understand(3)
By writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.(Real-world and mathematical problems)	Solve(2)
An inequality of the form $x>c$ or $x<c$ (representing a constraint or condition in a real-world of mathematical problem.)	Write(1)
Variables to represent two quantities(in real-world	Use(1) Write(1) Analyze(4)

problem that change in relationship to one another) An equation to express one quantity(dependent variable) another quantity(independent variable). The relationship between the dependent and independent variables(use graphs and tables to relate these to the equation)	

Essential Questions	**Big ideas**
● How are relationships represented mathematically? ● How can expressions, equations and inequalities be used to quantify, solve, model and/or analyze mathematical situations? ● What is the difference between an expression and an equation? ● What are the similarities and differences between equations and inequalities? ● How is the meaning of a variable different in an equation versus an inequality? ● What are the different ways to solve an equation or inequality? ● What does the equal sign mean in an equation? ● What does the inequality sign mean in an inequality?	Mathematical relationships can be represented as expressions, equations, and inequalities in mathematical situation. Algebra provides language through which we communicate the patterns in mathematics.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Exit tickets for pre-assessment of each lesson.	● **Application problems** ● **Exploratory Challenge** ● **Problem set**	**Exit Ticket** **Mid-Module Assessment Task*** **End-of-Module Assessment Task*** ***See Table Below.**

*Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic E	Constructed response with rubric	6.EE.A.1, 6.EE.A.2, 6.EE.A.3, 6.EE.A.4
End-of-Module Assessment Task	After Topic H	Constructed response with rubric	6.EE.A.2, 6.EE.B.5, 6.EE.B.6, 6.EE.B.7, 6.EE.B.8, 6.EE.C.9

Performance Assessment (*To be completed by grade level team)
Overview:

Engaging Learning Experiences
Task 1:
Task 2:
Task 3:
Task 4:

Instructional Resources
Useful Websites:

Engage NY Curriculum overview and guiding documents:
https://www.engageny.org/

Engage NY Grade 6 Resources:
https://www.engageny.org/resource/grade-6-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 6th Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/6th.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Suggested Tools and Representations

Bar model

Geometric figures

Protractors

Instructional Strategies	Meeting the Needs of All Students

<u>21st Century Skills</u> ● Critical thinking and problem solving ● Collaboration and leadership ● Agility and adaptability ● Initiative and entrepreneurialism ● Effective oral and written communication ● Accessing and analyzing information ● Curiosity and imagination **<u>Marzano's Nine Instructional Strategies for Effective Teaching and Learning</u>** 1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way 2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words 3. Reinforcing Effort and Providing Recognition: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. 4. Homework and Practice: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. 5. Nonlinguistic Representations: has recently been proven to stimulate and increase brain activity. 6. Cooperative Learning: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner. 7. Setting Objectives and Providing Feedback: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too	**Meeting the Needs of All Students** **The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students.** **Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations.** **Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.** **It It is important to note that the scaffold/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.** **Scaffolds for Students with Disabilities**

much positive feedback, however, the method in which you give that feedback should be varied. 8. Generating and Testing Hypotheses: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. 9. Cues, Questions, and Advanced Organizers: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	**Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.** **Provide Multiple Means of Representation** · **Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.** · **Clarify, compare, and make connections to math words in discussion, particularly during and after practice.** · **Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.** · **Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"** · **Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:** · **Enlarge sprint print for visually impaired learners.** · **Use student boards to work on one calculation at a time.** · **Invest in or make math picture dictionaries or word walls.** **Provide Multiple Means of Action and Expression** · **Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates.**

	Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. **· Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.** **· Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."** **· Adjust wait time for interpreters of deaf and hard-of-hearing students.** **· Select numbers and tasks that are "just right" for learners.** **· Model each step of the algorithm before students begin.** **· Give students a chance to practice the next day's sprint beforehand. (At home, for example.)** **· Give students a few extra minutes to process the information before giving the signal to respond.** **· Assess by multiple means, including "show and tell" rather than written.** **· Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"** **· Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"** **· Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.** **Provide Multiple Means of Engagement** **· Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.** **· Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use nonverbal signals, such as**

	“thumbs-up.” Assign a buddy or a group to clarify directions or process. **· Teach in small chunks so students get a lot of practice with one step at a time.** **· Know, use, and make the most of Deaf culture and sign language.** **· Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!”** **· Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.** **· Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently.** **· Follow predictable routines to allow students to focus on content rather than behavior.** **· Allow “everyday” and first language to express math understanding.** **· Re-teach the same concept with a variety of fluency games.** **· Allow students to lead group and pair-share activities.** **· Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding**

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

Equation (An equation is a statement of equality between two expressions.) **Equivalent Expressions** (Two simple expressions are equivalent if both evaluate to the same number for every substitution of numbers into all the letters in both expressions.) **Exponential Notation for Whole Number Exponents** (Let m be a non-zero whole number. For any number a, we define a mpower to be the product of m factors of a, i.e., $\underbrace{a\cdot a\cdot a\cdot a\ldots\cdot a}_{m\text{ times}}$. The number a is called the base, and m is called the exponent, or power of a.) **Linear Expression** (A linear expression is a product of two simple expressions where only one of the simple expressions has letters and only one letter in each term of that expression or sums and/or differences of such products.) **Simple Expression** (A simple expression is a number, a letter that represents a number, a product whose factors are either numbers or letters involving whole number exponents, or sums and/or differences of such products. Each product in a simple expression is called a term, and the evaluation of the numbers in the product is called the coefficient of the term.) **Truth Values of a Number Sentence** (A number sentence is said to be true if both numerical expressions are equivalent; it is said to be false otherwise. True and false are called truth values.)	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. "I do, we do, you do." ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' ● Encourage students to explain	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. ● Model and post conversation "starters," such as: "I agree because..." "Can you ● explain how you solved it?""I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and

	their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance

		and support

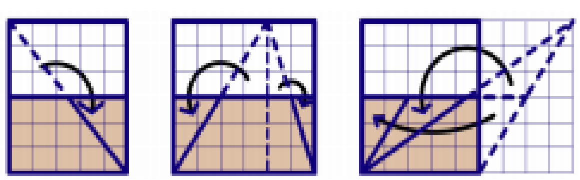
Grade 6 • Module 5

Area, Surface Area, and Volume Problems

OVERVIEW

Starting in Grade 1, students compose and decompose plane and solid figures (**1.G.A.2**). They move to spatial structuring of rectangular arrays in Grade 2 (**2.G.A.2**) and continually build upon their understanding of arrays to ultimately apply their knowledge to two- and three-dimensional figures in Grade 4 (**4.MD.A.3**) and Grade 5 (**5.MD.C.3**, **5.MD.C.5**). Students move from building arrays to using arrays to find area and eventually move to decomposing three-dimensional shapes into layers that are arrays of cubes. In this module, students utilize their previous experiences in shape composition and decomposition in order to understand and develop formulas for area, volume, and surface area.

In Topic A, students use composition and decomposition to determine the area of triangles, quadrilaterals, and other polygons. They determine that area is additive. Students learn through exploration that the area of a triangle is exactly half of the area of its corresponding rectangle. In Lesson 1, students discover through composition that the area of a parallelogram is the same as a rectangle. In Lesson 2, students compose rectangles using two copies of a right triangle. They extend their previous knowledge about the area formula for rectangles (**4.MD.A.3**) to evaluate the area of the rectangle using $A = bh$ and discover through manipulation that the area of a right triangle is exactly half that of its corresponding rectangle. In Lesson 3, students discover that any triangle may be decomposed into right triangles, and in Lesson 4, students further explore all triangles and discover through manipulation that the area of all triangles is exactly half the area of its corresponding rectangle. During this discovery process, students become aware that triangles have altitude, which is the length of the height of the triangle. The altitude is the perpendicular segment from a vertex of a triangle to the line containing the opposite side. The opposite side is called the base. Students understand that any side of the triangle can be a base, but the altitude always determines the base. They move from recognizing right triangles as categories (**4.G.A.2**) to determining that right triangles are constructed when altitudes are perpendicular and meet the base at one endpoint. Acute triangles are constructed when the altitude is perpendicular and meets within the length of the base, and obtuse triangles are constructed when the altitude is perpendicular and lies outside the length of the base. Students use this information to cut triangular pieces and rearrange them to fit exactly within one half of the corresponding rectangle to determine that the area formula for any triangle can be determined using $A = \frac{1}{2}bh$.



In Lesson 5, students apply their knowledge of the area of a triangular region, where they deconstruct parallelograms, trapezoids, and other quadrilaterals and polygons into triangles or rectangles in order to determine area. They intuitively decompose rectangles to determine the area of polygons. Topic A closes with Lesson 6 where students apply their learning from the topic to find areas of composite figures in real-life contexts, as well as to determine the area of missing regions (**6.G.A.1**).

In Module 3, students used coordinates and absolute value to find distances between points on a coordinate plane (**6.NS.C.8**). In Topic B, students extend this learning to Lessons 7 and 8 where they find edge lengths of polygons (the distance between two vertices using absolute value) and draw polygons given coordinates (**6.G.A.3**). From these drawings, students determine the area of polygons on the coordinate plane by composing and decomposing into polygons with known area formulas. In Lesson 9, students further investigate and calculate the area of polygons on the coordinate plane and also calculate the perimeter. They note that finding perimeter is simply finding the sum of the polygon's edge lengths (or finding the sum of the distances between vertices). Topic B concludes with students determining distance, perimeter, and area on the coordinate plane in real-world contexts.

In Grade 5, students recognized volume as an attribute of solid figures. They measured volume by packing right rectangular prisms with unit cubes and found that determining volume was the same as multiplying the edge lengths of the prism (**5.MD.C.3**, **5.MD.C.4**). Students extend this knowledge to Topic C where they continue packing right rectangular prisms with unit cubes; however, this time the right rectangular prism has fractional lengths (**6.G.A.2**). In Lesson 11, students decompose a one cubic unit prism in order to conceptualize finding the volume of a right rectangular prism with fractional edge lengths using unit cubes. They connect those findings to apply the formula $V = lwh$ and multiply fractional edge lengths (**5.NF.B.4**). In Lessons 12 and 13, students extend and apply the volume formula to $V = \text{The area of the base} \times \text{height}$ or simply $V = bh$, where b represents the area of the base. In Lesson 12, students explore the bases of right rectangular prisms and find the area of the base first, then multiply by the height. They determine that two formulas can be used to find the volume of a right rectangular prism. In Lesson 13, students apply both formulas to application problems. Topic C concludes with real-life application of the volume formula where students extend the notion that volume is additive (**5.MD.C.5c**) and find the volume of composite solid figures. They apply volume formulas and use their previous experience with solving equations (**6.EE.B.7**) to find missing volumes and missing dimensions.

Module 5 concludes with deconstructing the faces of solid figures to determine surface area. Students note the difference between finding the volume of right rectangular prisms and finding the surface area of such prisms. In Lesson 15, students build solid figures using nets. They note which nets compose specific solid figures and also understand when nets cannot compose a solid figure. From this knowledge, students deconstruct solid figures into nets to identify the measurement of the solids' face edges. With this knowledge from Lesson 16, students are prepared to use nets to determine the surface area of solid figures in Lesson 17. They find that adding the areas of each face of the solid will result in a combined surface area. In Lesson 18, students find that each right rectangular prism has a front, a back, a top, a bottom, and two sides. They determine that surface area is obtained by adding the areas of all the faces. They understand that the front and back of the prism have the same surface area, the top and bottom have the same surface area, and the sides have the same surface area. Thus, students develop the formula $SA = 2lw + 2lh + 2wh$ (**6.G.A.4**). To wrap up the module, students apply the surface area formula to real-life contexts and distinguish between the need to find surface area or volume within contextual situations.

Math Unit 5

Rigorous Curriculum Design Template

Unit 5-Area, Surface Area and Volume Problems

Subject:Math

Grade/Course: Grade 6

Pacing: 25 days

Unit of Study: Area, Surface Area, and Volume Problems

Priority Standards:

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers

6.EE.5 Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.

Foundational Standards:

Reason with shapes and their attributes.

1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape

.**2 2.G.A.2** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. 4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.) 5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.3 Geometric measurement: understand conceptual concepts of volume and relate volume to multiplication and to addition.

5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. 5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. Graph points on a coordinate plane to solve real-world and mathematical problems.

5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. Classify two-dimensional figures into categories based on their properties. 5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. Reason about and solve one-variable equations and inequalities.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them. Students make sense of real-world problems that involve area, volume, and surface area. One problem will involve multiple steps without breaking the problem into smaller, simpler questions. To solve surface area problems, students will have to find the area of different parts of the polygon before calculating the total area.

MP.3 Construct viable arguments and critique the reasoning of others. Students will develop different arguments as to why area formulas work for different polygons. Through this development, students may discuss and question their peers' thinking process. When students draw nets to represent right rectangular prisms, their representations may be different from their peers'. Although more than one answer may be correct, students will have an opportunity to defend their answers as well as question their peers. Students may also solve real-world problems using different methods; therefore, they may have to explain their thinking and critique their peers.

MP.4 Model with mathematics. Models will be used to demonstrate why the area formulas for different quadrilaterals are accurate. Students will use unit cubes to build right rectangular prisms and use these to calculate volume. The unit cubes will be used to model that v=lw$h$ and v=bh, where b represents the area of the base, are both accurate formulas to calculate the volume of a right rectangular prism. Students will use nets to model the process of calculating the surface area of a right rectangular prism.

MP.6 Attend to precision. Students will understand and use labels correctly throughout the module. For example, when calculating the area of a triangle, the answer will be labeled units 2 because the area is the product of two dimensions. When two different units are given within a problem, students know to use previous knowledge of conversions to make the units match before solving the problem. In multi-step problems, students solve each part of the problem separately and know when to round in order to calculate the most precise answer. Students will attend to precision of language when describing exactly how a region may be composed or decomposed to determine its area.

“Unwrapped” Standards

- 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers
- 6.EE.5 Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality.
- 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.

Concepts -What Students Need to Know(Context)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level (DOK)**
Expressions(in which letters stand for numbers)	Write(1) Read(1) Evaluate(4)
Solving an equation or inequality(Which value makes the equation or inequality true?)	Understand(3) Use(1))
Substitution to determine whether a given number makes an equation or inequality true.	
Variables to represent numbers	Use(1) Write(1)Understand(3)
Expressions when solving(real-world or mathematical problems)	
A variable can represent an unknown number(any number in a specified set)	
By writing and solving equations of the form x + P=q and px=q(p,q, and x are all nonnegative rational numbers.	Solve(2)

Essential Questions	**Big ideas**
• How can we best represent and verify geometric/algebraic relationships. • How do geometric relationships help to solve problems? • How can expressions, equations, and inequalities be used to quantify, solve, model and/or analyze mathematical situations?	Spatial sense and geometric relationships are a means to solve problems and make sense of a variety of phenomena.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Exit tickets for pre-assessment of each lesson.	• **Application problems** • **Exploratory Challenge** • **Problem set**	**Exit Ticket** **Mid-Module Assessment Task*** **End-of-Module Assessment Task*** ***See Table Below.**

*Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.G.A.1, 6.G.A.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	6.G.A.1, 6.G.A.2, 6.G.A.3, 6.G.A.4

Performance Assessment (*To be completed by grade level team)
Overview:

Engaging Learning Experiences
Task 1: Task 2: Task 3: Task 4:

Instructional Resources
Useful Websites: Engage NY Curriculum overview and guiding documents: https://www.engageny.org/ Engage NY Grade 6 Resources: https://www.engageny.org/resource/grade-6-mathematics Eureka Math Module PDFs: http://greatminds.net/maps/math/module-pdfs North Carolina 6th Grade Standards Unpacked: http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/6th.pdf Illustrative Mathematics – problems and tasks by grade and standard https://www.illustrativemathematics.org/ NCTM Illuminations – problems, tasks and interactives by grade and standard http://illuminations.nctm.org/Default.aspx Inside Mathematics – Problems of the Month and Performance Assessment tasks

http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Suggested Tools and Representations

Coordinate Planes

Nets

Prisms

Rulers

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills • Critical thinking and problem solving • Collaboration and leadership • Agility and adaptability • Initiative and entrepreneurialism • Effective oral and written communication • Accessing and analyzing information • Curiosity and imagination **Marzano's Nine Instructional Strategies for Effective Teaching and Learning** 1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way	**Meeting the Needs of All Students** **The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students.** **Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds,**

2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words 3. Reinforcing Effort and Providing Recognition: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. 4. Homework and Practice: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. 5. Nonlinguistic Representations: has recently been proven to stimulate and increase brain activity. 6. Cooperative Learning: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner. 7. Setting Objectives and Providing Feedback: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied. 8. Generating and Testing Hypotheses: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. 9. Cues, Questions, and Advanced Organizers: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	**utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations.** **Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.** **It It is important to note that the scaffold/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.** **Scaffolds for Students with Disabilities** **Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.** **Provide Multiple Means of Representation** **· Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.** **· Clarify, compare, and make connections to math words in discussion, particularly during and after practice.** **· Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define**

	"multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. **· Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"** **· Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:** **· Enlarge sprint print for visually impaired learners.** **· Use student boards to work on one calculation at a time.** **· Invest in or make math picture dictionaries or word walls.** **Provide Multiple Means of Action and Expression** **· Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.** **· Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.** **· Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."** **· Adjust wait time for interpreters of deaf and hard-of-hearing students.** **· Select numbers and tasks that are "just right" for learners.** **· Model each step of the algorithm before students begin.** **· Give students a chance to practice the next day's sprint beforehand. (At home, for example.)** **· Give students a few extra minutes to process the information before giving the signal to respond.** **· Assess by multiple means, including "show and tell" rather than written.**

	· Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" · Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" · Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. Provide Multiple Means of Engagement · Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. · Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use nonverbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. · Teach in small chunks so students get a lot of practice with one step at a time. · Know, use, and make the most of Deaf culture and sign language. · Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" · Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. · Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. · Follow predictable routines to allow students to focus on content rather than behavior. · Allow "everyday" and first language to express math understanding. · Re-teach the same concept with a variety of fluency

	games. · **Allow students to lead group and pair-share activities.** · **Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding**

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
Altitude and Base of a Triangle (An altitude of a triangle is a perpendicular segment from a vertex of a triangle to the line containing the opposite side. The opposite side is called the base. For every triangle, there are three choices for the altitude, and hence there are three base-altitude pairs. The height of a triangle is the length of the altitude. The length of the base is called either the base length or, more commonly, the base. Usually, context makes it clear whether the base refers to a number or a segment. These terms can mislead students: base suggests the bottom, while height usually refers to vertical distances. Do not reinforce these impressions by consistently displaying all triangles with horizontal bases.) Cube (A cube is a right rectangular prism all of whose edges are of equal length.) **Line Perpendicular to a Plane** (A line L intersecting a plane E at a point P is said to be perpendicular to the plane E if L is perpendicular to every line that (1) lies in E and (2) passes through the point P.. A segment is said to be perpendicular to a plane if the line that contains the segment is perpendicular to the plane. In Grade 6, a line perpendicular to a plane can be described using a	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. "I do, we do, you do." ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action</u>**	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. ● Model and post conversation "starters," such as: "I agree because..." "Can you ● explain how you solved it?""I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.

picture.) Parallel Planes (Two planes are parallel if they do not intersect. In Euclidean geometry, a useful test for checking whether two planes are parallel is if the planes are different and if there is a line that is perpendicular to both planes.) Pentagon (Given 5 different points A, B, C, D, and E in the plane, a 5-sided polygon, or pentagon, is the union of 5 segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}$, and $\overline{EA}$ such that (1) the segments intersect only at their endpoints, and (2) no two adjacent segments are collinear.) Right Rectangular Prism (Let E and E′ be two parallel planes. Let B be a rectangular region4 in the plane P. At each point P of B, consider the segment PP′ perpendicular to E, joining P to a point P′ of the plane E′. The union of all these segments is called a right rectangular prism. It can be shown that the region B′ in E′ corresponding to the region B is also a rectangular region whose sides are equal in length to the corresponding sides of B. The regions B and B′ are called the base faces (or just bases) of the prism. It can also be shown that the planar region between two corresponding sides of the bases is also a rectangular region called the lateral face of the prism. In all, the boundary of a right rectangular prism has 6 faces: the 2 base faces and 4 lateral faces. All adjacent faces intersect along segments called edges—base edges and lateral edges.) Surface of a Prism (The surface of a prism is the union of all of its faces—the base faces and lateral faces.) Triangular Region (A triangular region is the union of the triangle and its interior.)	**<u>and Expression</u>** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ ● Encourage students to explain their thinking and strategy for the solution. ● Choose numbers and tasks that are “just right” for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom’s Taxonomy with questions such as: “What would happen if...?”“Can you propose an alternative...?”“How would you evaluate...?”“What choice would you have made...?” Ask “Why?” and “What if?” questions.

		● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Grade 6 • Module 6

Statistics

OVERVIEW

In Grade 5, students used bar graphs and line plots to represent data and then solved problems using the information presented in the plots (**5.MD.B.2**). In this module, students move from simply representing data into analysis of data. In Topic A, students begin to think and reason statistically, first by recognizing a statistical question as one that can be answered by collecting data (**6.SP.A.1**). Students learn that the data collected to answer a statistical question has a distribution that is often summarized in terms of center, variability, and shape (**6.SP.A.2**). Beginning in Topic A, and throughout the module, students see and represent data distributions using dot plots and histograms (**6.SP.B.4**).

In Topics B and C, students study quantitative ways to summarize numerical data sets in relation to their context and to the shape of the distribution. The mean and mean absolute deviation (MAD) are used for data distributions that are approximately symmetric, and the median and interquartile range (IQR) are used for distributions that are skewed. Students apply their experience in writing, reading, and evaluating expressions in which letters stand for numbers (**6.EE.A.2**) as they learn to compute and interpret two pairs of statistical measures for center and spread (**6.SP.A.5**).

In Topic B, students study *mean* as a measure of center and *mean absolute deviation* as a measure of variability. Students learn that these measures are preferred when the shape of the distribution is roughly symmetric. Then, in Topic C, students study *median* as a measure of center and *interquartile range* as a measure of variability. Students learn that these measures are preferred when the shape of the distribution is skewed. Students develop in Topic B, and reinforce in Topic C, the idea that a measure of center provides a summary of all its values in a single number, while a measure of variation describes how values vary, also with a single number (**6.SP.A.3**).

In Topic D, students synthesize what they have learned as they connect the graphical, verbal, and numerical summaries to each other within situational contexts, culminating with a major project (**6.SP.B.4**, **6.SP.B.5**). Students implement the four-step investigative process with their projects by stating their statistical questions, explaining the plan they used to collect data, analyzing data numerically and with graphs, and interpreting their results as related to their questions. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Measures of center and variability for distributions that are approximately symmetric (mean and MAD) are covered before measures (median and IQR) for skewed data distributions. This choice was made because it is easier for students to understand measuring center and variability in the context of symmetric distributions.

For students, box plots are the most difficult of the graphical displays covered in this module. This is because they differ from dot plots and histograms in that they are not really a display of the data but rather a graph of five summary measures (minimum, lower quartile, median, upper quartile, and maximum). This graph conveys information on center and variability but is more difficult for students to interpret because, unlike histograms, where large area corresponds to many observations, in a box plot, large area indicates spread and small area indicates a large number of observations in a small interval. Box plots also require the calculation of quartiles and are best covered after quartiles have been introduced and used to calculate the IQR. For these reasons, box plots are introduced late in the module after the IQR and after students have already developed some fundamental understanding of data distributions, which is easier to do in the context of dot plots and histograms.

Rigorous Curriculum Design Template

Unit 6-Statistics

Subject:Math

Grade/Course: Grade 6

Pacing:25 days

Unit of Study: Statistics

Priority Standards:

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and boxplots.

6.SP.5 Summarize numerical data sets in relation to their context.

Foundational Standards:

Perform operations with multi-digit whole numbers and with decimals to hundredths.
5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm. 5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two digit divisors using strategies based on place value, the properties of operations and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Represent and interpret data.
5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s 2 to find the volume and surface area of a cube with sides of length s = 1/2.

Math Practice Standards:

MP.1 Make sense of problems and persevere in solving them. Students make sense of problems by defining them in terms of a statistical question and then determining what data might be collected in order to provide an answer to the question and therefore a solution to the problem.

MP.2 Reason abstractly and quantitatively. Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. Students use graphs to summarize the data and to answer statistical questions. MP.3 Construct viable arguments and critique the reasoning of others. Students examine the shape, center, and variability of a data distribution. They communicate the answer to a statistical question in the form of a poster presentation. Students also have an opportunity to critique poster presentations made by other students.

MP.4 Model with mathematics. Students create graphs of data distributions. They select an appropriate measure of center to describe a typical data value for a given data distribution. They also calculate and interpret an appropriate measure of variability based on the shape of the data distribution.

MP.6 Attend to precision. Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students use statistical terminology appropriately.

"Unwrapped" Standards

- 6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
- 6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- 6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- 6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and boxplots.
- 6.SP.5 Summarize numerical data sets in relation to their context.

Concepts -What Students Need to Know(Context)	Skills (What Students Need to Be Able to Do) Depth of Knowledge Level (DOK)

A statistical question(one that anticipates variability in data related and accounts for it in the answers) Data collected for a statistical question(described by its center, spread, and overall shape.) A measure of center for a numerical data set summarizes all of its values with a single number. A measure of variation describes how it values with a single number. Numerical data in plots(on a dot plots, histograms, and boxplots) Numerical data sets(in relation to their context)	Recognize(1) Understand(3) Recognize(1) Display(1) Summarize(2)

Essential Questions	**Big ideas**

● How does the type of data influence the choice of display? ● How can probability and data analysis be used to make predictions? ● How can each measure of center be used to describe data? ● What does the variation of data tell you? ● What measure is the most effective when analyzing a set of data? ● How can outliers skew a set of data? ● How can the collection, organization, interpretation, and display of data be used to answer questions?	Data can be modeled and used to make inferences.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Exit tickets for pre-assessment of each lesson.	● **Application problems** ● **Exploratory Challenge** ● **Problem set**	**Exit Ticket** **Mid-Module Assessment Task*** **End-of-Module Assessment Task*** ***See Table Below.**

*Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.SP.A.1, 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	6.SP.A.1, 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5
Project	Topic D: Lessons 17 and 22	Poster or other form of presentation	6.SP.A.1, 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5

Performance Assessment (*To be completed by grade level team)
Overview:
Engaging Learning Experiences
Task 1: Task 2: Task 3: Task 4:

Instructional Resources

Useful Websites:

Engage NY Curriculum overview and guiding documents:
https://www.engageny.org/

Engage NY Grade 6 Resources:
https://www.engageny.org/resource/grade-6-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 6th Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/6th.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

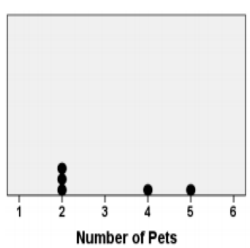
NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

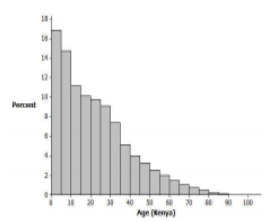
LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Suggested Tools and Representations



Dot Plot



Histogram

Dot Plots

Histograms

Box Plots

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills	Meeting the Needs of All Students
- Critical thinking and problem solving - Collaboration and leadership - Agility and adaptability - Initiative and entrepreneurialism - Effective oral and written communication - Accessing and analyzing information - Curiosity and imagination **<u>Marzano's Nine Instructional Strategies for Effective Teaching and Learning</u>** 1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way 2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words 3. Reinforcing Effort and Providing Recognition: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. 4. Homework and Practice: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. 5. Nonlinguistic Representations: has recently been proven to stimulate and increase brain activity. 6. Cooperative Learning: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner. 7. Setting Objectives and Providing Feedback: provide students with a direction. Objectives should not be too specific and should be adaptable to students’	**The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students.** **Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations.** **Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.** **It It is important to note that the scaffold/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.** **Scaffolds for Students with Disabilities** Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and

individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied.

8. Generating and Testing Hypotheses: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material.

9. Cues, Questions, and Advanced Organizers: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.

when they might be used.

Provide Multiple Means of Representation

· Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
· Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
· Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
· Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
· Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

· Enlarge sprint print for visually impaired learners.
· Use student boards to work on one calculation at a time.
· Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

· Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
· Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_______ is _____ hundreds, _____ tens, and _____ ones.
· Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count

	backwards in "Happy Counting." · Adjust wait time for interpreters of deaf and hard-of-hearing students. · Select numbers and tasks that are "just right" for learners. · Model each step of the algorithm before students begin. · Give students a chance to practice the next day's sprint beforehand. (At home, for example.) · Give students a few extra minutes to process the information before giving the signal to respond. · Assess by multiple means, including "show and tell" rather than written. · Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" · Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" · Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** · Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. · Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use nonverbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. · Teach in small chunks so students get a lot of practice with one step at a time. · Know, use, and make the most of Deaf culture and sign language. · Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" · Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.

<table>
<tr><td></td><td>· Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently.
· Follow predictable routines to allow students to focus on content rather than behavior.
· Allow “everyday” and first language to express math understanding.
· Re-teach the same concept with a variety of fluency games.
· Allow students to lead group and pair-share activities.
· Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding</td></tr>
</table>

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**

Statistical Question (A question that anticipates variability in the data that would be collected in order to answer the question.) Median (A measure of center appropriate for skewed data distributions. It is the middle value when the data are ordered from smallest to largest if there are an odd number of observations and half way between the middle two observations if the number of observations is even.) **Mean (**A measure of center appropriate for data distributions that are approximately symmetric. It is the average of the values in the data set. Two common interpretations of the mean are as a "fair share" and as the balance point of the data distribution.) **Dot Plot** (A plot of numerical data along a number line.) Histogram (A graphical representation of a numerical data set that has been grouped into intervals. Each interval is represented by a bar drawn above that interval that has a height corresponding to the number of observations in that interval.) **Box Plot** (A graph of five numerical summary measures: the minimum, lower quartile, median, upper quartile, and the maximum. It conveys information about center and variability in a data set.) **Variability** (Variability in a data set occurs when the observations in the data set are not all the same.) **Deviations from the Mean** (The differences calculated by subtracting the mean from the observations in a data set.) **Mean Absolute Deviation** (MAD) (A measure of variability appropriate for data distributions	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. "I do, we do, you do." ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' ● Encourage students to explain	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. ● Model and post conversation "starters," such as: "I agree because..." "Can you ● explain how you solved it?""I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and

that are approximately symmetric. It is the average of the absolute value of the deviations from the mean.) **Interquartile Range** (IQR) (A measure of variability appropriate for data distributions that are skewed. It is the difference between the upper quartile and the lower quartile of a data set and describes how spread out the middle 50% of the data are.)	their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance

		and support

Appendix A

1. Performance Task -Unit One
   http://map.mathshell.org/lessons.php?unit=6200&collection=8

PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Sharing Costs:
Travelling to School

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley Beta
Version

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics*:

1 Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

4. Model with mathematics.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics:*

6.RP Understand ratio concepts and use ratio reasoning to solve problems.

Resources

Lesson (complete)

- sharing costs - traveling to school.pdf (3907K PDF/Acrobat 19 Mar 2015)

Projector Resources

- l060_slides sharing costs equitably - traveling to school - gamma.ppt (3494K MS PowerPoint 19 Mar 2015)

Rubrics for Task 1-4

Task One

	Understanding
4	• Shows complete understanding of the required mathematical/scientific knowledge. • The solution completely addresses all mathematical/scientific components presented in the task.
3	• Shows nearly complete understanding of required mathematical/scientific knowledge. • The solution addresses almost all of the mathematical/scientific components presented in the task. There may be minor errors.
2	• Shows some understanding of the required mathematical/ scientific knowledge • The solution addresses some, but not all the mathematical/ scientific components presented in the task.
1	• Shows limited or no understanding of the problem, perhaps only re-copying the given data. • The solution addresses none of the mathematical/scientific components required to solve the task.

Task 2

	Understanding	Communication
4	• Shows complete understanding of the required mathematical/scientific knowledge. • The solution completely addresses all mathematical/scientific components presented in the task.	• There is a clear, effective explanation of the solution. All steps are included so the reader does not have to infer how the task was completed. • Mathematical/scientific representation is actively used as a means of communicating ideas. • There is precise and appropriate mathematical/scientific terminology and notation.
3	• Shows nearly complete understanding of required mathematical/scientific knowledge. • The solution addresses almost all of the mathematical/scientific components presented in the task. There may be minor errors.	• There is a clear explanation. • There is appropriate use of accurate mathematical/scientific representation. • There is effective use of mathematical/ scientific terminology and notation.
2	• Shows some understanding of the required mathematical/ scientific knowledge • The solution addresses some, but not all the mathematical/ scientific components presented in the task.	• There is an incomplete explanation; it may not be clearly represented. • There is some use of appropriate mathematical/scientific representation. • There is some use of mathematical/ scientific notation appropriate to the task.
1	• Shows limited or no understanding of the problem, perhaps only re-copying the given data. • The solution addresses none of the mathematical/scientific components required to solve the task.	• There is no explanation of the solution. The explanation cannot be understood, or is unrelated to the task. • There is no use or inappropriate use of mathematical/scientific representations. • There is no use, or mostly inappropriate use, of mathematical/scientific terminology and notation.

Task 3

	Planning and Execution	Communication
4	• Uses only the important elements of the task. • Uses an appropriate and complete strategy for solving the problem. • Uses only relevant information. • Uses clear and effective diagrams, tables, charts and graphs.	• There is a clear, effective explanation of the solution. All steps are included so the reader does not have to infer how the task was completed. • Mathematical/scientific representation is actively used as a means of communicating ideas. • There is precise and appropriate mathematical/scientific terminology and notation.
3	• Uses most of the important elements of the task. • Uses an appropriate but incomplete strategy for solving the problem. • Uses most of the relevant data. • Appropriate but incomplete use of diagrams, tables, charts and graphs.	• There is a clear explanation. • There is appropriate use of accurate mathematical/scientific representation. • There is effective use of mathematical/scientific terminology and notation.
2	• Uses some important elements of the task. • Uses an inappropriate strategy or application of strategy is unclear. • Uses some relevant data. • Limited use or misuse of diagrams, tables, charts, and graphs.	• There is an incomplete explanation; it may not be clearly represented. • There is some use of appropriate mathematical/scientific representation. • There is some use of mathematical/scientific notation appropriate to the task.
1	• Uses none of the important elements of the task. • Works haphazardly with no particular strategy for solving the problem. • Uses irrelevant data. • Does not show use of diagrams, tables, charts or graphs.	• There is no explanation of the solution. The explanation cannot be understood, or is unrelated to the task. • There is no use or inappropriate use of mathematical/scientific representations. • There is no use, or mostly inappropriate use, of mathematical/scientific terminology and notation.

Task 4

	Persistence
4	• Works hard on the task and doesn't need much help. • Student may extend his thinking beyond the problem and make new connections or create new problems.
3	• Works hard on the task and only gets help after having tried many strategies given throughout. • Completes task, working dutifully at the harder parts also.
2	• Can do simple parts of the problem with little help. • Starts working on the harder parts, but unless there is help, gives up.
1	• Needs help, even for the very simple tasks. • Gives up quickly, often just wanting someone to give the answer.

Appendix B

Three Representative Sample CFAs

1. Exit Ticket Pre/Post
2. Mid-Module Assessment
3. End of Module Assessment

Name ____ Date____________________

Lesson 1: Ratios

Exit Ticket

1. Write a ratio for the following description: Kaleel made three times as many baskets as John during basketball practice.

2. Describe a situation that could be modeled with the ratio $4:1$.

3. Write a ratio for the following description: For every 6 cups of flour in a bread recipe, there are 2 cups of milk.

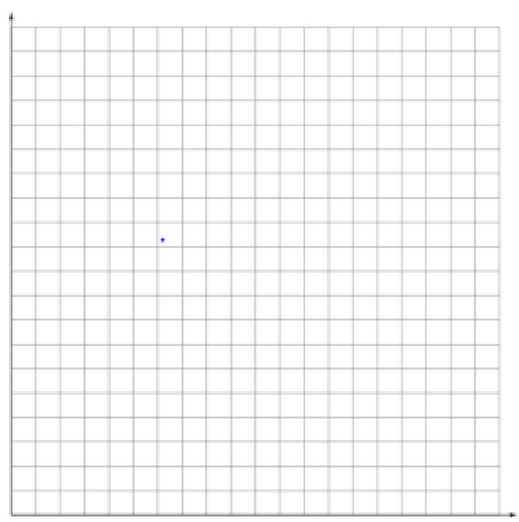
Name ____ Date ______________________

1. The most common women's shoe size in the U.S. is reported to be an $8\frac{1}{2}$. A shoe store uses a table like the one below to decide how many pairs of size $8\frac{1}{2}$ shoes to buy when it places a shoe order from the shoe manufacturers.

Total Number of Pairs of Shoes Being Ordered	Number of Pairs of Size $8\frac{1}{2}$ to Order
50	8
100	16
150	24
200	32

a. What is the ratio of the number of pairs of size $8\frac{1}{2}$ shoes the store orders to the total number of pairs of shoes being ordered?

b. Plot the values from the table on a coordinate plane. Label the axes. Then use the graph to find the number of pairs of size $8\frac{1}{2}$ shoes the store orders for a total order of 125 pairs of shoes.



2. Wells College in Aurora, New York was previously an all-girls college. In 2005, the college began to allow boys to enroll. By 2012, the ratio of boys to girls was 3 to 7. If there were 200 *more girls than boys* in 2012, how many boys were enrolled that year? Use a table, graph, or tape diagram to justify your answer.

3. Most television shows use 13 *minutes of every hour* for commercials, leaving the remaining 47 minutes for the actual show. One popular television show wants to change the ratio of commercial time to show time to be 3: 7. Create two ratio tables, one for the normal ratio of commercials to programming and another for the proposed ratio of commercials to programming. Use the ratio tables to make a statement about which ratio would mean fewer commercials for viewers watching 2 hours of television.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**a** 6.RP.A.1 6.RP.A.3a	Student provided an incorrect ratio and did not reflect an associated ratio. The student does not display an understanding of determining ratio using a ratio table.	Student provided an associated ratio, such as $25:4$. It may or may not have been expressed in the smallest unit possible. There is evidence that the student understood how to determine a ratio from a ratio table but lacked attentiveness to the precision for which the ratio was being asked.	Student provided the correct ratio, $4:25$, but may have been expressed using a larger unit, such as $8:50$. The notation or wording of the ratio statement may have had minor errors.	Student provided the correct ratio, $4:25$. The notation and/or wording of the ratio statement were correct.
	b 6.RP.A.1 6.RP.A.3a	Student did not produce a graph, or the graph did not accurately depict the pairs from the table. The student was unable to answer the question correctly.	Student depicted a graph, but the graph contained more than one error in its depiction, such as not going through the given points, not labeling the axes, or not depicting a line through the origin. The student may or may not have answered the question correctly.	Student depicted a graph, but the graph contained a minor error in its depiction, such as not accurately plotting the given points, not labeling the axes, or depicting a line that just missed going through the origin. The student answered the question correctly or incorrectly, but the answer would have been correct given the depiction of the graph.	Student depicted the graph correctly, including plotting the given points, labeling the axes, <u>AND</u> depicting a line that goes through the origin. The student answered the question correctly, and the answer was represented in the graph.

2	6.RP.A.3 (Stem Only)	Student was unable to answer the question. The student was not able to accurately depict the ratio of boys to girls or showed no evidence of moving beyond that basic depiction.	Student depicted the ratio of boys to girls and showed some evidence of using the depiction to solve the problem but was unable to come to a correct answer. The answer was either incomplete or incorrect.	Student was able to choose a depiction of the ratio and to incorporate the other information given into the depiction but made an error in arriving at the answer.	Student was able to choose a depiction of the ratio of boys to girls and incorporate into the depiction the additional information of the difference between the number of girls and the number of boys. The student was able to use the depiction to arrive at the correct answer.
3	6.RP.A.3a	Student was unable to complete the two tables or was unable to fill in at least one row in each table. The student was unable to compose a reasonably accurate comparison of which option would be better for viewers.	Student constructed ratio tables with at least one entry in each table and demonstrated some reasoning in making a statement of comparison, even if the statement did not match the table entries.	Student made two ratio tables with at least two entries in each table. There were one or more errors in the entries of the table. The student was able to make a statement of comparison of which option was better for viewers based on the entries provided in the table.	Student made two ratio tables with at least two entries in each table. The student was able to make an accurate comparison of which option was better for viewers and relate the comparison to a 2-hour show using accurate grade-level language.

Name ______________________________ Date ______________

1. The most common women's shoe size in the U.S. is reported to be an $8\frac{1}{2}$. A shoe store uses a table like the one below to decide how many pairs of size $8\frac{1}{2}$ shoes to buy when it places a shoe order from the shoe manufacturers.

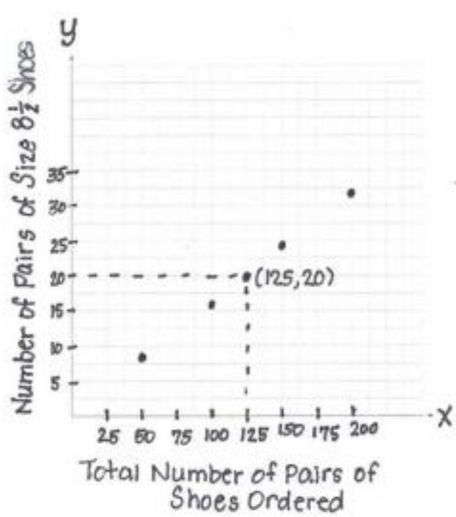
Total Number of Pairs of Shoes Being Ordered	Number of Pairs of Size $8\frac{1}{2}$ to Order
50	8
100	16
150	24
200	32

a. What is the ratio of the number of pairs of size $8\frac{1}{2}$ shoes the store orders to the total number of pairs of shoes being ordered?

~~The ratio of size $8\frac{1}{2}$ shoes to the total number~~

The ratio of the number of pairs of size $8\frac{1}{2}$ shoes to the total number of pairs of shoes ordered is 4:25

b. Plot the values from the table on a coordinate plane. Label the axes. Then use the graph to find the number of pairs of size $8\frac{1}{2}$ shoes the store orders for a total order of 125 pairs of shoes.



They should order 20 pairs of size $8\frac{1}{2}$ shoes if the total order is 125 pairs of shoes.

2. Wells College in Aurora, New York was previously an all-girls college. In 2005, the college began to allow boys to enroll. By 2012, the ratio of boys to girls was 3 to 7. If there were 200 *more girls than boys* in 2012, how many boys were enrolled that year? Use a table, graph, or tape diagram to justify your answer.

Girls | 50 | 50 | 50 | $\underbrace{50 \mid 50 \mid 50 \mid 50}_{200}$

Boys | $\underbrace{50 \mid 50 \mid 50}$

150 boys were enrolled in 2012.

3. Most television shows use 13 *minutes of every hour* for commercials, leaving the remaining 47 minutes for the actual show. One popular television show wants to change the ratio of commercial time to show time to be 3: 7. Create two ratio tables, one for the normal ratio of commercials to programming and another for the proposed ratio of commercials to programming. Use the ratio tables to make a statement about which ratio would mean fewer commercials for viewers watching 2 hours of television.

Normal

Total Time	Commercial Time	Show Time
60	13	47
120	26	94

Changed

Total Time	Commercial Time	Show Time
10	3	7
60	18	42
120	36	84

The normal way is better for viewers. In a 2 hour show, the normal way uses 26 minutes for commercials, but the proposed way would use 36 minutes for commercials.

Name ____________________________________ Date ____________________

1. Jasmine has taken an online boating safety course and is now completing her end of course exam. As she answers each question, the progress bar at the bottom of the screen shows what portion of the test she has finished. She has just completed question 16 and the progress bar shows she is 20% complete. How many total questions are on the test? Use a table, diagram, or equation to justify your answer.

2. Alisa hopes to play beach volleyball in the Olympics someday. She has convinced her parents to allow her to set up a beach volleyball court in their back yard. A standard beach volleyball court is approximately 26 feet by 52 feet. She figures that she will need the sand to be one foot deep. She goes to the hardware store to shop for sand and sees the following signs on pallets containing bags of sand.

a. What is the rate that Brand A is selling for? Give the rate and then specify the unit rate.

b. Which brand is offering the better value? Explain your answer.

c. Alisa uses her cell phone to search how many pounds of sand is required to fill 1 cubic foot and finds the answer is 100 pounds. Choose one of the brands and compute how much it will cost Alisa to purchase enough sand to fill the court. Identify which brand was chosen as part of your answer.

3. Loren and Julie have different part time jobs after school. They are both paid at a constant rate of dollars per hour. The tables below show Loren and Julie's total income (amount earned) for working a given amount of time.

Loren

Hours	2	4	6	8	10	12	14	16	18
Dollars	18	36	54	72	90	108			162

Julie

Hours	3	6	9	12	15	18	21	24	27
Dollars	36		108	144	180	216		288	324

a. Find the missing values in the two tables above.

b. Who makes more per hour? Justify your answer.

c. Write how much Julie makes as a rate. What is the unit rate?

d. How much money would Julie earn for working 16 hours?

e. What is the ratio between how much Loren makes per hour and how much Julie makes per hour?

f. Julie works $\frac{1}{12}$ hours/dollar. Write a one or two-sentence explanation of what this rate means. Use this rate to find how long it takes for Julie to earn $228.

4. Your mother takes you to your grandparents' house for dinner. She drives 60 minutes at a constant speed of 40 miles per hour. She reaches the highway and quickly speeds up and drives for another 30 minutes at constant speed of 70 miles per hour.

   a. How far did you and your mother travel altogether?

   b. How long did the trip take?

   c. Your older brother drove to your grandparents' house in a different car, but left from the same location at the same time. If he traveled at a constant speed of 60 miles per hour, explain why he would reach your grandparents house first. Use words, diagrams, or numbers to explain your reasoning.

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	6.RP.A.3c	Student was unable to depict the problem using a table, diagram, or equation, and the student either answered incorrectly or did not answer the question at all.	Student depicted the problem using a table, diagram, or equation, but had significant errors in their reasoning or calculations, leading to an incorrect answer.	Student was able to answer the question correctly, but was not able to explain their reasoning process with an accurate depiction using a table, diagram, or equation. OR Student gave an accurate depiction of the problem, but made a minor calculation or articulation error in arriving at the answer.	Student gave an accurate depiction of the problem with a table, diagram, or equation and connected that depiction to a correct answer to the question.
2	a 6.RP.A.2 6.RP.A.3d	Student was unable to answer the question. They were not able to accurately represent the rate or unit rate for Brand A. The student showed no evidence of moving beyond that representation.	Student was able to accurately represent the rate for Brand A, but was unable to determine the unit rate. The student is unable to apply the unit rate to further questioning in the problem.	Student correctly provided the unit rate as 12, but the work lacks connection to the original problem of 60 lb. per $5.	Student correctly provided the rate as 12 pounds per dollar and the unit rate is given as 12.

	b 6.RP.A.2 6.RP.A.3d	Student was unable to answer the question. They were not able to accurately represent the rate or unit rate for Brand B and showed no evidence of moving beyond that representation.	Student was able to accurately represent the rate for Brand B, but was unable to apply the unit rate in comparison to the unit rate of Brand A.	Student accurately represented the unit rate of Brand B as 12.5 lb. per $1 and compared the unit rate to being more than Brand A. However, the student did not make connections to the problem and did not determine that Brand B was a better deal because it gives more sand than Brand A.	Student accurately represented both unit rates of Brand A and Brand B. The student determined Brand B was a better unit rate and related the unit rates to the problem.
	c 6.RP.A.2 6.RP.A.3d	Student did not answer the question correctly. The total number of cubic feet was not found. The rate of 100 lb./1 ft. was not used to determine the total pounds of sand and the unit rate of the cost of either A or B was not used to determine the total cost of the project.	Student determined the total number of cubic feet. The rates to find the total pounds of sand needed were not used or were miscalculated. The unit rate of the cost of A or B was not used to determine the total cost of the project or was miscalculated.	Student accurately determined the number of cubic feet needed for the project. The rate of 100 lb./1 ft. was accurately calculated to determine the total pounds of sand needed; however, the rate of $1/the unit rate of A or B to determine the final cost was miscalculated.	Student accurately determined the total cubic feet needed, the total pounds of sand needed and used the appropriate rate to determine the final cost of the project. The student used labels accurately to support the reasoning of the final answer.
3	**a** 6.RP.A.1 6.RP.A.2 6.RP.A.3a 6.RP.A.3b	Student was unable to answer the question. The values were not placed in either table or incorrect values were provided.	Student was able to provide two to three correct values to portions of the tables, but did not support the answers mathematically.	Student was able to provide correct values for three to four portions of the tables, but did not support the answers mathematically.	Student was able to provide correct values for all portions of the tables. The student provided reasoning for the answers using additive patterns and unit rate conversion.
	b 6.RP.A.1 6.RP.A.2 6.RP.A.3a 6.RP.A.3b	Student did not calculate the hourly rate of either Loren or Julie correctly or did not answer the question. The rates to determine a final answer were not compared.	Student did not correctly calculate the hourly rate of either Loren or Julie and was unable to compare the rates and determine which girl made more money per hour.	Student correctly calculated the hourly rate of each girl, but did not compare the rates to determine which made more money per hour.	Student accurately answered the question and justified their reasoning through comparison of the hourly rates.

	c 6.RP.A.1 6.RP.A.2 6.RP.A.3a 6.RP.A.3b	Student was unable to answer the question. The rate or the unit rate was not accurately determined. The student did not make connections to the values in the table.	Student referenced values from the table. (e.g., $36/3 hrs.), but did not express the values as a rate or a unit rate.	Student correctly determined the rate of Julie's pay as $12 for every hour, but did not determine the unit rate to be 12.	Student accurately answered the question by representing the unit rate as 12 and by referencing the values from the table.
	d 6.RP.A.1 6.RP.A.2 6.RP.A.3a 6.RP.A.3b	Student was unable to answer the question. The correct rate with the amount of hours was not accurately computed. <u>OR</u> Student did not attempt the problem.	Student did not accurately compute the correct rate with the amount of hours, but was proficient in the process to find the correct answer.	Student computed the correct rate with the amount of hours. The student found the total amount of money Julie made in 16 hours. Student work lacked labeling and clear sequence in solving.	Student accurately derived the correct amount of money Julie made in 16 hours. The student used the correct rate and the work was labeled in order to justify the reasoning. The student's work is in logical progression.
	e 6.RP.A.1 6.RP.A.2 6.RP.A.3a 6.RP.A.3b	Student was unable to answer the question. The correct rate of pay for one or both of the girls was not found.	Student was able to compute the accurate rate of pay for the girls, but did not compare to determine which girl made more money per hour.	Student accurately computed the rate of pay for each girl and accurately compared the pay in ratio form. The student did not derive a simplified ratio from the rates of pay.	Student answered the problem accurately, with labels and simplified their final answer.
	f 6.RP.A.1 6.RP.A.2 6.RP.A.3a 6.RP.A.3b	Student explained what the rate meant in the problem, but did not accurately find the answer.	Student explained the meaning of the rate in detail using conversions, but made errors when deriving the plan to solve. *Example: The answer is not indicative of understanding cancellation of units and finds* $19 *instead of* 19 *hours.*	Student provided a lucid explanation with conversions and support. The student may have multiplied by minute conversion and found a final answer of 1,140 minutes instead of 19 hours.	Student answered the problem with precision and with coherent explanation of what the rate means. Calculations are accurate and the final answer is supported and justified through appropriate labeling.
4	**a** 6.RP.A.3b	Student was unable to answer the problem accurately. They student was not able to apply the rates to determine the amount of miles.	Student was able to show their intent to multiply the rate by the time to find the miles, but computed incorrectly.	Student multiplied the rates appropriately to the time for each section of the trip. The amount of separate miles was found, but the student did not combine them for a total amount of miles for the trip. <u>OR</u> Student showed	Student completed the entire problem accurately with appropriate labels. The student was able to derive a total distance with no computation errors.

				understanding of the concept, but made computation errors.	
	b 6.RP.A.3b	Student did not complete the problem or answered with an incorrect response.	Student used information from the original problem to determine the addends, but computed the total incorrectly.	Student used information from the original problem to determine addends and computed the sum correctly, but did not report the correct unit.	Student used information from the original problem to determine addends and computed the sum correctly. The student labeled work appropriately and converted the minutes into hours.
	c 6.RP.A.3b	Student did not use a diagram, words, or numbers to support the answer or used the diagram inappropriately. The student did not answer the problem with an accurate response.	Student provided an accurate response, but did not utilize a diagram, words, or numbers to support the answer.	Student provided a correct answer and used only words or numbers to support the answer.	Student used appropriate diagrams, words, and numbers to support the accurate answer.

Three Representative Model Lessons

Unit One

1. Lesson One
2. Lesson Two
3. Lesson Three

Lesson 1: Ratios

Student Outcomes

- Students understand that a *ratio* is an ordered pair of non-negative numbers, which are not both zero. Students understand that a ratio is often used instead of describing the first number as a multiple of the second.
- Students use the precise language and notation of ratios (e.g., $3:2$, 3 to 2). Students understand that the order of the pair of numbers in a ratio matters and that the description of the ratio relationship determines the correct order of the numbers. Students conceive of real-world contextual situations to match a given ratio.

Lesson Notes

The first two lessons of this module will develop the students' understanding of the term *ratio*. A ratio is always a pair of numbers, such as $2:3$ and never a pair of quantities such as 2 cm : 3 sec. Keeping this straight for students will require teachers to use the term ratio correctly and consistently. Students will be required to separately keep track of the units in a word problem. To help distinguish between ratios and statements about quantities that define ratios, we use the term *ratio relationship* to describe a phrase in a word problem that indicates a ratio. Typical examples of ratio relationship descriptions include "3 cups to 4 cups," "5 miles in 4 hours," etc. The ratios for these ratio relationships are $3:4$ and $5:4$, respectively.

Classwork

Example 1 (15 minutes)

Read the example aloud.

Example 1

The coed soccer team has four times as many boys on it as it has girls. We say the ratio of the number of boys to the number of girls on the team is $4:1$. We read this as "four to one."

- Let's create a table to show how many boys and how many girls could be on the team.

Create a table like the one shown below to show possibilities of the number of boys and girls on the soccer team. Have students copy the table into their student packet.

# of Boys	# of Girls	Total # of Players
4	1	5

- So, we would have four boys and one girl on the team for a total of five players. Is this big enough for a team?
  - *Adult teams require 11 players, but youth teams may have fewer. There is no right or wrong answer; just encourage the reflection on the question, thereby connecting their math work back to the context.*
- What are some other ratios that show four times as many boys as girls, or a ratio of boys to girls of 4 to 1?
  - *Have students add each ratio to their table.*

# of Boys	# of Girls	Total # of Players
4	1	5
8	2	10
12	3	15

- From the table, we can see that there are four boys for every one girl on the team.

Read the example aloud.

Suppose the ratio of the number of boys to the number of girls on the team is 3: 2.

Create a table like the one shown below to show possibilities of the number of boys and girls on the soccer team. Have students copy the table into their student materials.

# of Boys	# of Girls	Total # of Players
3	2	5

- What are some other team compositions where there are three boys for every two girls on the team?

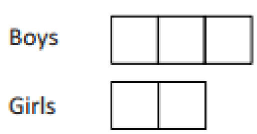
# of Boys	# of Girls	Total # of Players
3	2	5
6	4	10
9	6	15

- I can't say there are 3 times as many boys as girls. What would my multiplicative value have to be? There are ____ as many boys as girls.

Encourage the students to articulate their thoughts, guiding them to say there are $\frac{3}{2}$ as many boys as girls.

- Can you visualize $\frac{3}{2}$ as many boys as girls?

- Can we make a tape diagram (or bar model) that shows that there are $\frac{3}{2}$ as many boys as girls?



- Which description makes the relationship easier to visualize: saying the ratio is 3 to 2 or saying there are 3 halves as many boys as girls?
  - *There is no right or wrong answer. Have students explain why they picked their choices.*

Example 2 (8 minutes): Class Ratios

Discussion

Direct students:

- Find the ratio of boys to girls in our class.
- Raise your hand when you know: What is the ratio of boys to girls in our class?
- How can we say this as a multiplicative comparison without using ratios? Raise your hand when you know.

Allow for choral response when all hands are raised.

- Write the ratio of number of boys to number of girls in your student materials under Example 2, Question 1.
- Compare your answer with your neighbor's answer. Does everyone's ratio look exactly the same?

Allow for discussion of differences in what students wrote. Communicate the following in the discussions:

1. It is ok to use either the colon symbol or the word "to" between the two numbers of the ratio.
2. The ratio itself does not have units or descriptive words attached.

- Raise your hand when you know: What is the ratio of number of girls to number of boys in our class?
- Write the ratio down in your materials as number 2.
- Is the ratio of number of girls to number of boys the same as the ratio of number of boys to number of girls?
  - *Unless in this case there happens to be an equal number of boys and girls, then no, the ratios are not the same. Indicate that order matters.*
- Is this an interesting multiplicative comparison for this class? Is it worth commenting on in our class? If our class had 15 boys and 5 girls, might it be a more interesting observation?

For the exercise below, choose a way for students to indicate that they identify with the first statement (e.g., standing up or raising a hand). After each pair of statements below, have students create a ratio of the number of students who answered "yes" to the first statement to the number of students who answered "yes" to the second statement verbally, in writing, or both. Consider following each pair of statements with a discussion of whether it seems like an interesting ratio to discuss. Or alternatively, when you have finished all of these examples, ask students which ratio they found most interesting.

Students record a ratio for each of the examples you provide:

1. You traveled out of state this summer.
2. You did not travel out of state this summer.
3. You have at least one sibling.
4. You are an only child.
5. Your favorite class is math.
6. Your favorite class is not math.

Example 2: Class Ratios

Record a ratio for each of the examples the teacher provides.

1. *Answers will vary. One example is* $12:10$.
2. *Answers will vary. One example is* $10:12$.
3. *Answers will vary. One example is* $7:15$.
4. *Answers will vary. One example is* $15:7$.
5. *Answers will vary. One example is* $11:11$.
6. *Answers will vary. One example is* $11:11$.

Exercise 1 (2 minutes)

Have students look around the classroom to find quantities to compare. Have students create written ratio statements that represent their ratios in one of the summary forms.

Exercise 1

My own ratio compares *the number of students wearing jeans* **to** *the number of students not wearing jeans*.

My ratio is $16:6$.

Exercise 2 (10 minutes)

With a partner, students use words to describe a context that could be represented by each ratio given. Encourage students to be precise about the order in which the quantities are stated (emphasizing that order matters) and about the quantities being compared. That is, instead of saying the ratio of boys to girls, encourage them to say, the ratio of the number of boys to the number of girls. After students develop the capacity to be very precise about the quantities in the ratio, it is appropriate for them to abbreviate their communication in later lessons. Just be sure their abbreviations still accurately convey the meaning of the ratio in the correct order.

MP.6

Exercise 2

Using words, describe a ratio that represents each ratio below.

a. 1 to 12 *for every one year, there are twelve months*

b. $12:1$ *for every twelve months, there is one year*

c. 2 to 5 *for every two days of non-school days in a week, there are five school days*

d. 5 to 2 *for every five female teachers I have, there are two male teachers*

e. $10:2$ *for every ten toes, there are two feet*

f. $2:10$ *for every two problems I can finish, there are ten minutes that pass*

After completion, invite sharing and explanations of the chosen answers.

Point out the difference between ratios, such as, "for every one year, there are twelve months," and "for every five female teachers I have, there are two male teachers." The first type represents a constant relationship that will remain true as the number of years or months increases and the second one is somewhat arbitrary and will not remain true if the number of teachers increases.

Closing (5 minutes)

Provide students with this description:

A **ratio** is an ordered pair of non-negative numbers, which are not both zero. The *ratio* is denoted $A: B$ or A to B to indicate the order of the numbers. In this specific case, the number A is first, and the number B is second.

- What is a ratio? Can you verbally describe a ratio in your own words using this description?
  - *Answers will vary, but should include the description that a ratio is an ordered pair of numbers, which are both not zero.*
- How do we write ratios?
  - *A colon B ($A: B$) or A "to" B.*
- What are two quantities you would love to have in a ratio of $5: 2$ but hate to have in a ratio of $2: 5$?
  - *Answers will vary. For example, I would love to have a ratio of the number of hours of play time to the number of hours of chores be $5: 2$, but would hate to have a ratio of the number of hours of television time to the number of hours of studying be $2: 5$.*

Lesson Summary

A ratio is an ordered pair of non-negative numbers, which are not both zero.

The ratio is written $A: B$ or A to B to indicate the order of the numbers. The number A is first, and the number B is second.

The order of the numbers is important to the meaning of the ratio. Switching the numbers changes the relationship. The description of the ratio relationship tells us the correct order for the numbers in the ratio.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 1: Ratios

Exit Ticket

1. Write a ratio for the following description: Kaleel made three times as many baskets as John during basketball practice.

2. Describe a situation that could be modeled with the ratio $4:1$.

3. Write a ratio for the following description: For every 6 cups of flour in a bread recipe, there are 2 cups of milk.

Exit Ticket Sample Solutions

1. Write a ratio for the following description: Kaleel made three times as many baskets as John during basketball practice.

   A ratio of $3:1$ or 3 to 1 can be used.

2. Describe a situation that could be modeled with the ratio $4:1$.

   Answers will vary but could include the following: For every four teaspoons of cream in a cup of tea, there is one teaspoon of honey.

3. Write a ratio for the following description: For every 6 cups of flour in a bread recipe, there are 2 cups of milk.

   A ratio of $6:2$ or 6 to 2 can be used, or students might recognize and suggest the equivalent ratio of $3:1$.

Problem Set Sample Solutions

1. At the 6th grade school dance, there are 132 boys, 89 girls, and 14 adults.
   a. Write the ratio of the number of boys to the number of girls.

      $132:89$ *or* 132 *to* 89

   b. Write the same ratio using another form ($A:B$ vs. A to B).

      132 *to* 89 *or* $132:89$

   c. Write the ratio of the number of boys to the number of adults.

      $132:14$ *or* 132 *to* 14

   d. Write the same ratio using another form.

      132 *to* 14 *or* $132:14$

2. In the cafeteria, 100 milk cartons were put out for breakfast. At the end of breakfast, 27 remained.
   a. What is the ratio of the number of milk cartons taken to total number of milk cartons?

      $73:100$ *or* 73 *to* 100

   b. What is the ratio of the number of milk cartons remaining to the number of milk cartons taken?

      $27:73$ *or* 27 *to* 73

3. **Choose a situation that could be described by the following ratios, and write a sentence to describe the ratio in the context of the situation you chose.**

For example:

$3:2$. When making pink paint, the art teacher uses the ratio $3:2$. For every 3 cups of white paint she uses in the mixture, she needs to use 2 cups of red paint.

a. 1 to 2

For every one nose, there are two eyes (answers will vary).

b. 29 to 30

For every 29 girls in the cafeteria, there are 30 boys (answers will vary).

c. $52:12$

For every 52 weeks in the year, there are 12 months (answers will vary).

Lesson 2: Ratios

Student Outcomes

- Students reinforce their understanding that a ratio is an ordered pair of non-negative numbers, which are not both zero. Students continue to learn and use the precise language and notation of ratios (e.g., $3:2$, 3 to 2). Students demonstrate their understanding that the order of the pair of numbers in a ratio matters.
- Students create multiple ratios from a context in which more than two quantities are given. Students conceive of real-world contextual situations to match a given ratio.

Classwork

Exercise 1 (5 minutes)

Allow students time to complete the exercise. Students can work in small groups or in partners for the exercise.

Exercise 1

Come up with two examples of ratio relationships that are interesting to you.

1. *My brother watches twice as much television as I do. The ratio of number of hours he watches in a day to number of hours I watch in a day is usually* $2:1$.
2. *For every* 2 *chores my mom gives my brother, she gives* 3 *to me. The ratio is* $2:3$.

Allow students to share by writing the examples on the board, being careful to include some of the verbal clues that indicate a ratio relationship: *to, for each, for every*.

- What are the verbal cues that tell us someone is talking about a ratio relationship?

Exploratory Challenge (30 minutes)

Have students read and study the description of the data in the chart provided in their student materials. Ask students to explain what the chart is about (if possible, without looking back at the description). This strategy encourages students to really internalize the information given as opposed to jumping right into the problem without knowing the pertinent information.

MP.6

- Based on the survey, should the company order more pink fabric or more orange fabric?
- What is the ratio of the number of bolts of pink fabric to number of bolts of orange fabric you think the company should order?
- Someone said 5 to 3, and another person said (or my friend said) it would be 3 to 5. Are those the same? Is a ratio of 3 to 5 the same as a ratio of 5 to 3?
- Write a statement that describes the ratio relationship of this 3 to 5 ratio that we have been talking about.

Review the statements written by the students, checking and reinforcing their understanding that the ordering of the words in the description of the ratio relationship is what determines the order of the numbers in the ratio.

Allow students to work individually or in pairs to complete Exercises 2 and 3 for this Exploratory Challenge.

Exploratory Challenge

A t-shirt manufacturing company surveyed teen-aged girls on their favorite t-shirt color to guide the company's decisions about how many of each color t-shirt they should design and manufacture. The results of the survey are shown here.

Favorite T-Shirt Colors of Teen-Aged Girls Surveyed

			x			
			x			
			x	x		
	x		x	x		x
	x		x	x	x	x
	x	x	x	x	x	x
x	x	x	x	x	x	x
Red	Blue	Green	White	Pink	Orange	Yellow

Exercises for Exploratory Challenge

1. **Describe a ratio relationship, in the context of this survey, for which the ratio is $3:5$.**

   The number of girls who answered orange to the number of girls who answered pink.

2. **For each ratio relationship given, fill in the ratio it is describing.**

Description of the Ratio Relationship (Underline or highlight the words or phrases that indicate the description is a ratio.)	Ratio
For every 7 white t-shirts they manufacture, they should manufacture 4 yellow t-shirts. The ratio of the number of white t-shirts to the number of yellow t-shirts should be…	$7:4$
For every 4 yellow t-shirts they manufacture, they should manufacture 7 white t-shirts. The ratio of the number of yellow t-shirts to the number of white t-shirts should be…	$4:7$
The ratio of the number of girls who liked a white t-shirt best to the number of girls who liked a colored t-shirt best was…	$7:19$
For each red t-shirt they manufacture, they should manufacture 4 blue t-shirts. The ratio of the number of red t-shirts to the number of blue t-shirts should be…	$1:4$
They should purchase 4 bolts of yellow fabric for every 3 bolts of orange fabric. The ratio of the number of bolts of yellow fabric to the number of bolts of orange fabric should be…	$4:3$
The ratio of the number of girls who chose blue or green as their favorite to the number of girls who chose pink or red as their favorite was …	$6:6$ *or* $1:1$
Three out of every 26 t-shirts they manufacture should be orange. The ratio of the number of orange t-shirts to the total number of t-shirts should be…	$3:26$

3. For each ratio given, fill in a description of the ratio relationship it could describe, using the context of the survey.

Description of the Ratio Relationship (Underline or highlight the words or phrases that indicate your example is a ratio.)	Ratio
They should make 4 yellow t-shirts for every 3 orange t-shirts. The ratio of the number of yellow t-shirts to the number of orange t-shirts should be…	4 to 3
They should make 3 orange t-shirts for every 4 blue t-shirts. The ratio of the number of orange t-shirts to the number of blue t-shirts should be…	$3:4$
For every 19 colored t-shirts, there should be 7 white t-shirts. The ratio of the number of colored t-shirts to the number of white t-shirts should be…	$19:7$
7 out of 26 t-shirts should be white. The ratio of the number of white t-shirts to the number of total t-shirts should be…	7 to 26

If time permits, allow students to share some of their descriptions for the ratios in Exercise 3.

Closing (5 minutes)

- Are the ratios $2:5$ and $5:2$ the same? Why or why not?

Lesson Summary

- Ratios can be written in two ways: A to B or $A:B$.
- We describe ratio relationships with words, such as *to*, *for each*, *for every*.
- The ratio $A:B$ is not the same as the ratio $B:A$ (unless A is equal to B).

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 2: Ratios

Exit Ticket

Give two different ratios with a description of the ratio relationship using the following information:

There are 15 male teachers in the school. There are 35 female teachers in the school.

Exit Ticket Sample Solutions

Give two different ratios with a description of the ratio relationship using the following information:

There are 15 male teachers in the school. There are 35 female teachers in the school.

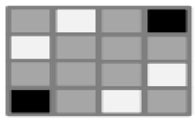
Possible solutions:

- *The ratio of the number of male teachers to the number of female teachers is* $15:35$.
- *The ratio of the number of female teachers to the number of male teachers is* $35:15$.
- *The ratio of the number of female teachers to the total number of teachers at the school is* $35:50$.
- *The ratio of the number of male teachers to the total number of teachers at the school is* $15:50$.

**Please note that some students may write other equivalent ratios as answers. For example,* $3:7$ *is equivalent to* $15:35$.

Problem Set Sample Solutions

1. **Using the floor tiles design shown below, create 4 different ratios related to the image. Describe the ratio relationship and write the ratio in the form $A:B$ or the form A to B.**



For every 16 *tiles, there are* 4 *white tiles.*

The ratio of the number of black tiles to the number of white tiles is 2 *to* 4.

(Answers will vary.)

2. **Billy wanted to write a ratio of the number of apples to the number of peppers in his refrigerator. He wrote $1:3$. Did Billy write the ratio correctly? Explain your answer.**

Billy is incorrect. There are 3 *apples and* 1 *pepper in the picture. The ratio of the number of apples to the number of peppers is* $3:1$.

Lesson 3: Equivalent Ratios

Student Outcomes

- Students develop an intuitive understanding of equivalent ratios by using tape diagrams to explore possible quantities of each part when given the part-to-part ratio. Students use tape diagrams to solve problems when the part-to-part ratio is given and the value of one of the quantities is given.
- Students formalize a definition of equivalent ratios: Two ratios, $A: B$ and $C: D$, are equivalent ratios if there is a positive number, c, such that $C = cA$ and $D = cB$.

Classwork

Exercise 1 (5 minutes)

This exercise continues to reinforce the students' ability to relate ratios to the real world, as practiced in Lessons 1 and 2. Provide students with time to think of a one-sentence story problem about a ratio.

Exercise 1

Write a one-sentence story problem about a ratio.

Answers will vary. The ratio of the number of sunny days to the number of cloudy days in this town is $3: 1$.

Write the ratio in two different forms.

$3: 1$ 3 *to* 1

Have students share their sentences with each other in pairs or trios. Ask a few students to share with the whole class.

Exercise 2 (15 minutes)

Ask students to read the problem, and then describe in detail what the problem is about without looking back at the description, if possible. This strategy encourages students to really internalize the information given as opposed to jumping right into the problem without knowing the pertinent information.

- Let's represent this ratio in a table.

The Length of Shanni's Ribbon (in inches)	The Length of Mel's Ribbon (in inches)
7	3
14	6
21	9

- We can use a tape diagram to represent the ratio of the lengths of ribbon. Let's create one together.

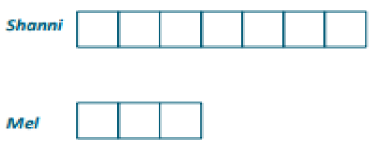
Walk through the construction of the tape diagram with students as they record with you.

- How many units should we draw for Shanni's portion of the ratio?
  - *Seven*
- How many units should we draw for Mel's portion of the ratio?
  - *Three*

Exercise 2

Shanni and Mel are using ribbon to decorate a project in their art class. The ratio of the length of Shanni's ribbon to the length of Mel's ribbon is $7:3$.

Draw a tape diagram to represent this ratio.



- What does each unit on the tape diagram represent?
  - *Allow students to discuss; they should conclude that they do not really know yet, but each unit represents some unit that is a length.*
- What if each unit on the tape diagrams represents 1 inch? What are the lengths of the ribbons?
  - *Shanni's ribbon is 7 inches; Mel's ribbon is 3 inches.*
- What is the ratio of the lengths of the ribbons?
  - $7:3$ *(Make sure that the students feel comfortable expressing the ratio itself as simply the pair of numbers $7:3$ without having to add units.)*
- What if each unit on the tape diagrams represents 2 meters? What are the lengths of the ribbons?
  - *Shanni's ribbon is 14 meters; Mel's ribbon is 6 meters.*
- How did you find that?
  - *Allow students to verbalize and record using a tape diagram.*
- What is the ratio of the length of Shanni's ribbon to the length of Mel's ribbon now? Students may disagree; some may say it is $14:6$, and others may say it is still $7:3$.
  - *Allow them to debate and justify their answers. If there is no debate, initiate one:* A friend of mine told me the ratio would be (provide the one that no one said, either $7:3$ or $14:6$). Is she right?
- What if each unit represents 3 inches? What are the lengths of the ribbons? Record. Shanni's ribbon is 21 inches; Mel's ribbon is 9 inches. Why?
  - *7 times 3 equals 21; 3 times 3 equals 9.*
- If each of the units represents 3 inches, what is the ratio of the length of Shanni's ribbon to the length of Mel's ribbon?
  - *Allow for discussion as needed.*

> *Scaffolding:*
>
> If students do not see that each unit represents a given length, write the length of each unit within the tape diagram units, and have students add them to find the total.

- We just explored three different possibilities for the length of the ribbon; did the number of units in our tape diagrams ever change?
  - *No.*
- What did these three ratios, $7:3$, $14:6$, $21:9$, all have in common?
  - *Write the ratios on the board. Allow students to verbalize their thoughts without interjecting a definition. Encourage all to participate by asking questions of the class with respect to what each student says, such as, "Does that sound right to you?"*
- Mathematicians call these ratios *equivalent*. What ratios can we say are equivalent to $7:3$?

Shanni and Mel are using ribbon to decorate a project in their art class. The ratio of the length of Shanni's ribbon to the length of Mel's ribbon is $7:3$.

Draw a tape diagram to represent this ratio.

Shanni								*7 inches*
								3 inches
								$7:3$
Mel								

Shanni	*2 m*	*2 m*	*2 m*	*2 m*	*2 m*	*2 m*	*2 m*	*14 meters*
								6 meters
								$14:6$
Mel	*2 m*	*2 m*	*2 m*					

Shanni	*3 in*	*3 in*	*3 in*	*3 in*	*3 in*	*3 in*	*3 in*	*21 inches*
								9 inches
								$21:9$
Mel	*3 in*	*3 in*	*3 in*					

Exercise 3 (8 minutes)

Work as a class or allow students to work independently first and then go through as a class.

Exercise 3

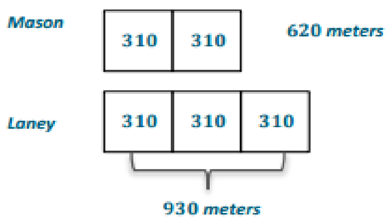
Mason and Laney ran laps to train for the long-distance running team. The ratio of the number of laps Mason ran to the number of laps Laney ran was 2 to 3.

a. **If Mason ran 4 miles, how far did Laney run? Draw a tape diagram to demonstrate how you found the answer.**

4 miles

Mason	*2 mi*	*2 mi*		
Laney	*2 mi*	*2 mi*	*2 mi*	*6 miles*

b. If Laney ran 930 meters, how far did Mason run? Draw a tape diagram to determine how you found the answer.



c. What ratios can we say are equivalent to $2:3$?

$4:6$ *and* $620:930$

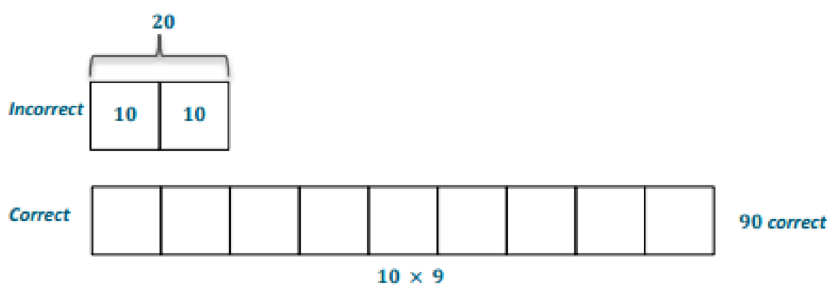
Exercise 4 (7 minutes)

Allow students to work the exercise independently and then compare their answers with a neighbor's answer.

Exercise 4

Josie took a long multiple-choice, end-of-year vocabulary test. The ratio of the number of problems Josie got incorrect to the number of problems she got correct is $2:9$.

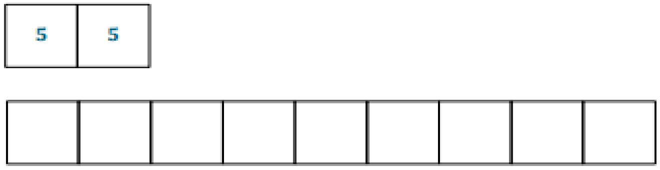
a. If Josie missed 8 questions, how many did she get correct? Draw a tape diagram to demonstrate how you found the answer.



c. What ratios can we say are equivalent to $2:9$?

$8:36$ *and* $20:90$

d. **Come up with another possible ratio of the number Josie got incorrect to the number she got correct.**



$10: 45$

$5 \times 9 = 45$

e. **How did you find the numbers?**

Multiplied 5×2 and 5×9

f. **Describe how to create equivalent ratios.**

Multiply both numbers of the ratio by the same number (any number you choose).

Closing (5 minutes)

Ask students to share their answers to part (f); then summarize by presenting the following definition:

Two ratios $A: B$ and $C: D$ are **equivalent** if there is a positive number, c, such that $C = cA$ and $D = cB$. Ratios are equivalent if there is a positive number that can be multiplied by both quantities in one ratio to find the corresponding quantities in the second ratio.

Note that if students do not have a sufficient grasp of algebra, they should not use the algebraic definition. It is acceptable to use only the second definition.

Lesson Summary

Two ratios $A: B$ and $C: D$ are equivalent ratios if there is a positive number, c, such that $C = cA$ and $D = cB$.

Ratios are equivalent if there is a positive number that can be multiplied by both quantities in one ratio to equal the corresponding quantities in the second ratio.

Exit Ticket (5 minutes)

Name ______________________________ Date________________

Lesson 3: Equivalent Ratios

Exit Ticket

Pam and her brother both open savings accounts. Each begin with a balance of zero dollars. For every two dollars that Pam saves in her account, her brother saves five dollars in his account.

1. Determine a ratio to describe the money in Pam's account to the money in her brother's account.

2. If Pam has 40 dollars in her account, how much money does her brother have in his account? Use a tape diagram to support your answer.

3. Record the equivalent ratio.

4. Create another possible ratio that describes the relationship between the amount of money in Pam's account and the amount of money in her brother's account.

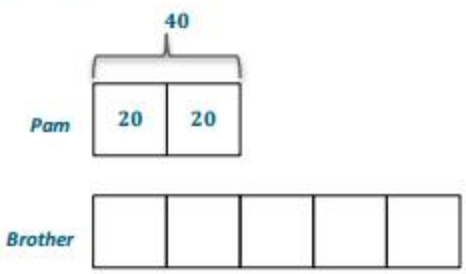
Exit Ticket Sample Solutions

Pam and her brother both open savings accounts. Each begin with a balance of zero dollars. For every two dollars that Pam saves in her account, her brother saves five dollars in his account.

1. Determine a ratio to describe the money in Pam's account to the money in her brother's account.

   $2:5$

2. If Pam has 40 dollars in her account, how much money does her brother have in his account? Use a tape diagram to support your answer.



3. Record the equivalent ratio.

   $40:100$

4. Create another possible ratio that describes the relationship between the amount of money in Pam's account and the amount of money in her brother's account.

   Answers will vary. $4:10$, $8:20$, *etc.*

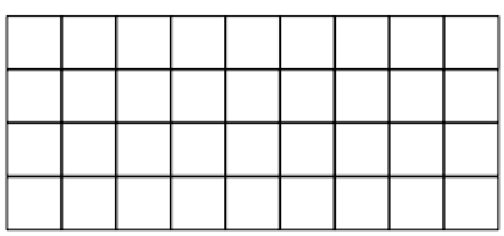
Problem Set Sample Solutions

1. Write two ratios that are equivalent to $1:1$.

   Answers will vary. $2:2$, $50:50$, *etc.*

3.

   a. The ratio of the width of the rectangle to the height of the rectangle is ___9___ to ___4___.



b. If each square in the grid has a side length of 8 mm, what is the width and height of the rectangle?

72 mm wide and 32 mm high.

4. For a project in their health class, Jasmine and Brenda recorded the amount of milk they drank every day. Jasmine drank 2 pints of milk each day, and Brenda drank 3 pints of milk each day.

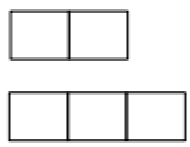
a. Write a ratio of the number of pints of milk Jasmine drank to the number of pints of milk Brenda drank each day.

$2:3$

b. Represent this scenario with tape diagrams.

Jasmine

Brenda



c. If one pint of milk is equivalent to 2 cups of milk, how many cups of milk did Jasmine and Brenda each drink? How do you know?

Jasmine drank 4 cups of milk, and Brenda drank 6 cups of milk. Since each pint represents 2 cups, I multiplied Jasmine's 2 pints by 2 and multiplied Brenda's 3 pints by 2.

d. Write a ratio of the number of cups of milk Jasmine drank to the number of cups of milk Brenda drank.

$4:6$

e. Are the two ratios you determined equivalent? Explain why or why not.

$2:3$ and $4:6$ are equivalent because they represent the same value. The diagrams never changed, only the value of each unit in the diagram.

<table>
<tr><th colspan="2">Instructional Strategies</th><th colspan="2">Meeting the Needs of All Students</th></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>New Vocabulary</td><td colspan="2">Students Achieving Below Standard</td><td>Students Achieving Above Standard</td></tr>
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