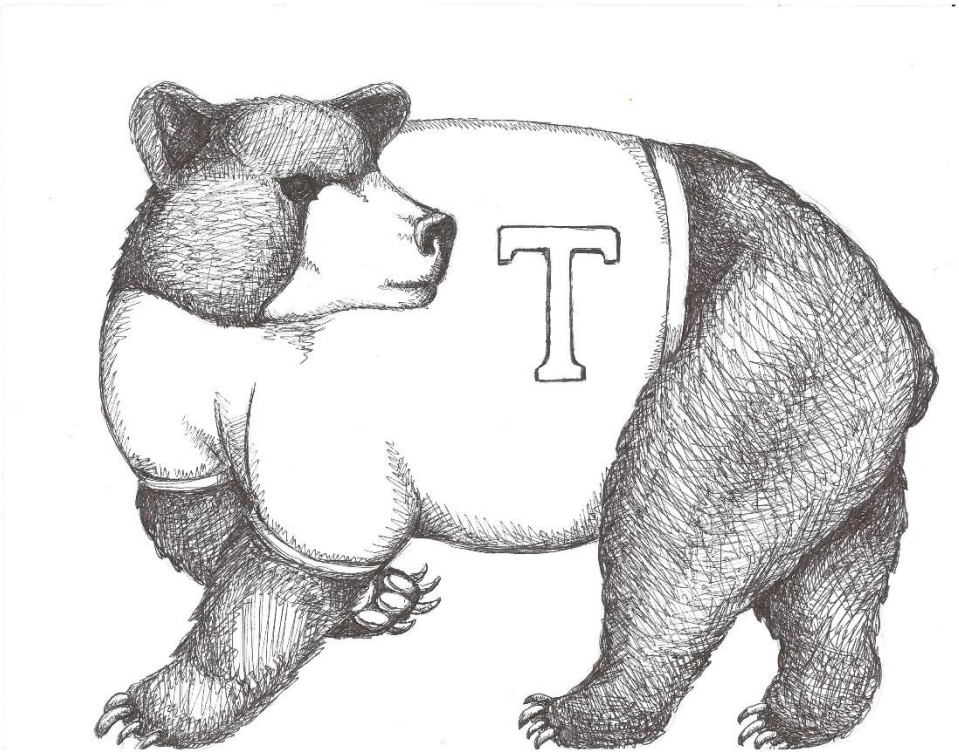
Thomaston Public Schools

158 Main Street
Thomaston, Connecticut 06787
www.thomastonschools.org – 860-283-4796



Thomaston Public Schools Curriculum
Thomaston Center School
Grade 4: Mathematics 2015

The Bridge to Adolescence

Acknowledgements

Curriculum Writer: Jean Nolte

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

_____*Alisha DiCorpo*_____
Alisha L. DiCorpo
Director of Curriculum and Professional Development

Date of Presentation to the Board of Education: August 2015

(Math Curriculum Grade 4)

Grade 4 Mathematics

Board of Education Mission Statement:

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:

The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main resource used when writing this curriculum:

NYS COMMON CORE MATHEMATICS CURRICULUM A Story of Units Curriculum. This work is licensed under a Creative Commons Attribution-Non Commercial-Share Alike 3.0 Unported License. A Story of Units: A Curriculum Overview for Grades P-5 Date: 7/31/13 5 © 2013 Common Core, Inc. Some rights reserved. commoncore.org

Course Description:

Unit 1: Place Value, Rounding, and Algorithms for Addition and Subtraction
Unit 2: Unit Conversions and Problem Solving with Metric Measurement
Unit 3: Multi-Digit Multiplication and Division
Unit 4: Angle Measure and Plane Figures
Unit 5: Fraction Equivalence, Ordering and Operations
Unit 6: Decimal Fractions
Unit 7: Exploring Multiplication

Fourth grade mathematics is about (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Key Areas of focus for 3-5: Multiplication and division of whole numbers and fractions-concepts, skills and problem solving.

Required Fluency: 4.NBT.4 Add and subtract within 1,000,000

CCS Major Emphasis Clusters:

Operations and Algebraic Thinking

• Use the four operations with whole numbers to solve problems.

Number and Operations in Base Ten

• Generalize place value understanding for multi-digit whole numbers.

• Use place value understanding and properties of operations to perform multi-digit arithmetic

. Number and Operations – Fractions

• Extend understanding of fraction equivalence and ordering.

• Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

• Understand decimal notation for fractions, and compare decimal fractions.

K-5 Pacing Guide

	Pre-Kindergarten	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	
20 days	M1: Numbers to 5 (45 days)	M1: Numbers to 10 (43 days)	M1: Sums and Differences to 10 (45 days)	M1: Sums and Differences to 20 (10 days)	M1: Properties of Multiplication and Division and Solving Problems with Units of 2-5 and 10 (25 days)	M1: Place Value, Rounding, and Algorithms for Addition and Subtraction (25 days)	M1: Place Value and Decimal Fractions (20 days)	20 days
				M2: Addition and Subtraction of Length Units (12 days)				
20 days				M3: Place Value, Counting, and Comparison of Numbers to 1000 (25 days)	M2: Place Value and Problem Solving with Units of Measure (25 days)	*M2: Unit Conversions (7 days)	M2: Multi-Digit Whole Number and Decimal Fraction Operations (35 days)	20 days
20 days	M2: Two-Dimensional and Three-Dimensional Shapes (15 days)	*M2: 2D and 3D Shapes (12 days)	M2: Introduction to Place Value Through Addition and Subtraction Within 20 (35 days)	M4: Addition and Subtraction Within 200 with Word Problems to 100 (35 days)	M3: Multiplication and Division with Units of 0, 1, 6-9, and Multiples of 10 (25 days)	M3: Multi-Digit Multiplication and Division (43 days)		20 days
20 days	M3: Counting to Answer Questions of How Many (50 days)	M3: Comparison of Length, Weight, Capacity, and Numbers to 10 (38 days)					M3: Addition and Subtraction of Fractions (22 days)	20 days
20 days			M3: Ordering and Comparing Length Measurements as Numbers (15 days)	M5: Addition and Subtraction Within 1000 with Word Problems to 100 (24 days)	M4: Multiplication and Area (20 days)	M4: Angle Measure and Plane Figures (20 days)	M4: Multiplication and Division of Fractions and Decimal Fractions (38 days)	20 days
20 days		M4: Number Pairs, Addition and Subtraction to 10 (47 days)	M4: Place Value, Comparison, Addition and Subtraction to 40 (35 days)	M6: Foundations of Multiplication and Division (24 days)	M5: Fractions as Numbers on the Number Line (35 days)	M5: Fraction Equivalence, Ordering, and Operations (45 days)		20 days
20 days	M4: Comparison of Length, Weight, and Capacity (35 days)						M5: Addition and Multiplication with Volume and Area (25 days)	20 days
			M5: Identifying, Composing, and Partitioning Shapes (15 days)	M7: Problem Solving with Length, Money, and Data (30 days)	M6: Collecting and Displaying Data (10 days)			
20 days	M5: Numerals to 5, Addition and Subtraction Stories, Counting to 20 (35 days)	M5: Numbers 10-20 and Counting to 100 (30 days)	M6: Place Value, Comparison, Addition and Subtraction to 100 (35 days)		M7: Geometry and Measurement Word Problems (40 days)	M6: Decimal Fractions (20 days)	M6: Problem Solving with the Coordinate Plane (40 days)	20 days
20 days		M6: Analyzing, Comparing, and Composing Shapes (10 days)		M8: Time, Shapes, and Fractions as Equal Parts of Shapes (20 days)		M7: Exploring Multiplication (20 days)		20 days

Approx. test date for grades 3-5

*Please refer to grade-level descriptions to identify partially labeled modules and the standards corresponding to all modules.

Key:	Geometry	Number	Number and Geometry, Measurement	Fractions

GRADE 4 • Unit 1 (MODULE 1)

Place Value, Rounding, and Algorithms for Addition and Subtraction

OVERVIEW

In this 25-day Grade 4 module, students extend their work with whole numbers. They begin with large numbers using familiar units (hundreds and thousands) and develop their understanding of millions by building knowledge of the pattern of *times ten* in the base ten system on the place value chart (**4.NBT.1**). They recognize that each sequence of three digits is read as hundreds, tens, and ones followed by the naming of the corresponding base thousand unit (thousand, million, billion).[1]

The place value chart is fundamental to Topic A. Building upon their previous knowledge of bundling, students learn that 10 hundreds can be composed into 1 thousand and, therefore, 30 hundreds can be composed into 3 thousands because a digit's value is 10 times what it would be one place to its right (**4.NBT.1**). Students learn to recognize that in a number such as 7,777 each 7 has a value that is 10 times the value of its neighbor to the immediate right. One thousand can be decomposed into 10 hundreds, therefore 7 thousands can be decomposed into 70 hundreds.

Similarly, multiplying by 10 shifts digits one place to the left, and dividing by 10 shifts digits one place to the right.

$$3{,}000 = 10 \times 300 \qquad 3{,}000 \div 10 = 300$$

In Topic B, students use place value as a basis for comparing whole numbers. Although this is not a new concept, it becomes more complex as the numbers become larger. For example, it becomes clear that 34,156 is 3 thousands greater than 31,156.

34,156 > 31,156

Comparison leads directly into rounding, where their skill with isolating units is applied and extended. Rounding to the nearest ten and hundred was mastered with three-digit numbers in Grade 3. Now, Grade 4 students moving into Topic C learn to round to any place value (**4.NBT.3**), initially using the vertical number line, though ultimately moving away from the visual model altogether. Topic C also includes word problems where students apply rounding to real life situations.

[1] Grade 4 expectations in the NBT standards domain are limited to whole numbers less than or equal to 1,000,000.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
4.NBT.1 **4.NBT.2** 4.OA.1	A	**Place Value of Multi-Digit Whole Numbers** Lesson 1: Interpret a multiplication equation as a comparison. Lesson 2: Recognize a digit represents 10 times the value of what it represents in the place to its right. Lesson 3: Name numbers within 1 million by building understanding of the place value chart and placement of commas for naming base thousand units. Lesson 4: Read and write multi-digit numbers using base ten numerals, number names, and expanded form.	4
4.NBT.2	B	**Comparing Multi-Digit Whole Numbers** Lesson 5: Compare numbers based on meanings of the digits using >, <, or = to record the comparison. Lesson 6: Find 1, 10, and 100 thousand more and less than a given number.	2
4.NBT.3	C	**Rounding Multi-Digit Whole Numbers** Lesson 7: Round multi-digit numbers to the thousands place using the vertical number line. Lesson 8: Round multi-digit numbers to any place using the vertical number line. Lesson 9: Use place value understanding to round multi-digit numbers to any place value. Lesson 10: Use place value understanding to round multi-digit numbers to any place value using real world applications.	4
		Mid-Module Assessment: Topics A–C (review content 1 day, assessment ½ day, return ½ day, remediation or further applications 1 day)	3
4.OA.3 **4.NBT.4** 4.NBT.1	D	**Multi-Digit Whole Number Addition** Lesson 11: Use place value understanding to fluently add multi-digit	2

4.NBT.2		whole numbers using the standard addition algorithm, and apply the algorithm to solve word problems using tape diagrams. Lesson 12: Solve multi-step word problems using the standard addition algorithm modeled with tape diagrams, and assess the reasonableness of answers using rounding.	

4.OA.3 **4.NBT.4** 4.NBT.1 4.NBT.2	E	**Multi-Digit Whole Number Subtraction** Lesson 13: Use place value understanding to decompose to smaller units once using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams. Lesson 14: Use place value understanding to decompose to smaller units up to three times using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams. Lesson 15: Use place value understanding to fluently decompose to smaller units multiple times in any place using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams. Lesson 16: Solve two-step word problems using the standard subtraction algorithm fluently modeled with tape diagrams, and assess the reasonableness of answers using rounding.	4
4.OA.3 4.NBT.1 4.NBT.2 4.NBT.4	F	**Addition and Subtraction Word Problems** Lesson 17: Solve *additive compare* word problems modeled with tape diagrams. Lesson 18: Solve multi-step word problems modeled with tape diagrams, and assess the reasonableness of answers using rounding. Lesson 19: Create and solve multi-step word problems from given tape diagrams and equations.	3
		End-of-Module Assessment: Topics A–F (review content 1 day, assessment ½	3

		day, return ½ day, remediation or further application 1 day)	
Total Number of Instructional Days			**25**

Math Unit -Unit: 1

Subject: Mathematics

Grade/Course: Grade 4

Pacing: 25 days

Unit of Study: Unit1: Place Value, Rounding, and Algorithms for Addition and Subtraction

Priority Standards:

Use the four operations with whole numbers to solve problems.[2]

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Generalize place value understanding for multi-digit whole numbers. (Grade 4 expectations are limited to whole numbers less than or equal to 1,000,000.)

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

---

[2] Only addition and subtraction multi-step word problems are addressed in this module. The balance of this cluster is addressed in Modules 3 and 7.

Use place value understanding and properties of operations to perform multi-digit arithmetic.[3]

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Foundational Standards

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. [4]

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Math Practice Standards:

MP.1 **Make sense of problems and persevere in solving them.** Students use the place value chart to draw diagrams of the relationship between a digit's value and what it would be one place to its right, for instance, by representing 3 thousands as 30 hundreds. Students also use the place value chart to compare large numbers.

MP.2 **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships as they use both special strategies and the standard addition algorithm to add and subtract multi-digit numbers. Students decontextualize when they represent problems symbolically and contextualize when they consider the value of the units used and understand the meaning of the quantities as they compute.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students construct arguments as they use the place value chart and model single- and multi-step problems. Students also use the standard algorithm as a general strategy to add and subtract multi-digit numbers when a special strategy is not suitable.

MP.5 **Use appropriate tools strategically.** Students decide on the appropriateness of using special strategies or the standard algorithm when adding and subtracting multi-digit numbers.

MP.6 **Attend to precision.** Students use the place value chart to represent digits and their values as

---

[3] The balance of this cluster is addressed in Modules 3 and 7.

[4] This standard is limited to problems with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., the order of operations.

they compose and decompose base ten units.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do) Depth of knowledge levels**
(4.OA.3	
multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems with remainders must be interpreted.	Solve (L-1)
problems using equations with a letter standing for the unknown quantity.	Represent (L-1)
reasonableness of answers using mental computation and estimation strategies including rounding.	Assess(L-3)
(4. NBT.1)	
that a digit in one place represents ten times what it represents in the place to its right in a multi-digit whole number.	Recognize (L-1)

(4. NBT.2) multi-digit whole numbers using base-ten numerals, number names, and expanded form. two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (4. NBT.3) place value understanding to round multi-digit whole numbers to any place (4. NBT.4) multi-digit whole numbers using the standard algorithm fluently .	Read/ Write (L-1) Compare (L-2) Use (L-2) Add and subtract (L-1)

Essential Questions	**Big ideas**

How are place value patterns repeated in large numbers? How can the same number be represented in different ways? When is estimation more appropriate than finding the exact answer? How can I use place value to decompose numbers to solve addition and subtraction problems? How does understanding place value help you solve addition and subtraction problems? How does the position of a digit affect its value?	Relationships among numbers and number systems form the foundations of number sense and mathematical communication. A digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. Mental math and rounding is useful in checking the reasonableness of an answer and for providing an estimate.

Assessments

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Exit tickets or sprints used as pre-assessment for each lesson	**Application problem** **Student Debriefs** **Problem set data**	**Assessment Summary**

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	4.NBT.1 4.NBT.2 4.NBT.3
End-of-Module Assessment Task	After Topic F	Constructed response with rubric	4.NBT.1 4.NBT.2 4.NBT.3 4.NBT.4 4.OA.3

Performance Task
To be created during the year
Engaging Learning Experiences
To be created during the year

Instructional Resources

Useful Websites:
Engage NY K-5 Curriculum overview and guiding documents:
https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents
Engage NY Grade 4 Resources:
https://www.engageny.org/resource/grade-4-mathematics
Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs
North Carolina 4th Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/4th.pdf
Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/
NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx
Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/
LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132
SBAC Digital Library

Suggested Tools and Representations

- Number lines (vertical to represent rounding *up* and rounding *down*)
- Personal white boards (one per student; see explanation on the following pages)
- Place value cards (one large set per classroom including 7 units to model place value)
- Place value chart (templates provided in lessons to insert into personal white boards)
- Place value disks (can be concrete manipulatives or pictorial drawings, such as the chip model, to represent numbers)
- Tape diagrams (drawn to model a word problem)

Instructional Strategies

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability

- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

- **1. Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way
- **2. Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words
- **3. Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.
- **4. Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.
- **5. Nonlinguistic Representations:** has recently been proven to stimulate and increase brain activity.
- **6. Cooperative Learning:** has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.
- **7. Setting Objectives and Providing Feedback**: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied.
- **8. Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material.
- **9. Cues, Questions, and Advanced Organizers:** helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.

Meeting the Needs of All Students

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for

Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

Scaffolds for Students with Disabilities

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.

- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"

- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow “everyday” and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary

- Ten thousands, hundred thousands (as places on the place value chart)
- Millions, ten millions, hundred millions (as places on the place value chart)
- Variable (letters that stand for numbers and can be added, subtracted, multiplied, and divided as numbers are)

Familiar Terms and Symbols[5]

- =, <, > (equal to, less than, greater than)
- Addend (e.g., in 4 + 5, the numbers 4 and 5 are the addends)
- Algorithm (a step-by-step procedure to solve a particular type of problem)
- Bundling, making, renaming, changing, exchanging, regrouping, trading (e.g., exchanging 10 ones for 1 ten)
- Compose (e.g., to make 1 larger unit from 10 smaller units)
- Decompose (e.g., to break 1 larger unit into 10 smaller units)
- Difference (answer to a subtraction problem)
- Digit (any of the numbers 0 to 9; e.g., What is the value of the digit in the tens place?)
- Endpoint (used with rounding on the number line; the numbers that mark the beginning and end of a given interval)
- Equation (e.g., 2,389 + 80,601 = _____)
- Estimate (an approximation of a quantity or number)

[5] These are terms and symbols students have used or seen previously.

- Expanded form (e.g., 100 + 30 + 5 = 135)
- Expression (e.g., 2 thousands × 10)
- Halfway (with reference to a number line, the midpoint between two numbers, e.g., 5 is halfway between 0 and 10)
- Number line (a line marked with numbers at evenly spaced intervals)
- Number sentence (e.g., 4 + 3 = 7)
- Place value (the numerical value that a digit has by virtue of its position in a number)
- Rounding (approximating the value of a given number)
- Standard form (a number written in the format 135)
- Sum (answer to an addition problem)
- Tape diagram (bar diagram)
- Unbundling, breaking, renaming, changing, regrouping, trading (e.g., exchanging 1 ten for 10 ones)
- Word form (e.g., one hundred thirty-five)

Students Achieving Below Standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.
- Guide students as they select and practice using their own graphic organizers and models to solve.
- Use direct instruction for vocabulary with visual or concrete representations.
- Use explicit directions with steps and procedures enumerated.
- Guide students through initial practice promoting gradual independence. “I do, we do, you do.”
- Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.
- Scaffold complex concepts and provide leveled problems for multiple entry points.

Provide Multiple Means of Action and Expression

- First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.
- Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’

- Encourage students to explain their thinking and strategy for the solution.
- Choose numbers and tasks that are “just right” for learners but teach the same concepts.
- Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2.

Provide Multiple Means of Engagement

- Clearly model steps, procedures, and questions to ask when solving.
- Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).
- Have students work together to solve and then check their solutions.
- Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?
- Practice routine to ensure smooth transitions.
- Set goals with students regarding the type of math work students should complete in 60 seconds.
- Set goals with the students regarding next steps and what to focus on next.

Students Achieving Above Standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations.
- Model and post conversation “starters,” such as: “I agree because...” “Can you
- explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...”
- Incorporate written reflection, evaluation, and synthesis.
- Allow creativity in expression and modeling solutions.

Provide Multiple Means of Action and Expression

- Encourage students to explain their reasoning both orally and in writing.
- Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.
- Offer choices of independent or group assignments for early finishers.
- Encourage students to notice and explore patterns and to identify rules and relationships in math.
- Have students share their observations in discussion and writing (e.g., journaling).
- Foster their curiosity about numbers and mathematical ideas.
- Facilitate research and exploration through discussion, experiments, internet searches, trips, etc.

- Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.
- Let students choose their mode of response: written, oral, concrete, pictorial, or abstract.
- Increase the pace. Offer two word problems to solve, rather than one.
- Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem).
- Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.
- Let students write word problems to show mastery and/or extension of the content.

Provide Multiple Means of Engagement

- Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.
- Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds).
- Make the most of the fun exercises for practicing skip-counting.
- Accept and elicit student ideas and suggestions for ways to extend games.

- Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

GRADE 4 • Unit 2 (MODULE 2)

Unit Conversions and Problem Solving with Metric Measurement

OVERVIEW

In this 12-day Grade 2 module, students engage in activities designed to deepen their conceptual understanding of measurement and to relate addition and subtraction to length. Their work in Module 2 is

exclusively with metric units in order to support place value concepts. Customary units are introduced in Module 7.

Topic A opens with students exploring concepts related to the centimeter ruler. In the first lesson, they are guided to connect measurement with physical units as they find the total number of unit lengths by laying multiple copies of centimeter cubes (physical units) end-to-end along various objects. Through this, students discover that to get an accurate measurement, there must be no gaps or overlaps between consecutive length units.

Next, students measure by iterating with one physical unit, using the *mark and advance* technique, also known as *mark and move forward*. Students then repeat the process by laying both multiple copies and a single cube along a centimeter ruler. This helps students create a mental benchmark for the centimeter. It also helps them realize that the distance between 0 and 1 on the ruler indicates the amount of space already covered. Hence 0, not 1, marks the beginning of the total length. Students use this understanding to create their own centimeter rulers using a centimeter cube and the mark and advance technique. Topic A ends with students using their unit rulers to measure lengths (**2.MD.1**), thereby connecting measurement with a ruler.

Students build skill in measuring using centimeter rulers and meter sticks in Topic B. They learn to see that a length unit is not a cube, or a portion of a ruler (which has width), but is a segment of a line. By measuring a variety of objects, students build a bank of known measurements or benchmark lengths, such as a doorknob being a meter from the floor, or the width of a finger being a centimeter. Then, students learn to estimate length using knowledge of previously measured objects and benchmarks. This enables students to internalize the mental rulers[6] of a centimeter or meter, empowering them to mentally iterate units relevant to measuring a given length (**2.MD.3**). The knowledge and experience signal that students are determining which tool is appropriate to make certain measurements (**2.MD.1**).

In Topic C, students measure and compare to determine how much longer one object is than another (**2.MD.4**). They also measure objects twice using different length units, both standard and non-standard, thereby developing their understanding of how the total measurement relates to the size of the length unit (**2.MD.2**). Repeated experience and explicit comparisons help students recognize that the smaller the length unit, the larger the number of units, and the larger the length unit, the smaller the number of units.

The module culminates as students relate addition and subtraction to length. They apply their conceptual understanding to choose appropriate tools and strategies, such as the ruler as a number line, benchmarks for estimation, and tape diagrams for comparison, to solve word problems (**2.MD.5**, **2.MD.6**). The problems progress from concrete (i.e., measuring objects and using the ruler as a number line to add and subtract) to abstract (i.e., representing lengths with tape diagrams to solve *start unknown* and two-step problems).

---

[6] See the Progression Document "Geometric Measurement," page 14.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
4.MD.1 **4.MD.2**	A	**Metric Unit Conversions** Lesson 1: Express metric length measurements in terms of a smaller unit; model and solve addition and subtraction word problems involving metric length. Lesson 2: Express metric mass measurements in terms of a smaller unit; model and solve addition and subtraction word problems involving metric mass. Lesson 3: Express metric capacity measurements in terms of a smaller unit; model and solve addition and subtraction word problems involving metric capacity.	3
4.MD.1 **4.MD.2**	B	**Application of Metric Unit Conversions** Lesson 4: Know and relate metric units to place value units in order to express measurements in different units. Lesson 5: Use addition and subtraction to solve multi-step word problems involving length, mass, and capacity.	2
		End-of-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
Total Number of Instructional Days			**7**

Subject: Mathematics

Grade/Course: Grade 4

Pacing: 7 days

Unit of Study: Unit: 2 Unit Conversion

Priority Standards:

Focus Grade Level Standards

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[7]

4.MD.1[8] Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),*

4.MD.2[9] Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Foundational Standards

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special case:

a. 100 can be thought of as a bundle of ten tens—called a "hundred."

---

[7] 4.MD.3 is addressed in Module 3.

[8] Pounds, ounces, and time are addressed in Module 7. This is a non-tested standard, but expressing metric measurements of length, mass, and capacity from larger to smaller units strengthens the upcoming modules.

[9] Time and money are addressed in Module 7. This is a non-tested standard, but the contexts of operating on distance, volume, and mass strengthen the upcoming modules.

2.MD.5 **Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.**

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (L). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems, i.e., problems involving notions of "times as much.")

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Math Practice Standards:

MP.1 **Make sense of problems and persevere in solving them.** Students use place value knowledge to convert larger units to smaller units before adding and subtracting. They fluently add and subtract metric units of length, weight, and capacity using the standard algorithm. Tape diagrams and number lines help students conceptualize a problem before it is solved and are used to assess the reasonableness of an answer.

MP.7 **Look for and make use of structure.** Students use knowledge of place value and mixed units to find patterns when converting from a larger unit to a smaller unit. They recognize that 1 thousand equals 1,000 ones and relate that to 1 kilometer equals 1,000 meters. Using this pattern, they might extend thinking to convert smaller to larger units when making a conversion chart.

MP.8 **Look for and express regularity in repeated reasoning.** Students find that metric unit conversions share a relationship on the place value chart. For example, 1,000 ones equals 1 thousand, 1,000 g equals 1 kg, 1,000 mL equals 1 L, and 1,000 m equals 1 km. Knowing and using these conversions and similarities allows for quick and easy conversion and calculation.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do) Depth of Knowledge levels**
(4.MD.1)relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec.	Know (L-1)
measurements in a larger unit in terms of a smaller unit within a single system of measurement.	Express (L-2)
measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in.	Record (L-2)
the length of a 4 ft snake as 48 in.	Express (L-2)
a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …	Generate (L-3)
(4.MD.2) the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.	Use (L-1)
measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	Represent (L-2)

Essential Questions	Big ideas
Why are units important in measurement? Why are standard units important? Why do we need to be able to measure distance? How are the units of linear measurement within a standard system related? What happens to a measurement when we change units? When should we measure with meters? Kilometers? When should we measure with inches? Feet? Yards?	3. Measuring sense involves an understanding of appropriate measurement units in various situations, the appropriate use of measurement tools, and of estimation in measurement.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Exit tickets for pre-assessment for each lesson	**Application problem** **Student Debriefs** **Problem set data**	**Assessment Summary**

Type	Administered	Format	Standards Addressed
End-of-Module Assessment Task	After Topic B	Constructed response with rubric	4.MD.1 4.MD.2

Performance Task
To be created during the year
Engaging Learning Experiences
To be created during the year

Instructional Resources
Useful Websites: Engage NY K-5 Curriculum overview and guiding documents: https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents Engage NY Grade 4 Resources: https://www.engageny.org/resource/grade-4-mathematics Eureka Math Module PDFs: http://greatminds.net/maps/math/module-pdfs North Carolina 4th Grade Standards Unpacked: http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/4th.pdf Illustrative Mathematics – problems and tasks by grade and standard https://www.illustrativemathematics.org/ NCTM Illuminations – problems, tasks and interactives by grade and standard http://illuminations.nctm.org/Default.aspx Inside Mathematics – Problems of the Month and Performance Assessment tasks http://www.insidemathematics.org/ LearnZillion –lesson plans/some with embedded tasks https://learnzillion.com/resources/17132 SBAC Digital Library ▪ **Suggested Tools and Representations**

- Balance scale, weights (masses)
- Centimeter ruler, meter stick
- Liter containers with millimeter scale
- Number line
- Tape diagram
- Two-column table

Instructional Strategies

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

1. **Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way
2. **Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words
3. **Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.
4. **Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.
5. **Nonlinguistic Representations:** has recently been proven to stimulate and increase brain activity.
6. **Cooperative Learning:** has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.
7. **Setting Objectives and Providing Feedback**: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied.
8. **Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material.

9. Cues, Questions, and Advanced Organizers: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.

Meeting the Needs of All Students

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

Scaffolds for Students with Disabilities

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to

calculate and explain. For example, a student searching to define “multiplication” may model groups of 6 with drawings or concrete objects and write the number sentence to match.

- Teach students how to ask questions (such as “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…”
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.”
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are “just right” for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day’s sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including “show and tell” rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?”
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?”
- Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

<u>Provide Multiple Means of Engagement</u>

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary

New or Recently Introduced Terms

- Convert (express a measurement in a different unit; rename units)
- Kilometer (km, a unit of measure for length)
- Mass (the measure of the amount of matter in an object)
- Milliliter (mL, a unit of measure for liquid volume)
- Mixed units (e.g., 3 m 43 cm)

Familiar Terms and Symbols[10]

- Centimeter (standard length unit within the metric system)
- Combine (join or put together)
- Compare (specifically using direct comparison)
- Difference (to find the difference between two numbers, subtract the smaller number from the greater number)
- Height (vertical distance measurement from bottom to top)
- Length (distance measurement from end to end; in a rectangular shape, length can be used to describe any of the four sides)
- Length unit (e.g., centimeters, inches)

Students Achieving Below Standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.
- Guide students as they select and practice using their own graphic organizers and models to solve.
- Use direct instruction for vocabulary with visual or concrete representations.
- Use explicit directions with steps and procedures enumerated.
- Guide students through initial practice promoting gradual independence. “I do, we do, you do.”
- Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.
- Scaffold complex concepts and provide leveled problems for multiple entry points.

Provide Multiple Means of Action and Expression

- First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.
- Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’
- Encourage students to explain their thinking and strategy for the solution.
- Choose numbers and tasks that are “just right” for learners but teach the same concepts.
- Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2.

[10] These are terms and symbols students have used or seen previously.

Provide Multiple Means of Engagement

- Clearly model steps, procedures, and questions to ask when solving.
- Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).
- Have students work together to solve and then check their solutions.
- Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?
- Practice routine to ensure smooth transitions.
- Set goals with students regarding the type of math work students should complete in 60 seconds.
- Set goals with the students regarding next steps and what to focus on next.

Students Achieving Above Standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach students how to ask questions (such as, “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations.
- Model and post conversation “starters,” such as: “I agree because...” “Can you
- explain how you solved it? ”I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...”
- Incorporate written reflection, evaluation, and synthesis.
- Allow creativity in expression and modeling solutions.

Provide Multiple Means of Action and Expression

- Encourage students to explain their reasoning both orally and in writing.
- Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.
- Offer choices of independent or group assignments for early finishers.
- Encourage students to notice and explore patterns and to identify rules and relationships in math.
- Have students share their observations in discussion and writing (e.g., journaling).
- Foster their curiosity about numbers and mathematical ideas.
- Facilitate research and exploration through discussion, experiments, internet searches, trips, etc.
- Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.
- Let students choose their mode of response: written, oral, concrete, pictorial, or abstract.
- Increase the pace. Offer two word problems to solve, rather than one.

- Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem).
- Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.
- Let students write word problems to show mastery and/or extension of the content.

Provide Multiple Means of Engagement

- Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.
- Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds).
- Make the most of the fun exercises for practicing skip-counting.
- Accept and elicit student ideas and suggestions for ways to extend games.

- Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

GRADE 4 • Unit 3 (MODULE 3)

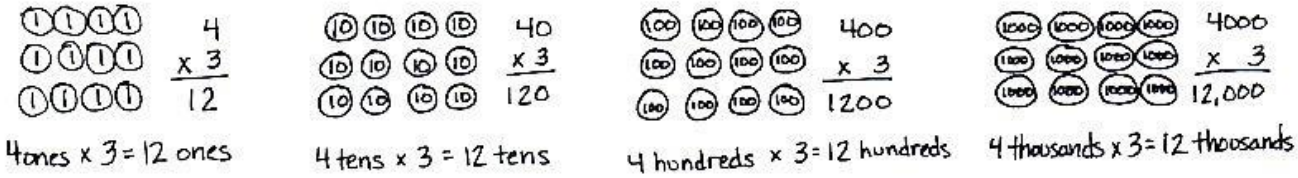
Multi-Digit Multiplication and Division

OVERVIEW

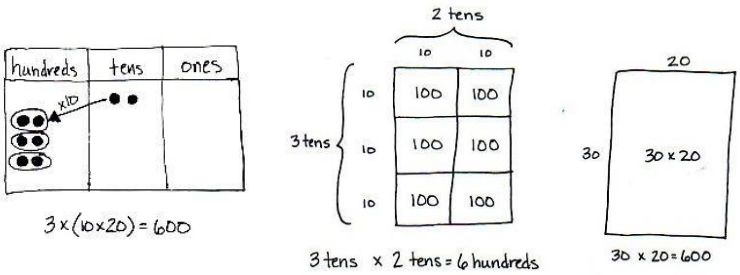
In this 43-day module, students use place value understanding and visual representations to solve multiplication and division problems with multi-digit numbers. As a key area of focus for Grade 4, this module moves slowly but comprehensively to develop students' ability to reason about the methods and models chosen to solve problems with multi-digit factors and dividends.

Students begin in Topic A by investigating the formulas for area and perimeter. They then solve multiplicative comparison problems including the language of *times as much as* with a focus on problems using area and perimeter as a context (e.g., "A field is 9 feet wide. It is 4 times as long as it is wide. What is the perimeter of the field?"). Students create diagrams to represent these problems as well as write equations with symbols for the unknown quantities (**4.OA.1**). This is foundational for understanding multiplication as scaling in Grade 5 and sets the stage for proportional reasoning in Grade 6. This Grade 4 module, beginning with area and perimeter, allows for new and interesting word problems as students learn to calculate with larger numbers and interpret more complex problems (**4.OA.2**, **4.OA.3**, **4.MD.3**).

In Topic B, students use place value disks to multiply single-digit numbers by multiples of 10, 100, and 1,000 and two-digit multiples of 10 by two-digit multiples of 10 (**4.NBT.5**). Reasoning between arrays and written numerical work allows students to see the role of place value units in multiplication (as pictured below). Students also practice the language of units to prepare them for multiplication of a single-digit factor by a factor with up to four digits and multiplication of two two-digit factors.



In preparation for two-digit by two-digit multiplication, students practice the new complexity of multiplying two two-digit multiples of 10. For example, students have multiplied 20 by 10 on the place value chart and know that it shifts the value one place to the left, 10 × 20 = 200. To multiply 20 by 30, the associative property allows for simply tripling the product, 3 × (10 × 20), or multiplying the units, 3 tens × 2 tens = 6 hundreds (alternatively, (3 × 10) × (2 × 10) = (3 × 2) × (10 × 10)). Introducing this early in the module allows students to practice during fluency so that, by the time it is embedded within the two-digit by two-digit multiplication in Topic H, understanding and skill are in place.



Building on their work in Topic B, students begin in Topic C decomposing numbers into base ten units in order to find products of single-digit by multi-digit numbers. Students use the distributive property and multiply using place value disks to model. Practice with place value disks is used for two-, three-, and four-digit by

one-digit multiplication problems with recordings as partial products. Students bridge partial products to the recording of multiplication via the standard algorithm.[11] Finally, the partial products method, the standard algorithm, and the area model are compared and connected by the distributive property (**4.NBT.5**).

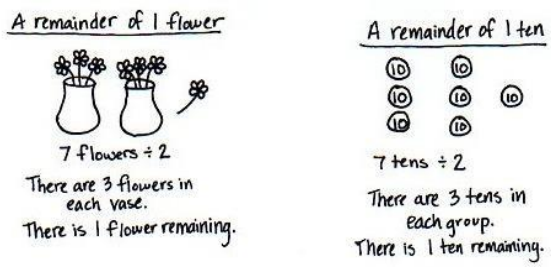
1,423 x 3

Topic D gives students the opportunity to apply their new multiplication skills to solve multi-step word problems (**4.OA.3**, **4.NBT.5**) and multiplicative comparison problems (**4.OA.2**). Students write equations from statements within the problems (**4.OA.1**) and use a combination of addition, subtraction, and multiplication to solve.

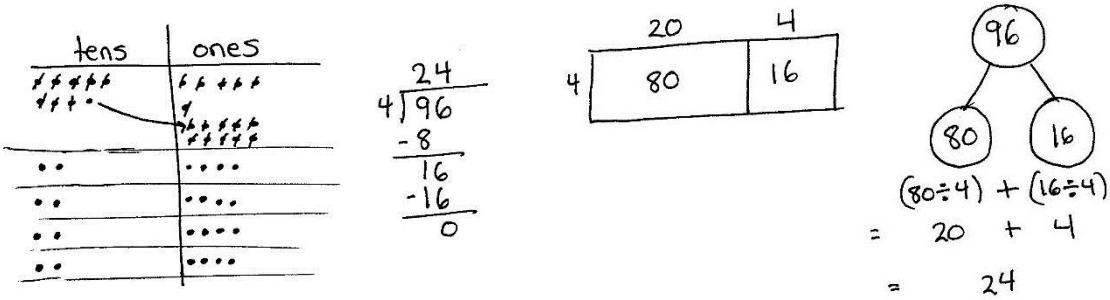
In Topic E, students synthesize their Grade 3 knowledge of division types (*group size unknown* and *number of groups unknown*) with their new, deeper understanding of place value.

[11] Students become fluent with the standard algorithm for multiplication in Grade 5 (5.NBT.5). Grade 4 students are introduced to the standard algorithm in preparation for fluency and as a general method for solving multiplication problems based on place value strategies, alongside place value disks, partial products, and the area model. Students are not assessed on the standard algorithm in Grade 4.

Students focus on interpreting the remainder within division problems, both in word problems and long division (**4.OA.3**). A remainder of 1, as exemplified below, represents a leftover flower in the first situation and a remainder of 1 ten in the second situation.[12]



While we have no reason to subdivide a remaining flower, there are good reasons to subdivide a remaining ten. Students apply this simple idea to divide two-digit numbers unit by unit: dividing the tens units first, finding the remainder (the number of tens unable to be divided), and decomposing remaining tens into ones to then be divided. Students represent division with single-digit divisors using arrays and the area model before practicing with place value disks. The standard division algorithm[13] is practiced using place value knowledge, decomposing unit by unit. Finally, students use the area model to solve division problems, first with and then without remainders (**4.NBT.6**).



In Topic F, armed with an understanding of remainders, students explore factors, multiples, and prime and composite numbers within 100 (**4.OA.4**), gaining valuable insights into patterns of divisibility as they test for primes and find factors and multiples. This prepares them for Topic G's work with multi-digit dividends.

Topic G extends the practice of division with three- and four-digit dividends using place value understanding. A connection to Topic B is made initially with dividing multiples of 10, 100, and 1,000 by single-digit numbers. Place value disks support students visually as they decompose each unit before dividing. Students then

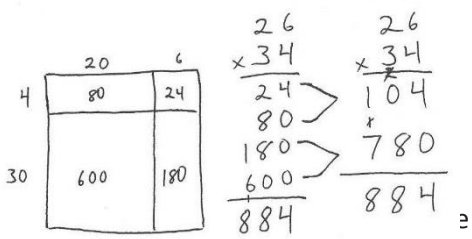
---

[12] Note that care must be taken in the interpretation of remainders. Consider the fact that 7 ÷ 3 is not equal to 5 ÷ 2 because the remainder of 1 is in reference to a different whole amount (213 is not equal to 212).

[13] Students become fluent with the standard division algorithm in Grade 6 (6.NS.2). For adequate practice in reaching fluency, students are introduced to, but not assessed on, the division algorithm in Grade 4 as a general method for solving division problems.

practice using the standard algorithm to record long division. They solve word problems and make connections to the area model as was done with two-digit dividends (**4.NBT.6**, **4.OA.3**).

The module closes as students multiply two-digit by two-digit number understanding and understanding of the area model to empower ther pictured to the right). Topic H culminates at the most abstract level by explicitly connecting the partial products appearing in the area model to the distributive property and recording the calculation vertically (**4.NBT.5**). Students see that p same as those obtained via the distributive property: 4 twenty-sixes +



As students progress through this module, they are able to apply the multiplication and division algorithms because of their in-depth experience with the place value system and multiple conceptual models. This helps to prepare them for fluency with the multiplication algorithm in Grade 5 and the division algorithm in Grade 6. Students are encouraged in Grade 4 to continue using models to solve when appropriate.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives			Days
4.OA.1 **4.OA.2** **4.MD.3** 4.OA.3	A	**Multiplicative Comparison Word Problems**		3
		Lesson 1:	Investigate and use the formulas for area and perimeter of rectangles.	
		Lesson 2:	Solve multiplicative comparison word problems by applying the area and perimeter formulas.	
		Lesson 3:	Demonstrate understanding of area and perimeter formulas by solving multi-step real world problems.	
4.NBT.5 4.OA.1 4.OA.2 4.NBT.1	B	**Multiplication by 10, 100, and 1,000**		3
		Lesson 4:	Interpret and represent patterns when multiplying by 10, 100, and 1,000 in arrays and numerically.	
		Lesson 5:	Multiply multiples of 10, 100, and 1,000 by single digits, recognizing patterns.	
		Lesson 6:	Multiply two-digit multiples of 10 by two-digit multiples of 10	

		with the area model.	

Standards		Topics and Objectives	Days
4.NBT.5 4.OA.2 4.NBT.1	C	**Multiplication of up to Four Digits by Single-Digit Numbers** Lesson 7: Use place value disks to represent two-digit by one-digit multiplication. Lesson 8: Extend the use of place value disks to represent three- and four-digit by one-digit multiplication. Lessons 9–10: Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm. Lesson 11: Connect the area model and the partial products method to the standard algorithm.	5
4.OA.1 **4.OA.2** **4.OA.3** **4.NBT.5**	D	**Multiplication Word Problems** Lesson 12: Solve two-step word problems, including multiplicative comparison. Lesson 13: Use multiplication, addition, or subtraction to solve multi-step word problems.	2
		Mid-Module Assessment: Topics A–D (review 1 day, assessment ½ day, return ½ day)	2
4.NBT.6 4.OA.3	E	**Division of Tens and Ones with Successive Remainders** Lesson 14: Solve division word problems with remainders. Lesson 15: Understand and solve division problems with a remainder using the array and area models. Lesson 16: Understand and solve two-digit dividend division problems with a remainder in the ones place by using place value disks. Lesson 17: Represent and solve division problems requiring decomposing a remainder in the tens. Lesson 18: Find whole number quotients and remainders.	8

		Lesson 19: Explain remainders by using place value understanding and models. Lesson 20: Solve division problems without remainders using the area model. Lesson 21: Solve division problems with remainders using the area model.	

4.OA.4	F	**Reasoning with Divisibility** Lesson 22: Find factor pairs for numbers to 100, and use understanding of factors to define prime and composite. Lesson 23: Use division and the associative property to test for factors and observe patterns. Lesson 24: Determine if a whole number is a multiple of another number. Lesson 25: Explore properties of prime and composite numbers to 100 by using multiples.	4
4.OA.3 **4.NBT.6** 4.NBT.1	G	**Division of Thousands, Hundreds, Tens, and Ones** Lesson 26: Divide multiples of 10, 100, and 1,000 by single-digit numbers. Lesson 27: Represent and solve division problems with up to a three-digit dividend numerically and with place value disks requiring decomposing a remainder in the hundreds place. Lesson 28: Represent and solve three-digit dividend division with divisors of 2, 3, 4, and 5 numerically. Lesson 29: Represent numerically four-digit dividend division with divisors of 2, 3, 4, and 5, decomposing a remainder up to three times. Lesson 30: Solve division problems with a zero in the dividend or with a zero in the quotient. Lesson 31: Interpret division word problems as either *number of groups unknown* or *group size unknown*. Lesson 32: Interpret and find whole number quotients and remainders to solve one-step division word problems with larger divisors of 6, 7, 8, and 9. Lesson 33: Explain the connection of the area model of division to the long	8

		division algorithm for three- and four-digit dividends.	
4.NBT.5 4.OA.3 4.MD.3	H	**Multiplication of Two-Digit by Two-Digit Numbers** Lesson 34: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart. Lesson 35: Multiply two-digit multiples of 10 by two-digit numbers using the area model. Lesson 36: Multiply two-digit by two-digit numbers using four partial products. Lessons 37–38: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.	5
		End-of-Module Assessment: Topics A–H (review 1 day, assessment ½ day, return ½ day, remediation or further application 1 day)	3
Total Number of Instructional Days			**43**

Math Unit - Unit: 3

Subject: Mathematics

Grade/Course: Grade 4

Pacing: 43 days

Unit of Study: Unit: 3 Multi-digit Multiplication and Division

Priority Standards:

Focus Grade Level Standards

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using

drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCLS Glossary, Table 2.)

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Foundational Standards

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See CCLS Glossary, Table 2.)

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?.*

3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

3.MD.7 Relate area to the operations of multiplication and addition.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Use place value understanding and properties of operations to perform multi-digit arithmetic.[14]

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[15]

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical

---

[14] 4.NBT.4 is addressed in Module 1 and is then reinforced throughout the year.

[15] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.2 is addressed in Modules 2, 5, 6, and 7.

problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Math Practice Standards:

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students solve multi-step word problems using the four operations by writing equations with a letter standing in for the unknown quantity.

MP.4 **Model with mathematics.** Students apply their understanding of place value to create area models and rectangular arrays when performing multi-digit multiplication and division. They use these models to illustrate and explain calculations.

MP.5 **Use appropriate tools strategically.** Students use mental computation and estimation strategies to assess the reasonableness of their answers when solving multi-step word problems. They draw and label bar and area models to solve problems as part of the RDW process. Additionally, students select an appropriate place value strategy when solving multiplication and division problems.

MP.8 **Look for and express regularity in repeated reasoning.** Students express the regularity they notice in repeated reasoning when they apply place value strategies in solving multiplication and division problems. They move systematically through the place values, decomposing or composing units as necessary, applying the same reasoning to each successive unit.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**

Use the four operations with whole numbers to solve problems.	
4.OA.1 a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.	Interpret (L-2)
verbal statements of multiplicative comparisons as multiplication equations.	Represent (L-1) Multiply or divide (L-2)
4.OA.2 to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCLS Glossary, Table 2.)	Solve (L-1)
4.OA.3 a multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.	Represent (L-1) Assess (L-3)
these problems using equations with a letters standing for the unknown quantity.	Find (L-1)
the reasonableness of answers using mental computation and estimation strategies including rounding.	Recognize (L-2) Determine (L-2)
Gain familiarity with factors and multiples.	

4.OA.4 all factor pairs for a whole number in the range 1–100.	Determine(L-2)
that a whole number is a multiple of each of its factors.	Determine (L-2)
whether a given whole number in the range 1–100 is a multiple of a given one-digit number.	Multiply (L-1)
whether a given whole number in the range 1–100 is prime or composite.	
Use place value understanding and properties of operations to perform multi-digit arithmetic.[16]	Illustrate and explain (L-2)
4.NBT.5 a whole number of up to four digits by a one-digit whole number, and two two-digit numbers, using strategies based on place value and the properties of operations.	Find (L-1)
the calculation by using equations, rectangular arrays, and/or area models.	Illustrate and explain (L-2)
4.NBT.6 whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.	Apply (L-1)
the calculation by using equations, rectangular arrays, and/or area models.	
Solve problems involving measurement and	

[16] 4.NBT.4 is addressed in Module 1 and is then reinforced throughout the year.

conversion of measurements from a larger unit to a smaller unit.[17] **4.MD.3** the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	

Essential Questions	**Big ideas**
When is it more efficient to use multiplication and division to solve problems? Why does knowing fair shares and equal groups help you explain multiplication and division problems? How are factors and multiples related? How do I identify prime numbers? How do I identify composite numbers? What is the difference between a prime and a composite number?	Equal groups of numbers can be joined to multiply and decomposed into equal groups to divide. Sometimes decomposing numbers into equal groups results in a remainder.

[17] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.2 is addressed in Modules 2, 5, 6, and 7.

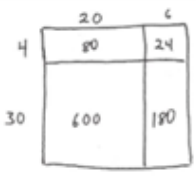
Assessments		
Common Formative Pre-Assessments **Exit tickets for pre-assessment for each lesson**	Progress Monitoring Checks – "Dipsticks" **Application Problems** **Student Debriefs** **Problem set data**	Common Formative Mid and or Post-Assessments Resources
Performance Task		
To be created during the year		
Engaging Learning Experiences		
To be created during the year		

Instructional Resources
Useful Websites: Engage NY K-5 Curriculum overview and guiding documents: https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents Engage NY Grade 4 Resources: https://www.engageny.org/resource/grade-4-mathematics Eureka Math Module PDFs: http://greatminds.net/maps/math/module-pdfs North Carolina 4th Grade Standards Unpacked: http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/4th.pdf Illustrative Mathematics – problems and tasks by grade and standard https://www.illustrativemathematics.org/ NCTM Illuminations – problems, tasks and interactives by grade and standard http://illuminations.nctm.org/Default.aspx Inside Mathematics – Problems of the Month and Performance Assessment tasks http://www.insidemathematics.org/ LearnZillion –lesson plans/some with embedded tasks https://learnzillion.com/resources/17132 SBAC Digital Library

Suggested Tools and Representations

- Area model
- Grid paper
- Number bond
- Place value disks: suggested minimum of 1 set per pair of students (18 ones, 18 tens, 18 hundreds, 18 thousands, 1 ten thousand)

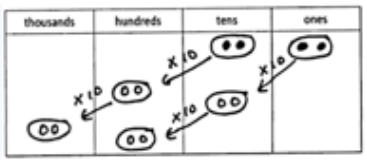
Area Model



Number Bond

Place Value Disks

Thousands Place Value Chart



- Tape diagram
- Ten thousands place value chart (Lesson 7 Template)
- Thousands place value chart (Lesson 4 Template)

Instructional Strategies

<u>21st Century Skills</u>

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

<u>Marzano's Nine Instructional Strategies for Effective Teaching and Learning</u>

1. **Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way
2. **Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words
3. **Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.

4. **Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.
5. **Nonlinguistic Representations:** has recently been proven to stimulate and increase brain activity.
6. **Cooperative Learning:** has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.
7. **Setting Objectives and Providing Feedback**: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied.
8. **Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material.
9. **Cues, Questions, and Advanced Organizers:** helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.

Meeting the Needs of All Students

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

Scaffolds for Students with Disabilities

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these

students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:
- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask

questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?”

- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?”
- Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

<u>Provide Multiple Means of Engagement</u>

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!”
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow “everyday” and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary

New or Recently Introduced Terms

- Associative property (e.g., $96 = 3 \times (4 \times 8) = (3 \times 4) \times 8$)
- Composite number (positive integer having three or more whole number factors)
- Distributive property (e.g., $64 \times 27 = (60 \times 20) + (60 \times 7) + (4 \times 20) + (4 \times 7)$)
- Divisible
- Divisor (the number by which another number is divided)

- Formula (a mathematical rule expressed as an equation with numbers and/or variables)
- Long division (process of dividing a large dividend using several recorded steps)
- Partial product (e.g., 24 × 6 = (20 × 6) + (4 × 6) = 120 + 24)
- Prime number (positive integer greater than 1 having whole number factors of only 1 and itself)
- Remainder (the number left over when one integer is divided by another)

Familiar Terms and Symbols[18]

- Algorithm (steps for base ten computations with the four operations)
- Area (the amount of two-dimensional space in a bounded region)
- Area model (a model for multiplication and division problems that relates rectangular arrays to area, in which the length and width of a rectangle represent the factors for multiplication, and for division the width represents the divisor and the length represents the quotient)
- Array (a set of numbers or objects that follow a specific pattern, a matrix)
- Bundling, grouping, renaming, changing (compose or decompose a 10, 100, etc.)
- Compare (to find the similarity or dissimilarity between)
- Distribute (decompose an unknown product in terms of two known products to solve)
- Divide, division (e.g., 15 ÷ 5 = 3)
- Equation (a statement that the values of two mathematical expressions are equal using the = sign)
- Factors (numbers that can be multiplied together to get other numbers)
- Mixed units (e.g., 1 ft 3 in, 4 lb 13 oz)
- Multiple (product of a given number and any other whole number)
- Multiply, multiplication (e.g., 5 × 3 = 15)
- Perimeter (length of a continuous line forming the boundary of a closed geometric figure)
- Place value (the numerical value that a digit has by virtue of its position in a number)
- Product (the result of multiplication)
- Quotient (the result of division)
- Rectangular array (an arrangement of a set of objects into rows and columns)
- Rows, columns (e.g., in reference to rectangular arrays)
- ___ *times as many* ___ *as* ___ (multiplicative comparative sentence frame)

Students Achieving Below Standard

[18] These are terms and symbols students have used or seen previously.

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.
- Guide students as they select and practice using their own graphic organizers and models to solve.
- Use direct instruction for vocabulary with visual or concrete representations.
- Use explicit directions with steps and procedures enumerated.
- Guide students through initial practice promoting gradual independence. “I do, we do, you do.”
- Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.
- Scaffold complex concepts and provide leveled problems for multiple entry points.

Provide Multiple Means of Action and Expression

- First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.
- Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’
- Encourage students to explain their thinking and strategy for the solution.
- Choose numbers and tasks that are “just right” for learners but teach the same concepts.
- Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2.

Provide Multiple Means of Engagement

- Clearly model steps, procedures, and questions to ask when solving.
- Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).
- Have students work together to solve and then check their solutions.
- Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?
- Practice routine to ensure smooth transitions.
- Set goals with students regarding the type of math work students should complete in 60 seconds.
- Set goals with the students regarding next steps and what to focus on next.

Students Achieving Above Standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations.
- Model and post conversation "starters," such as: "I agree because..." "Can you
- explain how you solved it?""I noticed that..."""Your solution is different from/ the same as mine because...""My mistake was to..."
- Incorporate written reflection, evaluation, and synthesis.
- Allow creativity in expression and modeling solutions.

Provide Multiple Means of Action and Expression

- Encourage students to explain their reasoning both orally and in writing.
- Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.
- Offer choices of independent or group assignments for early finishers.
- Encourage students to notice and explore patterns and to identify rules and relationships in math.
- Have students share their observations in discussion and writing (e.g., journaling).
- Foster their curiosity about numbers and mathematical ideas.
- Facilitate research and exploration through discussion, experiments, internet searches, trips, etc.
- Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.
- Let students choose their mode of response: written, oral, concrete, pictorial, or abstract.
- Increase the pace. Offer two word problems to solve, rather than one.
- Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem).
- Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.
- Let students write word problems to show mastery and/or extension of the content.

Provide Multiple Means of Engagement

- Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.
- Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds).
- Make the most of the fun exercises for practicing skip-counting.
- Accept and elicit student ideas and suggestions for ways to extend games.

- Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

GRADE 4 • Unit 4 (MODULE 4)

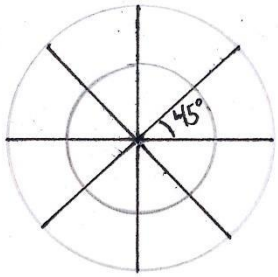
Angle Measure and Plane Figures

OVERVIEW

This 20-day module introduces points, lines, line segments, rays, and angles, as well as the relationships between them. Students construct, recognize, and define these geometric objects before using their new knowledge and understanding to classify figures and solve problems. With angle measure playing a key role in the work throughout the module, students learn how to create and measure angles, as well as how to create and solve equations to find unknown angle measures. In these problems, where the unknown angle is represented by a letter, students explore both measuring the unknown angle with a protractor and reasoning through the solving of an equation. This connection between the measurement tool and the numerical work lays an important foundation for success with middle school geometry and algebra. Through decomposition and composition activities, as well as an exploration of symmetry, students recognize specific attributes present in two-dimensional figures. They further develop their understanding of these attributes as they classify two-dimensional figures.

Topic A begins with students drawing points, lines, line segments, and rays, as well as identifying these in various contexts and within familiar figures. Students recognize that two rays sharing a common endpoint form an angle (**4.MD.5**). They create right angles through a paper-folding activity, identify right angles in their environment, and see that one angle can be greater (obtuse) or less (acute) than a right angle. Next, students use their understanding of angles to explore relationships between pairs of lines as they define, draw, and recognize intersecting, perpendicular, and parallel lines (**4.G.1**).

In Topic B, students explore the definition of degree measure, beginning with a circular protractor. By dividing the circumference of a circle into 360 equal parts, they recognize one part as representing 1 degree (**4.MD.5**). Through exploration, students realize that, although the size of a circle may change, an angle spans an arc, representing a constant fraction of the circumference. By carefully distinguishing the attribute of degree measure from that of length measure, the common misconception that degrees are a measure of length is avoided. Armed with their understanding of the degree as a unit of measure, students use various types of protractors to measure angles to the nearest degree and sketch angles of a given measure (**4.MD.6**). The idea that an angle measures the amount of *turning* in a particular direction is explored as students recognize familiar angles in varied contexts (**4.G.1**, **4.MD.5**).



Topic C begins by decomposing 360° using pattern blocks, allowing students to see that a group of angles meeting at a point with no spaces or overlaps add up to 360°. With this new understanding, students now discover that the combined measure of two adjacent angles on a line is 180° (supplementary angles), that the combined measure of two adjacent angles meeting to form a right angle is 90° (complementary angles), and that vertically opposite angles have the same measure. These properties are then used to solve unknown angle problems (**4.MD.7**).

An introduction to symmetry opens Topic D as students recognize lines of symmetry for two-dimensional figures, identify line-symmetric figures, and draw lines of symmetry (**4.G.3**). Given one-half of a line-symmetric figure and the line of symmetry, students draw the other half of the figure. This leads to their work with triangles. Students are introduced to the precise definition of a triangle, and then classify triangles based on angle measure and side length (**4.G.2**). For isosceles triangles, a line of symmetry is identified, and a folding activity demonstrates that base angles are equal. Folding an equilateral triangle highlights multiple lines of symmetry and establishes that all interior angles are equal. Students construct triangles given a set of classifying criteria (e.g., create a triangle that is both right and isosceles). Finally, students explore the definitions of familiar quadrilaterals and classify them based on their attributes, including angle measure and parallel and perpendicular lines (**4.G.2**). This work builds on Grade 3 reasoning about the attributes of shapes and lays a foundation for hierarchical classification of two-dimensional figures in Grade 5. The topic concludes as students compare and analyze two-dimensional figures according to their properties and use grid paper to construct two-dimensional figures given a set of criteria.

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
4.G.1	A	**Lines and Angles** Lesson 1: Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures. Lesson 2: Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles. Lesson 3: Identify, define, and draw perpendicular lines. Lesson 4: Identify, define, and draw parallel lines.	4
4.MD.5 **4.MD.6**	B	**Angle Measurement** Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor. Lesson 6: Use varied protractors to distinguish angle measure from length measurement. Lesson 7: Measure and draw angles. Sketch given angle measures and verify with a protractor. Lesson 8: Identify and measure angles as turns and recognize them in various contexts.	4
		Mid-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further application 1 day)	2
4.MD.7	C	**Problem Solving with the Addition of Angle Measures** Lesson 9: Decompose angles using pattern blocks. Lessons 10–11: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.	3
4.G.1 **4.G.2** **4.G.3**	D	**Two-Dimensional Figures and Symmetry** Lesson 12: Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures and draw lines of symmetry.	5

⇨		Lesson 13: Analyze and classify triangles based on side length, angle measure, or both. Lesson 14: Define and construct triangles from given criteria. Explore symmetry in triangles. Lesson 15: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size. Lesson 16: Reason about attributes to construct quadrilaterals on square or triangular grid paper.	
		End-of-Module Assessment: Topics A–D (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days			**20**

Math Unit -Unit: 4

Subject: Mathematics

Grade/Course: Grade 4

Pacing: 20 days

Unit of Study: Unit: 4 Angle Measure and Plane figures

Priority Standards:

Focus Grade Level Standards

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed whenever two rays share a common endpoint, and understand concepts of angle measurement:

a. **An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a**

"one-degree angle," and can be used to measure angles.

b. **An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.**

4.MD.6 **Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.**

4.MD.7 **Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.**

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Foundational Standards

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. [19]

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Math Practice Standards:

MP.2 **Reason abstractly and quantitatively.** Students represent angle measures within equations,

[19] This standard is limited to problems with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., the order of operations.

and when determining the measure of an unknown angle, they represent the unknown angle with a letter or symbol both in the diagram and in the equation. They reason about the properties of groups of figures during classification activities.

MP.3 **Construct viable arguments and critique the reasoning of others.** Knowing and using the relationships between adjacent and vertical angles, students construct an argument for identifying the angle measures of all four angles generated by two intersecting lines when given the measure of one angle. Students explore the concepts of parallelism and perpendicularity on different types of grids with activities that require justifying whether completing specific tasks is possible on different grids.

MP.5 **Use appropriate tools strategically.** Students choose to use protractors when measuring and sketching angles, drawing perpendicular lines, and precisely constructing two-dimensional figures with specific angle measurements. They use right angle templates (set squares) and straightedges to construct parallel lines. They also choose to use straightedges for sketching lines, line segments, and rays.

MP.6 **Attend to precision.** Students use clear and precise vocabulary. They learn, for example, to cross-classify triangles by both angle size and side length (e.g., naming a shape as a right, isosceles triangle). They use right angle templates (set squares) and straightedges to construct parallel lines and become sufficiently familiar with a protractor to decide which set of numbers to use when measuring an angle whose orientation is such that it opens from either direction, or when the angle measures more than 180°.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do) Depth of Knowledge levels**
4.MD.5 angles as geometric shapes that are formed whenever two rays share a common endpoint.	Recognize(L-1)
concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the	Understand (L-1)

	circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees**.**	Measure (L-1)
4.MD.6	angles in whole-number degrees using a protractor.	Sketch (L-1)
	angles of specified measure.	**Recognize** (L-1)
4.MD.7	angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.	**Solve** (L-1)
	addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	**Draw** (L-1)
4.G.1	points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.	**Identify** (L-1) **Classify (L-2)**

these in two-dimensional figures.	
4.G.2 two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.	**Recognize (L-1)** **Recognize (L-1)**
right triangles as a category, and identify right triangles.	
4.G.3 a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts.	**Identify** (L-1)
line-symmetric figures and draw lines of symmetry.	

Essential Questions	**Big ideas**

What is an angle? How are a circle and an angle related? What do we actually measure when we measure an angle? What are benchmark angles and how can they be useful in estimating angle measures? What does half rotation and full rotation mean? How can angles be combined to create other angles? How can we use the relationship of angles measures of a shape to solve problems? How are the angles of a triangle related? What do we know about the measurement of angles in a triangle? How are geometric objects different from one another? How are quadrilaterals alike and different? How the types of sides are be used to classify quadrilaterals? How are triangles alike and different? How do angle and side measures help us create and classify triangles? How are symmetrical figures created? How are symmetrical figures used in artwork?	Angles are measured in the real world. There is a connection between geometric shapes and angles. There are specific labels that can be assigned to angles.

Assessments

<table>
<tr><td>Common Formative Pre-Assessments</td><td>Progress Monitoring Checks – “Dipsticks”</td><td>Common Formative Mid and or Post-Assessments</td></tr>
<tr><td>Exit tickets for pre-assessment for each lesson</td><td>Application problem
Student Debriefs
Problem set data</td><td>

Assessment Summary

<table>
<tr><th>Assessment Type</th><th>Administered</th><th>Format</th><th>Standards Addressed</th></tr>
<tr><td>Mid-Module Assessment Task</td><td>After Topic B</td><td>Constructed response with rubric</td><td>4.MD.5
4.MD.6
4.G.1</td></tr>
<tr><td>End-of-Module Assessment Task</td><td>After Topic D</td><td>Constructed response with rubric</td><td>4.MD.5
4.MD.6
4.MD.7
4.G.1
4.G.2
4.G.3</td></tr>
</table>

</td></tr>
</table>

<table>
<tr><th>Performance Task</th></tr>
<tr><td>To be created during the year</td></tr>
<tr><th>Engaging Learning Experiences</th></tr>
</table>

To be created during the year

Instructional Resources
Useful Websites: Engage NY K-5 Curriculum overview and guiding documents: https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents Engage NY Grade 4 Resources: https://www.engageny.org/resource/grade-4-mathematics Eureka Math Module PDFs: http://greatminds.net/maps/math/module-pdfs North Carolina 4th Grade Standards Unpacked: http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/4th.pdf Illustrative Mathematics – problems and tasks by grade and standard https://www.illustrativemathematics.org/ NCTM Illuminations – problems, tasks and interactives by grade and standard http://illuminations.nctm.org/Default.aspx Inside Mathematics – Problems of the Month and Performance Assessment tasks http://www.insidemathematics.org/ LearnZillion –lesson plans/some with embedded tasks https://learnzillion.com/resources/17132 SBAC Digital Library ## Suggested Tools and Representations ▪ Protractors of various diameters, including a 360° and 180° protractor ▪ Ruler (used to measure length), straightedge (used to draw straight lines) ▪ Right angle template (created in Lesson 2), set square ▪ Folded paper models ▪ Pattern blocks ▪ Rectangular and triangular grid paper

Instructional Strategies

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

1. **Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way
2. **Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words
3. **Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.
4. **Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.
5. **Nonlinguistic Representations:** has recently been proven to stimulate and increase brain activity.
6. **Cooperative Learning:** has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.
7. **Setting Objectives and Providing Feedback**: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied.
8. **Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material.
9. **Cues, Questions, and Advanced Organizers:** helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.

Meeting the Needs of All Students

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means

of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

Scaffolds for Students with Disabilities

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:
- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

<u>Provide Multiple Means of Engagement</u>

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most

of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.

- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary

New or Recently Introduced Terms

- Benchmark (e.g., "round" numbers like multiples of 10)
- Endpoint (point where something begins or ends)
- Estimate (an approximation of a quantity or number)
- Hash mark (marks on a ruler or other measurement tool)
- Meter (standard unit of length in the metric system)
- Meter stick or strip (tool used to measure length)
- Number line
- Overlap (extend over, or cover partly)
- Ruler (tool used to measure length)

Familiar Terms and Symbols[20]

- Centimeter (standard length unit within the metric system)
- Combine (join or put together)
- Compare (specifically using direct comparison)
- Difference (to find the difference between two numbers, subtract the smaller number from the greater number)
- Height (vertical distance measurement from bottom to top)
- Length (distance measurement from end to end; in a rectangular shape, length can be used to describe any of the four sides)Length unit (e.g., centimeters, inches)

[20] These are terms and symbols students have used or seen previously.

Students performing Below standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.
- Guide students as they select and practice using their own graphic organizers and models to solve.
- Use direct instruction for vocabulary with visual or concrete representations.
- Use explicit directions with steps and procedures enumerated.
- Guide students through initial practice promoting gradual independence. “I do, we do, you do.”
- Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.
- Scaffold complex concepts and provide leveled problems for multiple entry points.

Provide Multiple Means of Action and Expression

- First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.
- Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’
- Encourage students to explain their thinking and strategy for the solution.
- Choose numbers and tasks that are “just right” for learners but teach the same concepts.
- Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2.

Provide Multiple Means of Engagement

- Clearly model steps, procedures, and questions to ask when solving.
- Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).
- Have students work together to solve and then check their solutions.
- Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?
- Practice routine to ensure smooth transitions.
- Set goals with students regarding the type of math work students should complete in 60 seconds.
- Set goals with the students regarding next steps and what to focus on next.

Students performing Above standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations.
- Model and post conversation "starters," such as: "I agree because..." "Can you
- explain how you solved it?""I noticed that..."""Your solution is different from/ the same as mine because..."""My mistake was to..."
- Incorporate written reflection, evaluation, and synthesis.
- Allow creativity in expression and modeling solutions.

Provide Multiple Means of Action and Expression

- Encourage students to explain their reasoning both orally and in writing.
- Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.
- Offer choices of independent or group assignments for early finishers.
- Encourage students to notice and explore patterns and to identify rules and relationships in math.
- Have students share their observations in discussion and writing (e.g., journaling).
- Foster their curiosity about numbers and mathematical ideas.
- Facilitate research and exploration through discussion, experiments, internet searches, trips, etc.
- Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.
- Let students choose their mode of response: written, oral, concrete, pictorial, or abstract.
- Increase the pace. Offer two word problems to solve, rather than one.
- Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem).
- Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.
- Let students write word problems to show mastery and/or extension of the content.

Provide Multiple Means of Engagement

- Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.
- Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds).
- Make the most of the fun exercises for practicing skip-counting.
- Accept and elicit student ideas and suggestions for ways to extend games.

- Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

GRADE 4 • Unit 5 (MODULE 5)

Fraction Equivalence, Ordering, and Operations

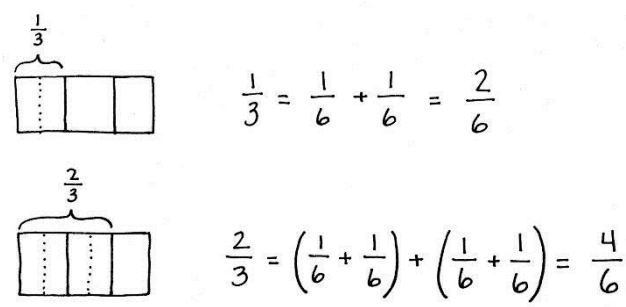
OVERVIEW

In this 45-day module, students build on their Grade 3 work with unit fractions as they explore fraction equivalence and extend this understanding to mixed numbers. This leads to the comparison of fractions and mixed numbers and the representation of both in a variety of models. Benchmark fractions play an important part in students' ability to generalize and reason about relative fraction and mixed number sizes. Students then have the opportunity to apply what they know to be true for whole number operations to the new concepts of fraction and mixed number operations.

Students begin Topic A by decomposing fractions and creating tape diagrams to represent them as sums of fractions with the same denominator in different ways (e.g., $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1}{5} + \frac{2}{5}$) (**4.NF.3b**). They proceed to see that representing a fraction as the repeated addition of a unit fraction is the same as multiplying that unit fraction by a whole number. This is already a familiar fact in other contexts.

For example, just as 3 twos = 2 + 2 + 2 = 3 × 2, so does 3 fourths $= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$.

The introduction of multiplication as a record of the decomposition of a fraction (**4.NF.4a**) early in the module allows students to become familiar with the notation before they work with more complex problems. As students continue working with decomposition, they represent familiar unit fractions as the sum of smaller unit fractions. A folded paper activity allows them to see that, when the number of



fractional parts in a whole increases, the size of the parts decreases. They proceed to investigate this concept with the use of tape diagrams and area models. Reasoning enables them to explain why two different fractions can represent the same portion of a whole (**4.NF.1**).

In Topic B, students use tape diagrams and area models to analyze their work from earlier in the module and begin using multiplication to create an equivalent fraction that comprises smaller units, e.g., $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ (**4.NF.1**). Based on the use of multiplication, they reason that division can be used to create a fraction that comprises larger units (or a single unit) equivalent to a given fraction (e.g., $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$). Their work is justified using area models and tape diagrams and, conversely, multiplication is used to test for and/or verify equivalence. Students use the tape diagram to transition to modeling equivalence on the number line. They see that, by multiplying, any unit fraction length can be partitioned into *n* equal lengths and that doing so multiplies both the total number of fractional units (the denominator) and number of selected units (the numerator) by *n*. They also see that there are times when fractional units can be grouped together, or divided, into larger fractional units. When that occurs, both the total number of fractional units and number of selected units are divided by the same number.

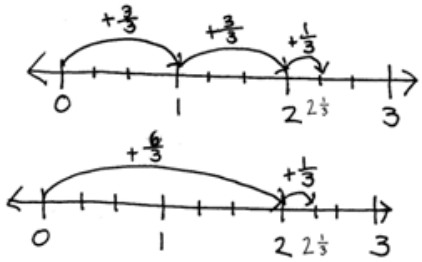
In Grade 3, students compared fractions using fraction strips and number lines with the same denominators. In Topic C, they expand on comparing fractions by reasoning about fractions with unlike denominators. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, $\frac{1}{2}$, or 1) on the number line. Alternatively, students compare using the same numerators. They find that the fraction with the greater denominator is the lesser fraction since the size of the fractional unit is smaller as the whole is decomposed into more equal parts (e.g., $\frac{1}{5} > \frac{1}{10}$, therefore $\frac{3}{5} > \frac{3}{10}$). Throughout the process, their reasoning is supported using tape diagrams and number lines in cases where one numerator or denominator is a factor of the other, such as $\frac{1}{5}$ and $\frac{1}{10}$ or $\frac{2}{3}$ and $\frac{5}{6}$. When the units are unrelated, students use area models and multiplication, the general method pictured below to the left, whereby two fractions are expressed in terms of the same denominators. Students also reason that comparing fractions can only be done when referring to the same whole, and they record their comparisons using the comparison symbols <, >, and = (**4.NF.2**).

Comparison Using Like

Comparison Using Like

In Topic D, students apply their understanding of whole number addition (the combining of like units) and subtraction (finding an unknown part) to work with fractions (**4.NF.3a**). They see through visual models that, if the units are the same, computation can be performed immediately, e.g., 2 bananas + 3 bananas = 5 bananas and 2 eighths + 3 eighths = 5 eighths. They see that, when subtracting fractions from one whole, the whole is decomposed into the same units as the part being subtracted, e.g., $1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{2}{5}$. Students practice adding more than two fractions and model fractions in word problems using tape diagrams (**4.NF.3d**). As an extension of the Grade 4 standards, students apply their knowledge of decomposition from earlier topics to add fractions with related units using tape diagrams and area models to support their numerical work. To find the sum of $\frac{1}{2}$ and $\frac{1}{4}$, for example, one simply decomposes 1 half into 2 smaller equal units, fourths, just as in Topics A and B. Now, the addition can be completed: $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$. Though not assessed, this work is warranted because, in Module 6, students are asked to add tenths and hundredths when working with decimal fractions and decimal notation.

At the beginning of Topic E, students use decomposition and visual models to add and subtract fractions less than 1 to or from whole numbers, e.g., $4 + \frac{3}{4} = 4\frac{3}{4}$ and $4 - \frac{3}{4} = (3 + 1) - \frac{3}{4}$. They use addition and multiplication to build fractions greater than 1 and represent them on the number line.



Students then use these visual models and decompositions to reason about the various forms in which a fraction greater than or equal to 1 may be presented, both as fractions and mixed numbers. They practice converting between these forms and begin understanding the usefulness of each form in different situations.

Through this understanding, the common misconception that every improper fraction must be converted to a mixed number is avoided. Next, students compare fractions greater than 1, building on their rounding skills and using understanding of benchmarks to reason about which of two fractions is greater (**4.NF.2**). This activity continues to build understanding of the relationship between the numerator and denominator of a fraction. Students progress to finding and using like denominators or numerators to compare and order mixed numbers. They apply their skills of comparing numbers greater than 1 by solving word problems (**4.NF.3d**) requiring the interpretation of data presented in line plots (**4.MD.4**). Students use addition and subtraction strategies to solve the problems, as well as decomposition and modeling to compare numbers in the data sets.

In Topic F, students estimate sums and differences of mixed numbers, rounding before performing the actual operation to determine what a reasonable outcome will be. They proceed to use decomposition to add and subtract mixed numbers (**4.NF.3c**). This work builds on their understanding of a mixed number being the sum of a whole number and fraction.

Using unit form, students add and subtract like units first (e.g., ones and ones, fourths and fourths). Students use decomposition, shown with number bonds, in mixed number addition to make one from fractional units before finding the sum. When subtracting, students learn to decompose the minuend or subtrahend when there are not enough fractional units from which to subtract. Alternatively, students can rename the subtrahend, giving more units to the fractional units, which connects to whole number subtraction when renaming 9 tens 2 ones as 8 tens 12 ones.

In Topic G, students build on the concept of representing repeated addition as multiplication, applying this familiar concept to work with fractions (**4.NF.4a, 4.NF.4b**). They use the associative property and their understanding of decomposition. Just as with whole numbers, the unit remains unchanged.

$$4 \times \frac{3}{5} = 4 \times (3 \times \frac{1}{5}) = (4 \times 3) \times \frac{1}{5} = \frac{4 \times 3}{5} = \frac{12}{5}$$

This understanding connects to students' work with place value and whole numbers. Students proceed to explore the use of the distributive property to multiply a whole number by a mixed number. They recognize that they are multiplying each part of a mixed number by the whole number and use efficient strategies to do so. The topic closes with solving multiplicative comparison word problems involving

$$\begin{aligned} 5 \times 3\tfrac{3}{4} &= 5 \times (3 + \tfrac{3}{4}) \\ &= (5 \times 3) + (5 \times \tfrac{3}{4}) \\ &= 15 + \tfrac{15}{4} \\ &= 15 + 3\tfrac{3}{4} \\ &= 18\tfrac{3}{4} \end{aligned}$$

fractions (**4.NF.4c**) as well as problems involving the interpretation of data presented on a line plot.

Topic H comprises an exploration lesson where students find the sum of all like denominators from $\frac{0}{\square}$ $\square\square$ $\frac{\square}{\square}$. Students first work in teams with fourths, sixths, eighths, and tenths. For example, they might find the sum of all sixths from $\frac{0}{6}$ to $\frac{6}{6}$. Students discover that they can make pairs with a sum of 1 to add more efficiently, e.g., $\frac{0}{6} + \frac{6}{6}, \frac{1}{6} + \frac{5}{6}, \frac{2}{6} + \frac{4}{6}$, and there will be one fraction, $\frac{3}{6}$, without a pair. They then extend this to similarly find sums of thirds, fifths, sevenths, and ninths, observing patterns when finding the sum of odd and even denominators (**4.OA.5**).

The Mid-Module Assessment follows Topic D, and the End-of-Module Assessment follows Topic H.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
4.NF.3b **4.NF.4a** 4.NF.3a	A	**Decomposition and Fraction Equivalence** Lessons 1–2: Decompose fractions as a sum of unit fractions using tape diagrams. Lesson 3: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams. Lesson 4: Decompose fractions into sums of smaller unit fractions using tape diagrams. Lesson 5: Decompose unit fractions using area models to show equivalence. Lesson 6: Decompose fractions using area models to show equivalence.	6
4.NF.1 4.NF.3b	B	**Fraction Equivalence Using Multiplication and Division** Lessons 7–8: Use the area model and multiplication to show the equivalence of two fractions. Lessons 9–10: Use the area model and division to show the equivalence of two fractions. Lesson 11: Explain fraction equivalence using a tape diagram and the number line,	5

		and relate that to the use of multiplication and division.	
4.NF.2	C	**Fraction Comparison** Lessons 12–13: Reason using benchmarks to compare two fractions on the number line. Lessons 14–15: Find common units or number of units to compare two fractions.	4
4.NF.3ad 4.NF.1 4.MD.2	D	**Fraction Addition and Subtraction** Lesson 16: Use visual models to add and subtract two fractions with the same units. Lesson 17: Use visual models to add and subtract two fractions with the same units, including subtracting from one whole. Lesson 18: Add and subtract more than two fractions. Lesson 19: Solve word problems involving addition and subtraction of fractions. Lessons 20–21: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12.	6
		Mid-Module Assessment: Topics A–D (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
4.NF.2 **4.NF.3** **4.MD.4** 4.NBT.6 4.NF.1 4.NF.4a	E	**Extending Fraction Equivalence to Fractions Greater Than 1** Lesson 22: Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models. Lesson 23: Add and multiply unit fractions to build fractions greater than 1 using visual models. Lessons 24–25: Decompose and compose fractions greater than 1 to express them in various forms. Lesson 26: Compare fractions greater than 1 by reasoning using benchmark fractions. Lesson 27: Compare fractions greater than 1 by creating common numerators or denominators. Lesson 28: Solve word problems with line plots.	7

4.NF.3c 4.MD.2	F	**Addition and Subtraction of Fractions by Decomposition** Lesson 29: Estimate sums and differences using benchmark numbers. Lesson 30: Add a mixed number and a fraction. Lesson 31: Add mixed numbers. Lesson 32: Subtract a fraction from a mixed number. Lesson 33: Subtract a mixed number from a mixed number. Lesson 34: Subtract mixed numbers.	6
4.NF.4 4.OA.2 4.MD.2 4.MD.4	G	**Repeated Addition of Fractions as Multiplication** Lessons 35–36: Represent the multiplication of n times a/b as $(n \times a)/b$ using the associative property and visual models. Lessons 37–38: Find the product of a whole number and a mixed number using the distributive property. Lesson 39: Solve multiplicative comparison word problems involving fractions. Lesson 40: Solve word problems involving the multiplication of a whole number and a fraction including those involving line plots.	6
4.OA.5	H	**Exploring a Fraction Pattern** Lesson 41: Find and use a pattern to calculate the sum of all fractional parts between 0 and 1. Share and critique peer strategies.	1
		End-of-Module Assessment: Topics A–H (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
Total Number of Instructional Days			**45**

Math Unit - Unit: 5

Subject: Mathematics

Grade/Course: Grade 4

Pacing: 45 days

Unit of Study: Unit: 5 Fraction Equivalence, Ordering, and Operations

Priority Standards:

Focus Grade Level Standards

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*.

Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Represent and interpret data.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Foundational Standards

3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Math Practice Standards:

MP.2 **Reason abstractly and quantitatively.** Students will reason both abstractly and quantitatively throughout this module. They will draw area models, number lines, and tape diagrams to represent fractional quantities, as well as word problems.

MP.3 **Construct viable arguments and critique the reasoning of others.** Much of the work in this module is centered on multiple ways to solve fraction and mixed number problems. Students explore various strategies and participate in many *turn and talk* and *explain to your partner* activities. By doing so, they construct arguments to defend their choice of strategy, as well as think about and critique the reasoning of others.

MP.4 **Model with mathematics.** Throughout this module, students represent fractions with various models. Area models are used to investigate and prove equivalence. The number line is used to compare and order fractions, as well as model addition and subtraction of fractions. Students also use models in problem solving as they create line plots to display given sets of fractional data and solve problems requiring the interpretation of data presented in line plots.

MP.7 **Look for and make use of structure.** As students progress through this fraction module, they search for and use patterns and connections that will help them build understanding of new concepts. They relate and apply what they know about operations with whole numbers to operations with fractions.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **DOK Levels**
Generate and analyze patterns.	
4.OA.5 a number or shape pattern that follows a given rule. Identify	Generate (L-3)
apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this*	Identify (L-1)

way.	
Extend understanding of fraction equivalence and ordering.	
4.NF.1 why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.	Explain (L-3)
this principle to recognize and generate equivalent fractions.	Use (L-1)
4.NF.2 two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2.	Compare (L-2)
that comparisons are valid only when the two fractions refer to the same whole.	Recognize (L-1)
the results of comparisons with symbols >, =, or <,	Record (L-1)
the conclusions, e.g., by using a visual fraction model.	Justify (L-3)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	
4.NF.3 a fraction a/b with $a > 1$ as a sum of fractions $1/b$.	Understand (L-1)
a. addition and subtraction of fractions as joining and separating parts referring to the same whole.	Understand (L-1)
b. a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.	Decompose (L-1)
decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2*	

	1/8 = *1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*	Justify (L-3)
	c. mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	Add and subtract (L-2)
	d. word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	Solve (L-1)
4.NF.4		
	previous understandings of multiplication to multiply a fraction by a whole number.	
	a. a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation* *5/4 = 5 × (1/4).*	Apply and extend (L-4)
	b. a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*	Understand(L-2)
	c. word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	Understand (L-2) Solve (L-2)

Represent and interpret data. **4.MD.4** a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	Make (L-2) Solve (L-2)

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Exit tickets for pre-assessment for each lesson	**Application problem** **Student Debriefs** **Problem set data**	**Assessment Summary**

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic D	Constructed response with rubric	4.NF.1 4.NF.2 4.NF.3abd 4.NF.4a
End-of-Module Assessment Task	After Topic H	Constructed response with rubric	4.OA.5 4.NF.1 4.NF.2 4.NF.3

						4.NF.4 4.MD.4

Performance Task
To be created during the year
Engaging Learning Experiences
To be created during the year

Instructional Resources
Useful Websites: Engage NY K-5 Curriculum overview and guiding documents: https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents Engage NY Grade 4 Resources: https://www.engageny.org/resource/grade-4-mathematics Eureka Math Module PDFs: http://greatminds.net/maps/math/module-pdfs North Carolina 4th Grade Standards Unpacked: http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/4th.pdf Illustrative Mathematics – problems and tasks by grade and standard https://www.illustrativemathematics.org/ NCTM Illuminations – problems, tasks and interactives by grade and standard http://illuminations.nctm.org/Default.aspx Inside Mathematics – Problems of the Month and Performance Assessment tasks http://www.insidemathematics.org/ LearnZillion –lesson plans/some with embedded tasks https://learnzillion.com/resources/17132 SBAC Digital Library ▪ **Suggested Tools and Representations**

- Area model
- Fraction strips (made from paper, folded, and used to model equivalent fractions)
- Line plot
- Number line
- Rulers
- Tape diagram

Instructional Strategies

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

1. **Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way
2. **Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words
3. **Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.
4. **Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.
5. **Nonlinguistic Representations:** has recently been proven to stimulate and increase brain activity.

6. **Cooperative Learning:** has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.

7. **Setting Objectives and Providing Feedback**: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied.
8. **Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material.
9. **Cues, Questions, and Advanced Organizers:** helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.

Meeting the Needs of All Students

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

Scaffolds for Students with Disabilities

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:
- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"

- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?”
- Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

<u>Provide Multiple Means of Engagement</u>

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!”
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow “everyday” and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary

- Benchmark (standard or reference point by which something is measured)
- Common denominator (when two or more fractions have the same denominator)
- Denominator (e.g., the 5 in $\frac{3}{5}$ names the fractional unit as fifths)
- Fraction greater than 1 (a fraction with a numerator that is greater than the denominator)
- Line plot (display of data on a number line, using an x or another mark to show frequency)
- Mixed number (number made up of a whole number and a fraction)
- Numerator (e.g., the 3 in $\frac{3}{5}$ indicates 3 fractional units are selected)

Familiar Terms and Symbols[21]

- =, <, > (equal to, less than, greater than)
- Compose (change a smaller unit for an equivalent of a larger unit, e.g., 2 fourths = 1 half, 10 ones = 1 ten; combining 2 or more numbers, e.g., 1 fourth + 1 fourth = 2 fourths, 2 + 2 + 1 = 5)
- Decompose (change a larger unit for an equivalent of a smaller unit, e.g., 1 half = 2 fourths, 1 ten = 10 ones; partition a number into 2 or more parts, e.g., 2 fourths = 1 fourth + 1 fourth, 5 = 2 + 2 + 1)
- Equivalent fractions (fractions that name the same size or amount)
- Fraction (e.g., $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$)
- Fractional unit (e.g., half, third, fourth)
- Multiple (product of a given number and any other whole number)
- Non-unit fraction (fractions with numerators other than 1)
- Unit fraction (fractions with numerator 1)
- Unit interval (e.g., the interval from 0 to 1, measured by length)
- Whole (e.g., 2 halves, 3 thirds, 4 fourths)

Students performing Below standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.
- Guide students as they select and practice using their own graphic organizers and models to solve.
- Use direct instruction for vocabulary with visual or concrete representations.
- Use explicit directions with steps and procedures enumerated.
- Guide students through initial practice promoting gradual independence. “I do, we do, you do.”
- Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.
- Scaffold complex concepts and provide leveled problems for multiple entry points.

Provide Multiple Means of Action and Expression

[21] These are terms and symbols students have seen previously.

- First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.
- Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?'
- Encourage students to explain their thinking and strategy for the solution.
- Choose numbers and tasks that are "just right" for learners but teach the same concepts.
- Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2.

Provide Multiple Means of Engagement

- Clearly model steps, procedures, and questions to ask when solving.
- Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).
- Have students work together to solve and then check their solutions.

- Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?
- Practice routine to ensure smooth transitions.
- Set goals with students regarding the type of math work students should complete in 60 seconds.
- Set goals with the students regarding next steps and what to focus on next.

Students performing Above standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations.
- Model and post conversation "starters," such as: "I agree because..." "Can you
- explain how you solved it?""I noticed that..."Your solution is different from/ the same as mine because..."My mistake was to..."
- Incorporate written reflection, evaluation, and synthesis.
- Allow creativity in expression and modeling solutions.

Provide Multiple Means of Action and Expression

- Encourage students to explain their reasoning both orally and in writing.
- Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.
- Offer choices of independent or group assignments for early finishers.

- Encourage students to notice and explore patterns and to identify rules and relationships in math.
- Have students share their observations in discussion and writing (e.g., journaling).
- Foster their curiosity about numbers and mathematical ideas.
- Facilitate research and exploration through discussion, experiments, internet searches, trips, etc.
- Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.
- Let students choose their mode of response: written, oral, concrete, pictorial, or abstract.
- Increase the pace. Offer two word problems to solve, rather than one.
- Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem).
- Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.
- Let students write word problems to show mastery and/or extension of the content.

Provide Multiple Means of Engagement

- Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.
- Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds).
- Make the most of the fun exercises for practicing skip-counting.
- Accept and elicit student ideas and suggestions for ways to extend games.

- Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

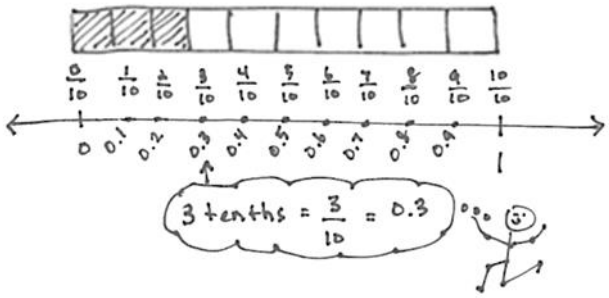
GRADE 4 • Unit 6 (MODULE 6)

Decimal Fractions

OVERVIEW

This 20-day module gives students their first opportunity to explore decimal numbers via their relationship to decimal fractions, expressing a given quantity in both fraction and decimal forms. Utilizing the understanding of fractions developed throughout Module 5, students apply the same reasoning to decimal numbers, building a solid foundation for Grade 5 work with decimal operations. Previously referred to as whole numbers, all numbers written in the base ten number system with place value units that are powers of 10 are henceforth referred to as decimal numbers, a set which now includes tenths and hundredths, e.g., 1, 15, 248, 0.3, 3.02, and 24.345.

In Topic A, students use their understanding of fractions to explore tenths. At the opening of the topic, they use metric measurement to see tenths in relation to different whole units: centimeters, meters, kilograms, and liters. Students explore, creating and identifying tenths of various wholes, as they draw lines of specified length, identify the weight of objects, and read the level of liquid measurements. Students connect these concrete experiences pictorially as tenths are represented on the number line and with tape diagrams as pictured to the right. Students express tenths as decimal fractions and are introduced to decimal notation. They write statements of equivalence in unit, fraction, and decimal forms, e.g., 3 tenths = $\frac{3}{10}$ = 0.3 (**4.NF.6**). Next, students return to the use of metric measurement to investigate decimal fractions greater than 1. Using a centimeter ruler, they draw lines that measure, for example, $2\frac{4}{10}$ or $6\frac{8}{10}$ centimeters. Using the area model, students see that numbers containing a whole number and fractional part, i.e., mixed numbers, can also be expressed using decimal notation provided that the fractional part can be converted to a decimal number (**4.NF.6**). Students use place value disks to represent the value of each digit in a decimal number. Just as they wrote whole numbers in expanded form using multiplication, students write the value of a decimal number in expanded form using fractions and decimals, e.g., 2 ones 4 tenths = $2\frac{4}{10}$ = $(2 \times 1) + (4 \times \frac{1}{10})$ and $2.4 = (2 \times 1) + (4 \times 0.1)$. Additionally, students plot decimal numbers on the number line.



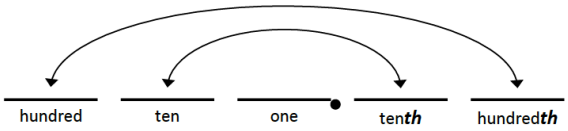
Students decompose tenths into 10 equal parts to create hundredths in Topic B. Through the decomposition of a meter, students identify 1 centimeter as 1 hundredth of a meter. As students count up by hundredths, they realize the equivalence of 10 hundredths and 1 tenth and go on to represent them as both decimal fractions and as decimal numbers (**4.NF.5**). Students use area models, tape diagrams, and number disks on a place value chart to see and model the equivalence of numbers involving units of tenths and hundredths. They express the value of the number in both decimal and fraction expanded forms.

$$31\frac{46}{100} = (3 \times 10) + (1 \times 1) + (4 \times \frac{1}{10}) + (6 \times \frac{1}{100})$$
$$31.46 = (3 \times 10) + (1 \times 1) + (4 \times 0.1) + (6 \times 0.01)$$

Close work with the place value chart helps students see that place value units are not symmetric about the decimal point—a common misconception that often leads students to mistakenly believe there is a *oneths* place. They explore the placement of decimal numbers to hundredths and recognize that the place value chart is symmetric about the ones column. This understanding helps students recognize that, even as we move to the units on the right side of the decimal on the place value chart, a column continues to represent a unit 10 times as large as that of the column to its right. This understanding builds on the place value work

done in Module 1 and enables students to understand that 3.2, for example, might be modeled as 3 ones 2 tenths, 32 tenths, or 320 hundredths. Topic B concludes with students using their knowledge of fraction equivalence to work with decimal numbers expressed in unit form, fraction form, and decimal form (**4.NF.6**).

Symmetry with respect to the ones place

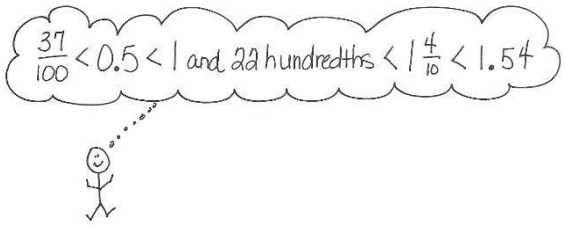


The focus of Topic C is comparison of decimal numbers (**4.NF.7**). To begin, students work with concrete representations of measurements. They see measurement of length on meter sticks, of mass using a scale, and of volume using graduated cylinders. In each case, students record the measurements on a place value chart and then compare them. They use their understanding of metric measurement and decimals to answer questions, such as, "Which is greater? Less? Which is longer? Shorter? Which is heavier? Lighter?" Comparing the decimals in the context of measurement supports students' justification of their comparisons and grounds their reasoning, while at the same time setting them up for work with decimal comparison at a more concrete level. Next, students use area models and number lines to compare decimal numbers and use the <, >, and = symbols to record their comparisons. All of their work with comparisons at the pictorial level helps to eradicate the common misconception that is often made when students assume a greater number of hundredths must be greater than a lesser number of tenths. For example, when comparing 7 tenths and 27 hundredths, students recognize that 7 tenths is greater than 27 hundredths because, as in any comparison, one must consider the *size of the units*. Students go on to arrange mixed groups of decimal fractions in unit, fraction, and decimal forms in order from greatest to least, or least to greatest. They use their understanding of different ways of expressing equivalent values to arrange a set of decimal fractions as pictured below.

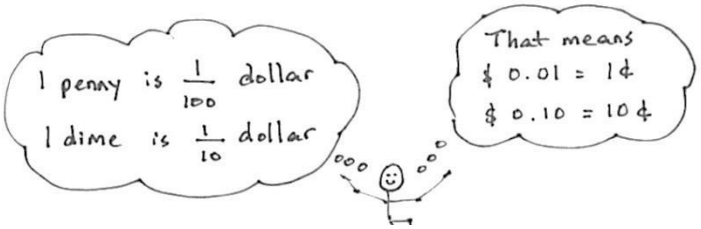
Rice Bag	ones (kilograms)	.	tenths	hundredths
A	0	.	1	0
B	0	.	6	5
C	0	.	7	
D	0	.	4	6

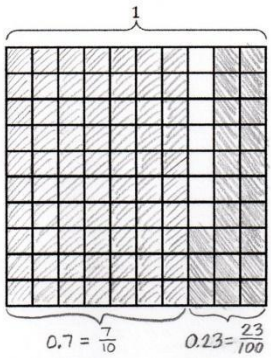
0.7 kg, 0.65 kg, 0.46 kg, 0.1 kg

Topic D introduces the addition of decimals by way of finding equivalent decimal fractions and adding fractions. Students add tenths and hundredths, recognizing that they must convert the addends to the same units (**4.NF.5**). The sum is then converted back into a decimal (**4.NF.6**). They use their knowledge of like denominators and understanding of fraction equivalence to do so. Students use the same process to add and subtract mixed numbers involving decimal units. They then apply their new knowledge to solve word problems involving metric measurements.



Students conclude their work with decimal fractions in Topic E by applying their knowledge to the real world context of money. They recognize 1 penny as $\frac{1}{100}$ dollar and apply their understanding of tenths and hundredths to decimal forms. To do this, students decompose pennies and express the amount as a decimal f numbers of coins and dollars using Grade 2 knc and subtraction word problems are solved usin adding dollars and cents. Multiplication and di problems are solved using cents as the unit (**4.MD.2**). The final answer in each word problem is converted from cents into a decimal using a dollar symbol for the unit. For example, *Jack has 2 quarters and 7 dimes. Jim has 1 dollar, 3 quarters, and 6 pennies. How much money do they have together? Write your answer as a decimal.*





$$\frac{7}{10} + \frac{23}{100} = \frac{70}{100} + \frac{23}{100} = \frac{93}{100}$$

$$\frac{93}{100} = 0.93$$

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
4.NF.6 4.NBT.1 4.MD.1	A	**Exploration of Tenths** Lesson 1: Use metric measurement to model the decomposition of one whole into tenths. Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers. Lesson 3: Represent mixed numbers with units of tens, ones, and tenths with number disks, on the number line, and in expanded form.	3
4.NF.5 **4.NF.6** 4.NBT.1 4.NF.1 4.NF.7 4.MD.1	B	**Tenths and Hundredths** Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths. Lesson 5: Model the equivalence of tenths and hundredths using the area model and number disks. Lesson 6: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms. Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart. Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.	5
		Mid-Module Assessment: Topics A–B (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
4.NF.7 4.MD.1 4.MD.2	C	**Decimal Comparison** Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions. Lesson 10: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.	3

		Lesson 11: Compare and order mixed numbers in various forms.	
4.NF.5 **4.NF.6** 4.NF.3c 4.MD.1	D	**Addition with Tenths and Hundredths** Lesson 12: Apply understanding of fraction equivalence to add tenths and hundredths. Lesson 13: Add decimal numbers by converting to fraction form. Lesson 14: Solve word problems involving the addition of measurements in decimal form.	3
4.MD.2 4.NF.5 4.NF.6	E	**Money Amounts as Decimal Numbers** Lesson 15: Express money amounts given in various forms as decimal numbers. Lesson 16: Solve word problems involving money.	2
		End-of-Module Assessment: Topics A–E (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
Total Number of Instructional Days			**20**

Unit: 6

Subject: Mathematics

Grade/Course: Grade 4

Pacing: 20 days

Unit of Study: Unit: 6 Decimal Fractions

Priority Standards:

Focus Grade Level Standards

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.7 **Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.** Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[22]

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

[22] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.3 is addressed in Module 3.

Foundational Standards

2. MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

3. NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

3. NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3. NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

c.

3. NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

b. Recognize and generate simple equivalent fractions, (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

3. MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems [problems involving notions of “times as much”; see CCSS Glossary, Table 2]).

Math Practice Standards:

MP.2 **Reason abstractly and quantitatively.** Throughout this module, students use area models, tape diagrams, number disks, and number lines to represent decimal quantities. When determining the equivalence of a decimal fraction and a fraction, students consider the units that are involved and attend to the meaning of the quantities of each. Further, students use metric measurement and money amounts to build an understanding of the decomposition of a whole into tenths and hundredths.

MP.4 **Model with mathematics.** Students represent decimals with various models throughout this module, including expanded form. Each of the models helps students to build understanding and to analyze the relationship and role of decimals within the number system. Students use a tape diagram to represent tenths and then to decompose one-tenth into hundredths. They use number disks and a place value chart to extend their understanding of place value to include decimal fractions. Further, students use a place value chart along with the area model to compare decimals. A number line models decimal numbers to the hundredths.

MP.6 **Attend to precision.** Students attend to precision as they decompose a whole into tenths and tenths into hundredths. They also make statements such as 5 ones and 3 tenths equals 53 tenths. Focusing on the units of decimals, students examine equivalence, recognize that the place value chart is symmetric around 1, and compare decimal numbers. In comparing decimal numbers, students are required to consider the units involved. Students communicate their knowledge of decimals through discussion and then apply their learning to add decimals, recognizing the need to convert to like units when necessary.

MP.8 **Look for and express regularity in repeated reasoning.** As they progress through this module, students have multiple opportunities to explore the relationships between and among units of ones, tenths, and hundredths. Relationships between adjacent place values, for example, are the same on the right side of the decimal point as they are on the left side, and students investigate this fact working with tenths and hundredths. Further, adding tenths and hundredths requires finding like units just as it does with whole numbers, such as when adding centimeters and meters. Students come to understand equivalence, conversions, comparisons, and addition involving decimal fractions.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **DOK Levels**
4.NF.5 a fraction with denominator 10 as an equivalent fraction with denominator 100,	**Express (L-1)**
this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*	**Use (L-1)**
4.NF.6 decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*	**Use (L-1)**
4.NF.7 two decimals to hundredths by reasoning about their size that comparisons are valid only when the two decimals refer to the same whole.	**Compare (L-2)**
the results of comparisons with the symbols >, =, or <.	**Record (L-1)**
the conclusions, e.g., by using a visual model.	**Justify (L-3)**
problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[23]	**Solve (L-2)**
4.MD.2 (the four operations to solve word	**Use (L-1)**

[23] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.3 is addressed in Module 3.

problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	**Represent (L-1)**

Essential Questions	**Big ideas**
What is a decimal fraction and how can it be represented? When can tenths and hundredths be used interchangeably? How can we add decimal fractions of different denominators? How are decimal fractions written using the decimal notation?	All Decimals can be represented as fractions and all fractions can be represented as decimals. For example The decimal .3 is equivalent to the fraction 3/10 which can also be represented as 30/100.) Common denominators are essential to add and subtract fractions and decimals.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Exit tickets for pre-assessment for each lesson	**Application problem** **Student Debriefs** **Problem set data**	**Assessment Summary**

Type	Administered	Format	Standards Addressed

		Mid-Module Assessment Task	After Topic B	Constructed response with rubric	4.NF.5 4.NF.6
		End-of-Module Assessment Task	After Topic E	Constructed response with rubric	4.NF.5 4.NF.6 4.NF.7 4.MD.2

Performance Task
To be created during the year
Engaging Learning Experiences
To be created during the year

Instructional Resources

Useful Websites:
Engage NY K-5 Curriculum overview and guiding documents:
https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents
Engage NY Grade 4 Resources:
https://www.engageny.org/resource/grade-4-mathematics
Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs
North Carolina 4th Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/4th.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/
NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx
Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/
LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132
SBAC Digital Library

- ## Suggested Tools and Representations

- 1-liter container with milliliter marks
- Area model
- Centimeter ruler
- Decimal place value disks (tenths and hundredths)
- Digital scale
- Meter stick
- Number line
- Place value chart with decimals to hundredths
- Tape diagram
- Whole number place value disks (hundreds, tens, and ones)

Instructional Strategies

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

1. **Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way
2. **Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words
3. **Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.
4. **Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.
5. **Nonlinguistic Representations:** has recently been proven to stimulate and increase brain activity.
6. **Cooperative Learning:** has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.
7. **Setting Objectives and Providing Feedback**: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied.
8. **Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material.
9. **Cues, Questions, and Advanced Organizers:** helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.

Meeting the Needs of All Students

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of

engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to

teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

Scaffolds for Students with Disabilities

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:
- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner

- share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

<u>Provide Multiple Means of Engagement</u>

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.

- Allow “everyday” and first language to express math understanding.

- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary

- Decimal expanded form (e.g., $(2 \times 10) + (4 \times 1) + (5 \times 0.1) + (9 \times 0.01) = 24.59$)
- Decimal fraction (fraction with a denominator of 10, 100, 1,000, etc.)
- Decimal number (number written using place value units that are powers of 10)
- Decimal point (period used to separate the whole number part from the fractional part of a decimal number)
- Fraction expanded form (e.g., $(2 \times 10) + (4 \times 1) + (5 \times \frac{1}{10}) + (9 \times \frac{1}{100}) = 24\frac{59}{100}$)
- Hundredth (place value unit such that 100 hundredths equals 1 one)
- Tenth (place value unit such that 10 tenths equals 1 one)

Familiar Terms and Symbols[24]

- Expanded form (e.g., 100 + 30 + 5 = 135)
- Fraction (numerical quantity that is not a whole number, e.g., $\frac{1}{3}$)
- Unit fraction (fractions with numerator 1)
- Unit interval (e.g., the interval from 0 to 1, measured by length)
- Whole (e.g., 2 halves, 3 thirds, 4 fourths)

Students performing Below standard

[24] These are terms and symbols students have seen previously.

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.
- Guide students as they select and practice using their own graphic organizers and models to solve.

- Use direct instruction for vocabulary with visual or concrete representations.
- Use explicit directions with steps and procedures enumerated.
- Guide students through initial practice promoting gradual independence. “I do, we do, you do.”
- Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.
- Scaffold complex concepts and provide leveled problems for multiple entry points.

Provide Multiple Means of Action and Expression

- First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.
- Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’
- Encourage students to explain their thinking and strategy for the solution.
- Choose numbers and tasks that are “just right” for learners but teach the same concepts.
- Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2.

Provide Multiple Means of Engagement

- Clearly model steps, procedures, and questions to ask when solving.
- Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).
- Have students work together to solve and then check their solutions.

- Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?
- Practice routine to ensure smooth transitions.
- Set goals with students regarding the type of math work students should complete in 60 seconds.
- Set goals with the students regarding next steps and what to focus on next.

Students performing Above standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations.
- Model and post conversation "starters," such as: "I agree because..." "Can you
- explain how you solved it?""I noticed that..."""Your solution is different from/ the same as mine because...""My mistake was to..."
- Incorporate written reflection, evaluation, and synthesis.
- Allow creativity in expression and modeling solutions.

Provide Multiple Means of Action and Expression

- Encourage students to explain their reasoning both orally and in writing.
- Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.
- Offer choices of independent or group assignments for early finishers.
- Encourage students to notice and explore patterns and to identify rules and relationships in math.
- Have students share their observations in discussion and writing (e.g., journaling).
- Foster their curiosity about numbers and mathematical ideas.
- Facilitate research and exploration through discussion, experiments, internet searches, trips, etc.
- Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.
- Let students choose their mode of response: written, oral, concrete, pictorial, or abstract.
- Increase the pace. Offer two word problems to solve, rather than one.
- Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem).
- Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.
- Let students write word problems to show mastery and/or extension of the content.

Provide Multiple Means of Engagement

- Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.
- Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds).
- Make the most of the fun exercises for practicing skip-counting.
- Accept and elicit student ideas and suggestions for ways to extend games.

- Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

GRADE 4 • Unit 7 (MODULE 7)

Exploring Measurement with Multiplication

OVERVIEW

In this module, students build their competencies in measurement as they relate multiplication to the conversion of measurement units. Throughout the module, students will explore multiple strategies for solving measurement problems involving unit conversion.

In Topic A, students build on the work they did in Module 2 with measurement conversions. Working heavily in customary units, students use two-column conversion tables (**4.MD.1**) to practice conversion rates. For example, following a discovery activity where students learn that 16 ounces make 1 pound, students generate a two-column conversion table listing the number of ounces in 1 to 10 pounds. Tables for other measurement units are then generated in a similar fashion. Students then reason about why they do not need to complete the tables beyond 10 of the larger units. They use their multiplication skills from Module 3 to complete the tables and are able to see and explain connections such as $(13 \times 16) = (10 \times 16) + (3 \times 16)$. One student could reason, for example, that "Since the table shows that there are 160 ounces in 10 pounds and 48 ounces in 3 pounds, I can add them together to tell that there are 208 ounces in 13 pounds." Another student might reason, "Since there are 16 ounces in each pound, I can use the rule of the table and multiply 13 pounds by 16 to find that there are 208 ounces in 13 pounds."

As the topic progresses, students solve multiplicative comparison word problems. They are then challenged to create and solve their own word problems and to critique the reasoning of their peers (**4.OA.1, 4.OA.2**). They share their solution strategies and original problems within small groups, as well as share and critique the problem solving strategies used by their peers. Through the use of guided questions, students discuss not only how the problems were solved, but also the advantages and disadvantages of using each strategy. They further discuss what makes one strategy more efficient than another. By the end of Topic A, students have started to internalize the conversion rates through fluency exercises and continued practice.

Topic B builds upon the conversion work from Topic A to add and subtract mixed units of capacity, length, weight, and time. Working with metric and customary units, students add like units, making comparisons to adding like fractional units, further establishing the importance of deeply understanding the unit. Just as 2 fourths + 3 fourths = 5 fourths, so does 2 quarts + 3 quarts = 5 quarts. 5 fourths can be decomposed into 1 one 1 fourth, and therefore, 5 quarts can be decomposed into 1 gallon 1 quart. Students realize the same situation occurs in subtraction. Just as $1 - \frac{3}{4}$ must be renamed to $\frac{4}{4} - \frac{1}{4}$ so that the units are alike, students must also rename units of measurements to make like units (1 quart – 3 cups = 4 cups – 3 cups). Students go on to add and subtract mixed units of measurements, finding multiple solution strategies, similar to the mixed number work in fractions. With focus on measurement units of capacity, length, weight, and time, students apply this work to solve multi-step word problems.

In Topic C, students reason how to convert larger units of measurements with fractional parts into smaller units by using hands-on measurements. For example, students convert $3\frac{1}{4}$ feet to inches by first finding the number of inches in $\frac{1}{4}$ foot. They partition a length of one foot into 4 equal parts and find $\frac{1}{4}$ foot equals 3 inches. They then convert 3 feet to 36 inches and add 3 inches to find that $3\frac{1}{4}$ feet = 39 inches. This work is directly analogous to earlier work with fraction equivalence using the tape diagram, area model, and number line in Topics A, B, and D of Module 5. Students partitioned a whole into 4 equal parts, decomposed 1 part into 3 smaller units and found 1 fourth to be equal to 3 twelfths. The foot ruler is partitioned with precisely the same reasoning. Students close the topic by using measurements to solve multi-step word problems that require converting larger units into smaller units.

The End-of-Module Assessment follows Topic C.

Students practice major skills and concepts learned throughout the year in these final four lessons, including measuring angles and drawing lines, multiplication and division, and addition and subtraction through guided group work, fluency activities, and vocabulary games.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
4.OA.1 **4.OA.2** **4.MD.1** 4.NBT.5 4.MD.2	A	**Measurement Conversion Tables** Lessons 1–2: Create conversion tables for length, weight, and capacity units using measurement tools, and use the tables to solve problems. Lesson 3: Create conversion tables for units of time, and use the tables to solve problems. Lesson 4: Solve multiplicative comparison word problems using measurement conversion tables. Lesson 5: Share and critique peer strategies.	5
4.OA.2 **4.OA.3** **4.MD.1** **4.MD.2** 4.NBT.5 4.NBT.6	B	**Problem Solving with Measurement** Lesson 6: Solve Problems involving mixed units of capacity. Lesson 7: Solve problems involving mixed units of length. Lesson 8: Solve problems involving mixed units of weight. Lesson 9: Solve problem involving mixed units of time. Lessons 10–11: Solve multi-step measurement word problems.	6
4.OA.3 **4.MD.1** **4.MD.2** 4.NBT.5 4.NBT.6	C	**Investigation of Measurements Expressed as Mixed Numbers** Lessons 12–13: Use measurement tools to convert mixed number measurements to smaller units. Lesson 14: Solve multi-step word problems involving converting mixed number measurements to a single unit.	3
		End-of-Module Assessment: Topics A–C (assessment 1 day, ½ day return, remediation or further application ½ day)	2
	D	**Year in Review** Lessons 15–16: Create and determine the area of composite figures. Lesson 17: Practice and solidify Grade 4 fluency.	4

		Lesson 18: Practice and solidify Grade 4 vocabulary.	
Total Number of Instructional Days			**20**

Math Unit -Unit: 7

Subject: Mathematics

Grade/Course: Grade 4

Pacing: 20 days

Unit of Study: Unit: 7 Exploring Multiplication

Priority Standards:

Focus Grade Level Standards

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCSS Glossary, Table 2.)

4.OA.3 Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[25]

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)….*

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Foundational Standards

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.5 **Apply properties of operations as strategies to multiply and divide.** ***Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)***

3.OA.7 **Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.**

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about

[25] The focus now is on customary units in word problems for application of fraction concepts. 4.MD.3 is addressed in Module 3.

their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and at the same point of a number line diagram.*

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Math Practice Standards:

MP.2 **Reason abstractly and quantitatively.** Students create conversion charts for related measurement units and use the information in the charts to solve complex real-world measurement problems. They also draw number lines and tape diagrams to represent word problems.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students work in groups to select appropriate strategies to solve problems. They present these strategies to the class and discuss the advantages and disadvantages of each strategy in different situations before deciding which ones are most efficient. Students also solve problems created by classmates and explain to the problem's creator how they solved it to see if it is the method the student had in mind when writing the problem.

MP.7 **Look for and make use of structure.** Students look for and make use of connections between measurement units and word problems to help them understand and solve related word problems. They choose the appropriate unit of measure when given the choice and see that the structure of the situations in the word problems dictates which units to measure with.

MP.8 **Look for an express regularity in repeated reasoning.** The creation and use of the measurement conversion tables is a focal point of this module. Students identify and use the patterns found in each table they create. Using the tables to solve various word problems gives students ample opportunities to apply the same strategy to different situations.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
4.OA.1 a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.	Interpret (L-2)
verbal statements of multiplicative comparisons as multiplication equations.	Represent(L-1)
4.OA.2 to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCLS Glossary, Table 2.)	Multiply or divide (L-2)
4.OA.3 a multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.	Solve (L-1)
these problems using equations with a letter standing for the unknown quantity.	Solve (L-1)
the reasonableness of answers using mental computation and estimation strategies including rounding.	Assess (L-3)
4.MD.1 relative sizes of measurement units within	

one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec.	Know (L-1)
measurements in a larger unit in terms of a smaller unit within a single system of measurement.	Express (L-2)
measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in.	Record (L-2)
the length of a 4 ft snake as 48 in.	Express (L-2)
a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …	Generate (L-3)
4.MD.2 the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.	Use (L-1)
measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	Represent (L-2)

Performance Task

See Snowman Construction Activity attached (Appendix A)
Engaging Learning Experiences
See Snowman Construction Activity attached (Appendix A)

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Exit tickets for pre-assessment for each lesson	**Application problem** **Student Debriefs** **Problem set data**	**Assessment Summary**

Type	Administered	Format	Standards Addressed
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	4.OA.1 4.OA.2 4.OA.3 4.MD.1 4.MD.2

Instructional Resources

Useful Websites:
Engage NY K-5 Curriculum overview and guiding documents:
https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents
Engage NY Grade 4 Resources:
https://www.engageny.org/resource/grade-4-mathematics
Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs
North Carolina 4th Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/4th.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/
NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx
Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/
LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132
SBAC Digital Library

- **Suggested Tools and Representations**
- Analog clock (with second hand)
- Balance scale with mass weights
- Beaker (marked for mL and L)
- Composite figure
- Digital scale (metric and customary units)
- Gallon, quart, pint, and cup containers
- Meter stick, yard stick, 12-inch ruler, centimeter ruler
- Number bond
- Number line
- Protractor
- Stopwatch
- Tape diagram
- Two-column table

Instructional Strategies

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

1. **Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way
2. **Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words
3. **Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.
4. **Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.
5. **Nonlinguistic Representations:** has recently been proven to stimulate and increase brain activity.
6. **Cooperative Learning:** has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.
7. **Setting Objectives and Providing Feedback**: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied.
8. **Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material.
9. **Cues, Questions, and Advanced Organizers:** helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.

Meeting the Needs of All Students

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of

engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with

particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

Scaffolds for Students with Disabilities

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

<u>Provide Multiple Means of Representation</u>

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:
- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner

- share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.

- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

<u>Provide Multiple Means of Engagement</u>

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.

- Re-teach the same concept with a variety of fluency games.

- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary

- Decimal expanded form (e.g., $(2 \times 10) + (4 \times 1) + (5 \times 0.1) + (9 \times 0.01) = 24.59$)
- Decimal fraction (fraction with a denominator of 10, 100, 1,000, etc.)
- Decimal number (number written using place value units that are powers of 10)
- Decimal point (period used to separate the whole number part from the fractional part of a decimal number)
- Fraction expanded form (e.g., $(2 \times 10) + (4 \times 1) + (5 \times \frac{1}{10}) + (9 \times \frac{1}{100}) = 24\frac{59}{100}$)
- Hundredth (place value unit such that 100 hundredths equals 1 one)
- Tenth (place value unit such that 10 tenths equals 1 one)

Familiar Terms and Symbols[26]

- Expanded form (e.g., 100 + 30 + 5 = 135)
- Fraction (numerical quantity that is not a whole number, e.g., $\frac{1}{3}$)
- Unit fraction (fractions with numerator 1)
- Unit interval (e.g., the interval from 0 to 1, measured by length)
- Whole (e.g., 2 halves, 3 thirds, 4 fourths)

Students Performing Below Standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

[26] These are terms and symbols students have seen previously.

Provide Multiple Means of Representation

- Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.
- Guide students as they select and practice using their own graphic organizers and models to solve.

- Use direct instruction for vocabulary with visual or concrete representations.
- Use explicit directions with steps and procedures enumerated.
- Guide students through initial practice promoting gradual independence. “I do, we do, you do.”
- Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.
- Scaffold complex concepts and provide leveled problems for multiple entry points.

Provide Multiple Means of Action and Expression

- First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.
- Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’
- Encourage students to explain their thinking and strategy for the solution.
- Choose numbers and tasks that are “just right” for learners but teach the same concepts.
- Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2.

Provide Multiple Means of Engagement

- Clearly model steps, procedures, and questions to ask when solving.
- Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).
- Have students work together to solve and then check their solutions.

- Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?
- Practice routine to ensure smooth transitions.
- Set goals with students regarding the type of math work students should complete in 60 seconds.
- Set goals with the students regarding next steps and what to focus on next.

Students Performing Above Standard

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations.
- Model and post conversation "starters," such as: "I agree because..." "Can you
- explain how you solved it?""I noticed that..."""Your solution is different from/ the same as mine because..."""My mistake was to..."
- Incorporate written reflection, evaluation, and synthesis.
- Allow creativity in expression and modeling solutions.

Provide Multiple Means of Action and Expression

- Encourage students to explain their reasoning both orally and in writing.
- Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.
- Offer choices of independent or group assignments for early finishers.
- Encourage students to notice and explore patterns and to identify rules and relationships in math.
- Have students share their observations in discussion and writing (e.g., journaling).
- Foster their curiosity about numbers and mathematical ideas.
- Facilitate research and exploration through discussion, experiments, internet searches, trips, etc.
- Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.
- Let students choose their mode of response: written, oral, concrete, pictorial, or abstract.
- Increase the pace. Offer two word problems to solve, rather than one.
- Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem).
- Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.
- Let students write word problems to show mastery and/or extension of the content.

Provide Multiple Means of Engagement

- Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.
- Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds).
- Make the most of the fun exercises for practicing skip-counting.
- Accept and elicit student ideas and suggestions for ways to extend games.

- Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Appendix A: Performance Assessment

Appendix B: Three representative sample CFAs

1) **Exit Ticket: Pre-assessment**
2) **Mid –Unit Assessment**
3) **Post- Assessment**

Appendix C: Three Representative lesson plans

Appendix A: Performance Assessment

Focus Standards Addressed:

4. NBT.4

4. OA.2

4. MD.2

Snowman Construction Company *

Mobile's Mayor Jones is seeking your help. The winter of 2015 has found Mobile a snowless town. Citizens and businessmen alike are desperately seeking a few good snowmen. Because it hasn't snowed in Mobile, Alabama this winter, the people of this friendly town miss the snowmen that always see on TV decorated the yards. Mayor Jones has asked your company, the

Snowman Construction Company, to build some snappy snowmen for the town to buy.

Your job is to build a snowman for your construction company to sell. You may spend no more than $25.00 in materials. You may use any of the materials listed to build the snowman as long as you stay within the price limit.

Performance tasks:

Task 1. Review the "Snowman Construction Company Materials List." To identify parts of your snowman and make a list of parts to use. (DOK L-1)

Task 2. Organize the materials you will use and check your calculations to stay within the price required. (DOK L-2)

Task 3. Draw a sketch of your snowman as you choose from the materials list to make sure you buy enough because you can't go back to the store! Then synthesize the information as you design and assemble your snowman. (DOK L-4)

Task 4. Write a brief sales pitch for your snowman that includes an explanation stating at least three reasons why the people of Mobile should buy it. (DOK L-3)

Financial Literacy & Fourth Grade Skills

What you should learn about money:

- It is important to plan carefully when you make a budget.
- You have to make choices when you make purchases.
- The biggest things can cost more, but it doesn't mean they are better.

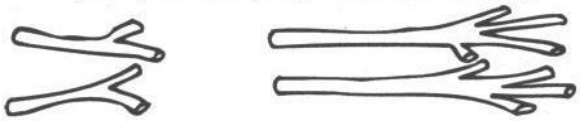
What you should learn that we cover in class:

- Adding and multiplying money with regrouping.
- Writing persuasively.
- Reading tables and charts to gain information.
- Presenting material in visual and written format.

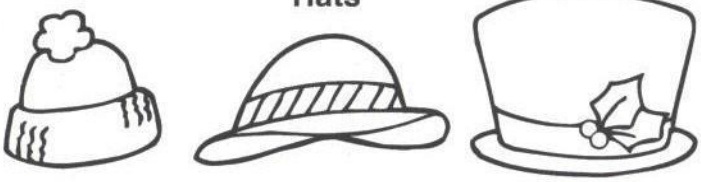
*Adapted from a lesson originally published in Mailbox Magazine December/January 2003

TOTAL

Snowballs: 5 lb. = $0.99 *(must use at least two)* small $0.99 · medium $2.97 · large $4.95 · extra large $5.94				
_____ x $0.99 = _____	_____ x $2.97 = _____	_____ x $4.95= _____	_____ x $5.94 = _____	
Carrots small $0.12 · medium $0.14 · large $0.20				
_____ x $0.12 = _____	_____ x $0.14 = _____	_____ x $0.20 = _____		

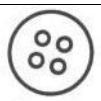
Arms *(free, donated by local tree farm)*  small pair large pair		
_____ x $0= _____	_____ x $0= _____	
Scarf  blue = $2.49 red = $3.72 green = $4.05 purple = $5.26		
_____x $_____= _____		

TOTAL

Hats  $4.25 $6.50 $8.00			
____ x $4.25= ____	____ x $6.50= ____	____ x $8.00= ____	

Buttons

regular
$0.03



large
$0.05

_____ x $0.03 = _____

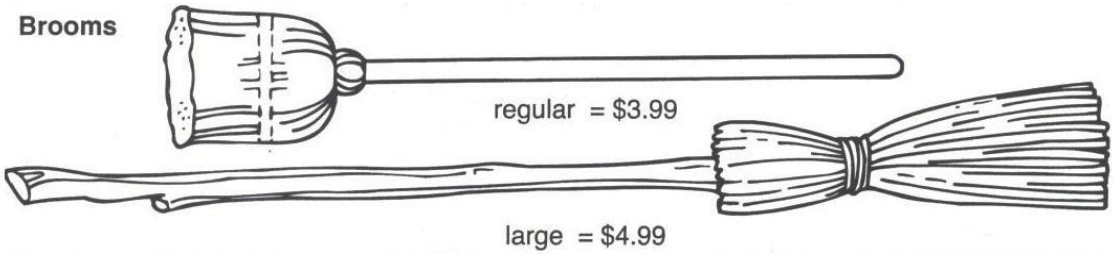
_____ x $0.05 = _____

Coal

1 oz. piece = $0.02

_____ x $0.02 = _____

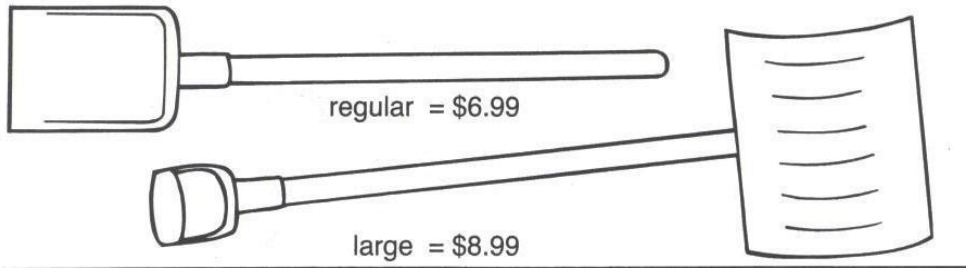
Brooms



_____ x $3.99 = _____

_____ x $4.99 = _____

Shovels



_____ x $6.99 = _____	_____ x $8.99 = _____	
		FINAL PRICE:

Sales Pitch

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Snowman

Construction

Company

Task 1

	Understanding
4	• Shows complete understanding of the required mathematical/scientific knowledge. • The solution completely addresses all mathematical/scientific components presented in the task.
3	• Shows nearly complete understanding of required mathematical/scientific knowledge. • The solution addresses almost all of the mathematical/scientific components presented in the task. There may be minor errors.
2	• Shows some understanding of the required mathematical/ scientific knowledge • The solution addresses some, but not all the mathematical/ scientific components presented in the task.
1	• Shows limited or no understanding of the problem, perhaps only re-copying the given data. • The solution addresses none of the mathematical/scientific components required to solve the task.

Task 2

	Understanding	Planning and Execution
4	• Shows complete understanding of the required mathematical/scientific knowledge. • The solution completely addresses all mathematical/scientific components presented in the task.	• Uses only the important elements of the task. • Uses an appropriate and complete strategy for solving the problem. • Uses only relevant information. • Uses clear and effective diagrams, tables, charts and graphs.
3	• Shows nearly complete understanding of required mathematical/scientific knowledge. • The solution addresses almost all of the mathematical/scientific components presented in the task. There may be minor errors.	• Uses most of the important elements of the task. • Uses an appropriate but incomplete strategy for solving the problem. • Uses most of the relevant data. • Appropriate but incomplete use of diagrams, tables, charts and graphs.
2	• Shows some understanding of the required mathematical/ scientific knowledge • The solution addresses some, but not all the mathematical/ scientific components presented in the task.	• Uses some important elements of the task. • Uses an inappropriate strategy or application of strategy is unclear. • Uses some relevant data. • Limited use or misuse of diagrams, tables, charts, and graphs.
1	• Shows limited or no understanding of the problem, perhaps only re-copying the given data. • The solution addresses none of the mathematical/scientific components required to solve the task.	• Uses none of the important elements of the task. • Works haphazardly with no particular strategy for solving the problem. • Uses irrelevant data. • Does not show use of diagrams, tables, charts or graphs.

Task 3

	Planning and Execution	Persistence
4	● Uses only the important elements of the task. ● Uses an appropriate and complete strategy for solving the problem. ● Uses only relevant information. ● Uses clear and effective diagrams, tables, charts, and graphs.	● Works hard on the task and doesn't need much help. ● Student may extend his thinking beyond the problem or make new connections or create more problems.
3	● Uses most of the important elements of the task ● Uses an appropriate but incomplete strategy for solving the problem. ● Uses most of the relevant data. ● Appropriate but incomplete use of diagrams, tables, charts, and graphs.	● Works hard on the task and only gets help after having tried many strategies given throughout. ● Completes task working dutifully on the harder parts also.
2	● Uses some important elements of the task. ● Uses an inappropriate strategy or application of strategy is unclear. ● Uses some relevant data. ● Limited use or misuse of diagrams, tables, charts, and graphs.	● Can do simple parts of the problem with little help. ● Starts working on the harder parts, but unless there is help, gives up.
1	● Uses none of the important elements of the task. ● Works haphazardly with no particular strategy for solving the problem. ● Uses irrelevant data. ● Does not show use of diagrams, tables, charts, or graphs.	● Needs help even for the very simple tasks. ● Gives up quickly, often just wanting someone to give the answer.

Task 4

	Communication
4	• There is a clear, effective explanation of the solution. All steps are included so the reader does not have to infer how the task was completed. • Mathematical/scientific representation is actively used as a means of communicating ideas. • There is precise and appropriate mathematical/scientific terminology and notation.
3	• There is a clear explanation. • There is appropriate use of accurate mathematical/scientific representation. • There is effective use of mathematical/ scientific terminology and notation.
2	• There is an incomplete explanation; it may not be clearly represented. • There is some use of appropriate mathematical/scientific representation. • There is some use of mathematical/ scientific notation appropriate to the task.
1	• There is no explanation of the solution. The explanation cannot be understood, or is unrelated to the task. • There is no use or inappropriate use of mathematical/scientific representations. • There is no use, or mostly inappropriate use, of mathematical/scientific terminology and notation.

Appendix B:

Pre-assessment Exit Ticket:

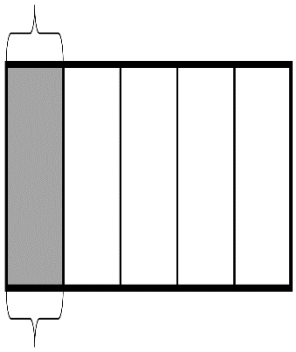
Name _____ Date _____

_____

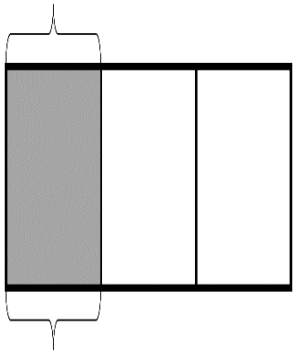
1. Draw horizontal lines to decompose each rectangle into the number of rows as indicated. Use the model to give the shaded area as both a sum of unit fractions and as a multiplication sentence.

a. 2 rows

b. 2 rows



c. 4 rows



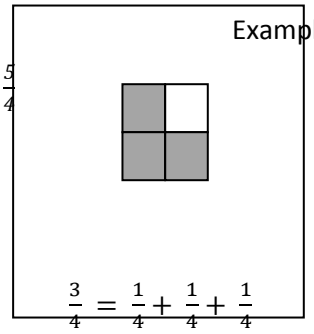
Mid-Module and End of Module Assessment Tasks:

Name _____ Date _____

1. Let each small square represent $\frac{1}{4}$.

   a. Using the same unit, draw and shade the following fractions.
   Represent each as a sum of unit fractions.

Example: $\frac{3}{4}$



i. 1

ii. $\frac{2}{4}$

iii. $\frac{5}{4}$

   b. Record the decompositions of Parts (i) and (iii) using only 2 addends.

      i.

      iii.

   c. Rewrite the equations from Part (a) as the multiplication of a whole number by a unit fraction.

      i.

ii.

2. a.Using the fractional units shown, identify the fraction of the rectangle that is shaded. Continue this pattern by drawing the next area model in the sequence and identifying the fraction shaded.

b.Use multiplication to explain why the first two fractions are equivalent.

iii.

3. Cross out the fraction that is not equivalent to the other three. Show how you know.

a. $\frac{3}{5}$ $\frac{60}{100}$ $\frac{6}{10}$ $\frac{6}{5}$

b. $\frac{6}{4}$ $\frac{3}{2}$ $\frac{12}{8}$ $\frac{8}{4}$

c. $\frac{6}{4}$ $\frac{16}{12}$ $\frac{9}{6}$ $\frac{3}{2}$

4. Fill in the circle with <, =, or > to make a true number sentence. Justify each response by drawing a model (such as an area model or number line), creating common denominators or numerators, or explaining a comparison to a benchmark fraction.

a. $\frac{6}{5}$ $\frac{4}{5}$

b. $\frac{5}{8}$ $\frac{5}{10}$

c. $\frac{5}{5}$ $\frac{12}{12}$

d. $\frac{5}{12}$ $\frac{6}{10}$

. $\frac{5}{6}$ $\frac{3}{4}$

f. $\frac{8}{3}$ $\frac{16}{6}$

. $\frac{7}{4}$ ◯ $\frac{9}{5}$

h. $\frac{12}{8}$ ◯ $\frac{11}{6}$

5. Fill in the blanks to make each number sentence true. Draw a number line, tape diagram, or area model to represent each problem.

a. _____ = $\frac{5}{12} + \frac{6}{12}$

b. $\frac{53}{100} - \frac{27}{100} =$ _____

c. $\frac{8}{12} +$ _____ = 1

d. $\frac{3}{10} + \frac{6}{10} + \frac{2}{10} =$ _____

e. $1 - \frac{5}{8} =$ _____

f. $\frac{7}{8} - \frac{3}{8} =$ _____

6. Ray, Robin, and Freddy went fishing.

   a. They spent $\frac{1}{6}$ of their money on water, $\frac{4}{6}$ of their money on lunch, and the rest on worms. What fraction of their money was spent on worms? Draw a model and write an equation to solve.

   b. Robin noticed her water bottle was $\frac{1}{2}$ full and Freddy's was $\frac{3}{4}$ full. Robin said, "My $\frac{1}{2}$ full bottle has more water than your $\frac{3}{4}$ full bottle." Explain how $\frac{1}{2}$ bottle could be more than $\frac{3}{4}$ bottle.

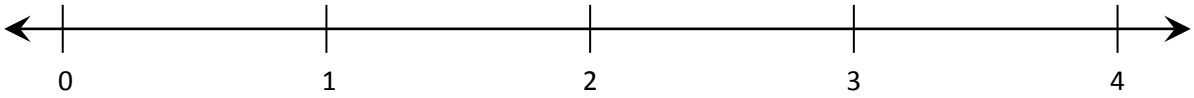
   c. Ray, Robin, and Freddy each had identical containers of worms. Ray used $\frac{3}{8}$ container. Robin used $\frac{6}{8}$ container, and Freddy used $\frac{7}{8}$ container. How many total containers of worms did they use?

   d. Express the number of remaining containers as a product of a whole number and a unit fraction.

   e. Six out of the eight fish they caught were trout. What is another fraction equal to 6 eighths? Write a number sentence and draw a model to show the two fractions are equal

Name _____ Date _____

1. a. Partition the tape diagram to show $5 \times \frac{2}{3}$. Partition the number line to show $10 \times \frac{1}{3}$.



b. Use the models above to explain why $5 \times \frac{2}{3} = 10 \times \frac{1}{3}$.

2. Fill in the circles below with <, =, or > to make true number sentences. Use decomposition or multiplication to justify your answer.

a. $7 \quad \bigcirc \quad \frac{43}{6}$

b. $11\frac{1}{3} \quad \bigcirc \quad \frac{34}{3}$

c. $\frac{13}{6}$ $\frac{38}{12}$

3. Generate a pattern of at least 13 fractions by adding $\frac{4}{3}$ to $\frac{1}{3}$ and then continuing to add $\frac{4}{3}$ to each fraction. Circle each fraction equal to a whole number. Write what you notice about the pattern of whole numbers. The first two fractions are written for you.

$\frac{1}{3}, \frac{5}{3},$

4. Find each sum or difference.

a. $6\frac{4}{10} + 7\frac{7}{10}$

b. $3\frac{3}{8} + 6\frac{5}{8} + 1\frac{7}{8}$

c. $1\frac{9}{12} - 1\frac{4}{12}$

d. $5\frac{2}{5} - 1\frac{3}{5}$

5. Rewrite $3 \times \frac{6}{8}$ as the product of a unit fraction and a whole number. Solve.

b. Rewrite $4 \times 6\frac{2}{3}$ as the product of a unit fraction and a whole number. Solve.

6. Determine if the following are true or false. Explain how you know using models or words. Make false problems true by rewriting the right side of the number sentence.

a. $7\frac{1}{3} = 7 + \frac{1}{3}$

b. $\frac{5}{3} = \frac{3}{3} + \frac{2}{3}$

c. $\frac{13}{6} - \frac{5}{6} = \frac{13-5}{6}$

d. $\frac{11}{3} = 11 + \frac{1}{3}$

e. $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} = 4 \times \frac{7}{8}$

f. $5 \times 3\frac{3}{4} = 15 + \frac{3}{4}$

7. The chart to the right shows data Amashi collected about butterfly wingspans.

Butterfly	Wingspan (inches)
Monarch	$3\frac{7}{8}$
Milbert's Tortoiseshell	$2\frac{5}{8}$
Zebra Swallowtail	$2\frac{1}{2}$
Viceroy	$2\frac{6}{8}$
Postman	$3\frac{3}{8}$
Purple Spotted Swallowtail	$2\frac{2}{8}$
Julia	$3\frac{2}{4}$
Southern Dogface	$2\frac{3}{8}$
Tiger Swallowtail	$3\frac{1}{2}$
Regal Fritillary	$3\frac{4}{8}$

a. At the bottom of this page, create a line plot to display the data in the table.

b. What is the difference in wingspan between the widest and narrowest butterflies on the chart?

c. Three butterflies have the same wingspan. Explain how you know the measurements are equal.

Solve each problem. Draw a model, write an equation, and write a statement for each.

d. Amashi wants to display a Postman and Viceroy side-by-side in a photo box with a width of 6 inches. Will these two butterflies fit? Explain how you know.

e. Compare the wingspan of the Milbert's Tortoiseshell and the Zebra Swallowtail using >, <, or =.

f. The Queen Alexandra Birdwing can have a wingspan that is 5 times as wide as the Southern Dogface's. How many inches can the Birdwing's wingspan be?

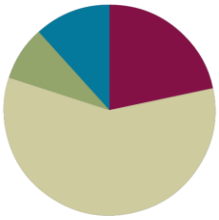
g. Amashi discovered a pattern. She started with $2\frac{2}{8}$ inches and added $\frac{1}{8}$ inch to each measurement. List the next four measurements in her pattern. Name the five butterflies whose wingspans match the measurements in her pattern.

Appendix C: Three representative model lessons

Lesson 1

Objective: Interpret a multiplication equation as a comparison.

Suggested Lesson Structure



NOTES ON FLUENCY PRACTICE:

Think of fluency as having three goals:

1. Maintenance (staying sharp on previously learned skills).
2. Preparation (targeted practice for the current lesson).
3. Anticipation (skills that ensure that students will be ready for the in-depth work of upcoming lessons).

Fluency Practice	(13 minutes)
Application Problem	(5 minutes)
Concept Development	(35 minutes)
Student Debrief	(7 minutes)
Total Time	**(60 minutes)**

Fluency Practice (13 minutes)

- Sprint: Multiply and Divide by 10 **4.NBT.1** (10 minutes)
- Place Value **4.NBT.2** (3 minutes)

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

For the Place Value fluency activity, students may represent ones, etc.,

using counters rather than drawing.

Others may benefit from the opportunity to practice simultaneously speaking and showing units (e.g., tens).

Provide sentence frames to support oral response, such as "_____tens_____ones is _____ (standard form) _____."

Sprint: Multiply and Divide by 10 (10 minutes)

Materials: (S) Multiply and Divide by 10 Sprint

Note: Reviewing this fluency activity acclimates students to the Sprint routine, a vital component of the fluency program.

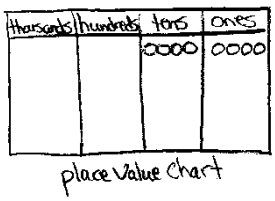
Place Value (3 minutes)

Materials: (S) Personal white board, unlabeled thousands place value chart (Template)

Note: Reviewing and practicing place value skills in isolation prepares students for success in multiplying different place value units during the lesson.

T: (Project place value chart to the thousands.) Show 4 ones as place value disks. Write the number below it.

S: (Draw 4 ones disks and write 4 below it.)



T: Show 4 ten disks and write the number below it.

S: (Draw 4 ten disks and write 4 at the bottom of the tens column.)

T: Say the number in unit form.

S: 4 tens 4 ones.

T: Say the number in standard form.

S: 44.

Continue for the following possible sequence: 2 tens 3 ones, 2 hundreds 3 ones, 2 thousands 3 hundreds, 2 thousands 3 tens, and 2 thousands 3 hundreds 5 tens and 4 ones.

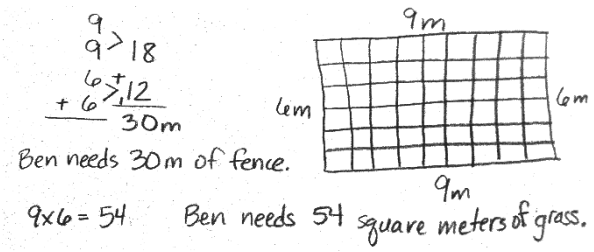
NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Enhance the relevancy of the Application Problem by substituting names, settings, and tasks to reflect students and their experiences.

Set individual student goals and expectations. Some students may successfully solve for area and perimeter in 5 minutes, others may solve for one, while others may solve for both and compose their own application problem.

Application Problem (5 minutes)

Ben has a rectangular area 9 meters long and 6 meters wide. He wants a fence that will go around it as well as grass sod to cover it. How many meters of fence will he need? How many square meters of grass sod will he need to cover the entire area?



Note: As the first lesson of the year, this Application Problem reviews area, perimeter, multiplication, and addition—all important concepts from Grade 3. This problem can be extended after the Concept Development by asking students to find an area 10 times as much as the grass sod or to find a perimeter 10 times as wide and 10 times as long.

Concept Development (35 minutes)

Materials: (T) Place value disks: ones, tens, hundreds, and thousands; unlabeled thousands place value chart (Template) (S) Personal white board, unlabeled thousands place value chart (Template)

Problem 1: 1 ten is 10 times as much as 1 one.

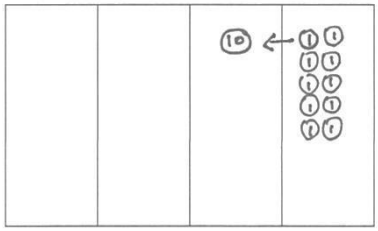
(Have a place value chart ready. Draw or place 1 unit into the ones place.)

T: How many units do I have?

S: 1.

T: What is the name of this unit?

S: A one.



MP.6 Count the ones with me. (Draw ones as they do so.)

S: 1 one, 2 ones, 3 ones, 4 ones, 5 ones...10 ones.

T: 10 ones. What larger unit can I make?

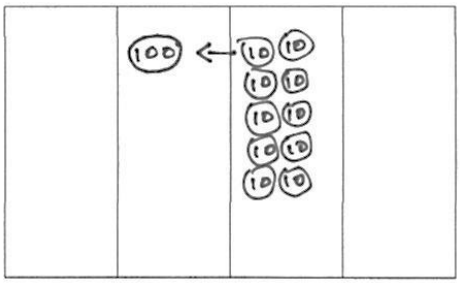
S: 1 ten.

T: I change 10 ones for 1 ten. We say, "1 ten is 10 times as much as 1 one." Tell your partner what we say and what that means. Use the model to help you.

S: 10 ones make 1 ten. ❼ 10 times 1 one is 1 ten or 10 ones. ❼ We say 1 ten is 10 times as many as 1 one.

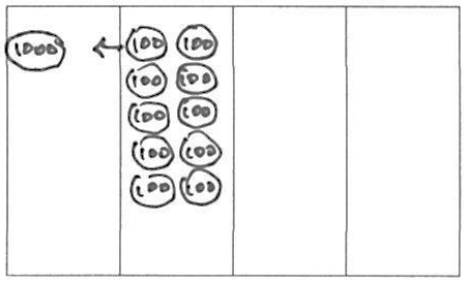
Problem 2: One hundred is 10 times as much as 1 ten.

Quickly repeat the process from Problem 1 with 10 copies of 1 ten.



Problem 3: One thousand is 10 times as much as 1 hundred.

Quickly repeat the process from Problem 1 with 10 copies of 1 hundred.



T: Discuss the patterns you have noticed with your partner.

S: 10 ones make 1 ten. 10 tens make 1 hundred. 10 hundreds make 1 thousand. ❼ Every time we get 10 we bundle and make a bigger unit. ❼ We copy a unit 10 times to make the next larger unit. ❼ If we take any of the place value units, the next unit on the left is ten times as many.

T: Let's review, in words, the multiplication pattern that matches our models and 10 times as many.

1 ten = 10 × 1 one (Say, "1 ten is 10 times as much as 1 one.")

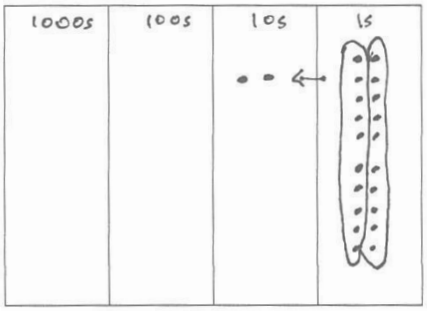
1 hundred = 10 × 1 ten (Say, "1 hundred is 10 times as much as 1 ten.")

1 thousand = 10 × 1 hundred (Say, "1 thousand is 10 times as much as 1 hundred.")

Display the following information for student reference:

Problem 4: Model *10 times as much as* on the place value chart with an accompanying equation.

Note: Place value disks are used as models throughout the curriculum and can be represented in two different ways. A disk with a value labeled inside of it, such as in Problem 1, should be drawn or placed on a place value chart with no headings. The value of the disk in its appropriate column indicates the column heading. A place value disk drawn as a dot should be used on place value charts with headings, as in Problem 4. This type of representation is called the chip model. The chip model is a faster way to represent place value disks and is used as students move away from a concrete stage of learning.



(Model 2 tens is 10 times as much as 2 ones on the place value chart and as an equation.)

T: Draw place value disks as dots. Because you are using dots, label your columns with the unit value.

T: Represent 2 ones. Solve to find 10 times as many as 2 ones. Work together.

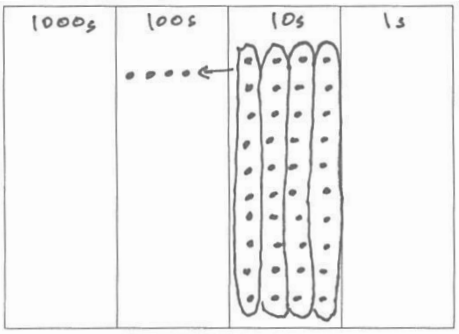
S: (Work together.)

T: 10 times as many as 2 ones is?

S: 20 ones ❼ 2 tens.

T: Explain this equation to your partner using your model.

S: 10 × 2 ones = 20 ones = 2 tens



Repeat the process with 10 times as many as 4 tens is 40 tens is 4 hundreds and 10 times as many as 7 hundreds is 70 hundreds is 7 thousands.

10 × 4 tens = 40 tens = 4 hundreds

10 × 7 hundreds = 70 hundreds = 7 thousands

Problem 5: Model as an equation 10 times as much as 9 hundreds is 9 thousands.

T: Write an equation to find the value of 10 times as many as 9 hundreds. (Circulate and assist students as necessary.)

T: Show me your boards. Read your equation.

S: 10 × 9 hundreds = 90 hundreds = 9 thousands..

T: Yes. Discuss whether this is true with your partner. (Write 10 × 9 hundreds = 9 thousands.)

S: Yes it is true because 90 hundreds equals 9 thousands, so this equation just eliminates that extra step. ❼ Yes, we know 10 of a smaller unit equals 1 of the next larger unit, so we just avoided writing that step.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the purposeful sequencing of the Problem Set guide your selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Consider assigning incomplete problems for homework or at another time during the day.

Challenge quick finishers to write their own 10 times as

many statements similar to Problems 2 and 5.

Student Debrief (7 minutes)

Lesson Objective: Interpret a multiplication equation as a comparison.

Invite students to review their solutions for the Problem Set and the totality of the lesson experience. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set.

You may choose to use any combination of the questions below to lead the discussion.

- What relationship do you notice between the problem of Matthew's stamps and Problems 1(a) and 1(b)?
- How did Problem 1(c) help you to solve Problem 4?
- In Problem 5 which solution proved most difficult to find? Why?
- How does the answer about Sarah's age and her grandfather's age relate to our lesson's objective?
- What are some ways you could model 10 times as many? What are the benefits and drawbacks of each way of modeling? (Money, base ten materials, disks, labeled drawings of disks, dots on a labeled place value chart, tape diagram.)
- Take 2 minutes to explain to your partner what we learned about the value of each unit as it moves from right to left on the place value chart.
- Write and complete the following statements:

_____ ten is _____ times as many as _____ one.

_____ hundred is _____ times as many as _____ ten.

_____ thousand is _____ times as many as _____ hundred.

Exit Ticket (3 minutes)

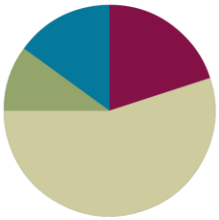
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Lesson 2

Objective: Recognize a digit represents 10 times the value of what it represents in the place to its right.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(33 minutes)
Student Debrief	(9 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Skip-Counting **3.OA.7** (4 minutes)
- Place Value **4.NBT.2** (4 minutes)
- Multiply by 10 **4.NB5.1** (4 minutes)

Skip-Counting (4 minutes)

Note: Practicing skip-counting on the number line builds a foundation for accessing higher order concepts throughout the year.

Direct students to count by threes forward and backward to 36, focusing on the crossing-ten transitions.

Example: (3, 6, 9, 12, 9, 12, 9, 12, 15, 18, 21, 18, 21, 24, 27, 30, 27, 30, 33, 30, 33, 30, 33, 36…) The purpose of focusing on crossing the ten transitions is to help students make the connection that, for example, when adding 3 to 9, 9 + 1 is 10 and then 2 more is 12.

We see a similar purpose in counting down by threes; 12 – 2 is 10 and subtracting 1 more is 9. This work builds on the fluency work of previous grade levels. Students should understand that when crossing the ten, , they are regrouping.

Direct students to count by fours forward and backward to 48, focusing on the crossing-ten transitions.

Place Value (4 minutes)

Materials: (S) Personal white board, unlabeled thousands place value chart (Lesson 1 Template)

Note: Reviewing and practicing place value skills in isolation prepares students for success in multiplying different place value units during the lesson.

T: (Project place value chart to the thousands place.) Show 5 tens as place value disks, and write the number below it.

S: (Draw 5 tens. Write 5 below the tens column and 0 below the ones column.)

T: (Draw to correct student misunderstanding.) Say the number in unit form.

S: 5 tens.

T: Say the number in standard form.

S: 50.

Continue for the following possible sequence: 3 tens 2 ones, 4 hundreds 3 ones, 1 thousand 2 hundreds, 4 thousands 2 tens, and 4 thousands 2 hundreds 3 tens 5 ones.

Multiply by 10 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews concepts learned in Lesson 1.

T: (Project 10 ones × 10 = 1 _____.) Fill in the blank.

S: (Write 10 ones × 10 = 1 hundred.)

T: Say the multiplication sentence in standard form.

S: 10 × 10 = 100.

Repeat for the following possible sequence: 10 × _____ = 2 hundreds; 10 × _____ = 3 hundreds; 10 × _____ = 7 hundreds; 10 × 1 hundred = 1 _____; 10 × ____ = 2 thousands; 10 × _____ = 8 thousands; 10 × 10 thousands = _____.

Application Problem (6 minutes)

Amy is baking muffins. Each baking tray can hold 6 muffins.

a. If Amy bakes 4 trays of muffins, how many muffins will she have in all?

b. The corner bakery produced 10 times as many muffins as Amy baked. How many muffins did the bakery produce?

Extension: If the corner bakery packages the muffins in boxes of 100, how many boxes of 100 could they make?

Note: This Application Problem builds on the concept from the previous lesson of *10 times as many*

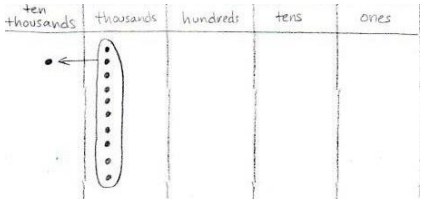
Concept Development (33 minutes)

Materials: (S) Personal white board, unlabeled millions place value chart (Template)

Problem 1: Multiply single units by 10 to build the place value chart to 1 million. Divide to reverse the process.

T: Label ones, tens, hundreds, and thousands on your place value chart.

T: On your personal white board, write the multiplication sentence that shows the relationship between 1 hundred and 1 thousand.



S: (Write: 10 × 1 hundred = 10 hundreds = 1 thousand.)

T: Draw place value disks on your place value chart to find the value of 10 times 1 thousand.

T: (Circulate.) I saw that Tessa drew 10 disks in the thousands column. What does that represent?

S: 10 times 1 thousand equals 10 thousands. (10 × 1 thousand = 10 thousands.)

T: How else can 10 thousands be represented?

S: 10 thousands can be bundled because, when you have 10 of one unit, you can bundle them and move the bundle to the next column.

T: (Point to the place value chart.) Can anyone think of what the name of our next column after the thousands might be? (Students share. Label the **ten thousands** column.)

T: Now write a complete multiplication sentence to show 10 times the value of 1 thousand. Show how you regroup.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Scaffold student understanding of the place value pattern by recording the following sentence frames:

- 10 × 1 one is 1 ten
- 10 × 1 ten is 1 hundred
- 10 × 1 hundred is 1 thousand
- 10 × 1 thousand is 1 ten thousand
- 10 × 1 ten thousand is 1 hundred thousand

Students may benefit from speaking this pattern chorally. Deepen understanding with prepared visuals (perhaps using an interactive whiteboard).

S: (Write 10 × 1 thousand = 10 thousands = 1 ten thousand.)

T: On your place value chart, show what 10 times the value of 1 ten thousand equals. (Circulate and assist students as necessary.)

T: What is 10 times 1 ten thousand?

S: 10 ten thousands. ❼ 1 **hundred thousand**.

T: That is our next larger unit. (Write 10 × 1 ten thousand = 10 ten thousands = 1 hundred thousand.)

T: To move another column to the left, what would be my next 10 times statement?

S: 10 times 1 hundred thousand.

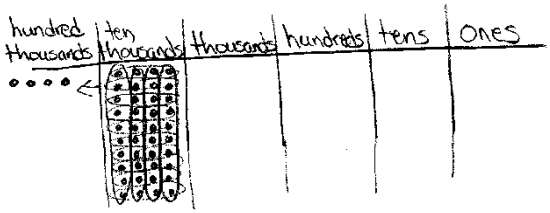
T: Solve to find 10 times 1 hundred thousand. (Circulate and assist students as necessary.)

T: 10 hundred thousands can be bundled and represented as **1 million**. Title your column and write the multiplication sentence.

S: (Write 10 × 1 hundred thousand = 10 hundred thousands = 1 million.)

After having built the place value chart by multiplying by ten, quickly review the process simply moving from right to left on the place value chart and then reversing and moving left to right. (e.g., 2 tens times 10 equals 2 hundreds; 2 hundreds times 10 equals 2 thousands; 2 thousands divided by 10 equals 2 hundreds; 2 hundreds divided by 10 equals 2 tens.)

Problem 2: Multiply multiple copies of one unit by 10.



T: Draw place value disks and write a multiplication sentence to show the value of 10 times 4 ten thousands.

T: 10 times 4 ten thousands is?

S: 40 ten thousands. ❼ 4 hundred thousands.

T: (Write 10 × 4 ten thousands = 40 ten thousands = 4 hundred thousands.) Explain to your partner how you know this equation is true.

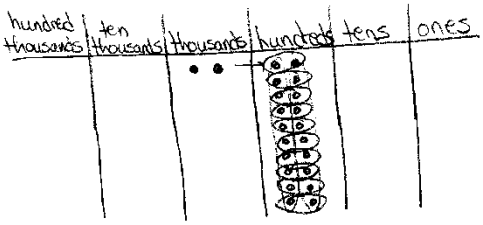
Repeat with 10 × 3 hundred thousands.

Problem 3: Divide multiple copies of one unit by 10.

T: (Write 2 thousands ÷ 10.) What is the process for solving this division expression?

S: Use a place value chart. ❼ Represent 2 thousands on a place value chart. Then change them for smaller units so we can divide.

T: What would our place value chart look like if we changed each thousand for 10 smaller units?



MP.1

S: 20 hundreds. ❼ 2 thousands can be changed to be 20 hundreds because 2 thousands and 20 hundreds are equal.

T: Solve for the answer.

S: 2 hundreds. ❼ 2 thousands ÷ 10 is 2 hundreds because 2 thousands unbundled becomes 20 hundreds. ❼ 20 hundreds divided by 10 is 2 hundreds. ❼ 2 thousands ÷ 10 = 20 hundreds ÷ 10 = 2 hundreds.

Repeat with 3 hundred thousands ÷ 10.

Problem 4: Multiply and divide multiple copies of two different units by 10.

T: Draw place value disks to show 3 hundreds and 2 tens.

T: (Write 10 × (3 hundreds 2 tens).) Work in pairs to solve this expression. I wrote 3 hundreds 2 tens in parentheses to show it is one number. (Circulate as students work. Clarify that both hundreds and tens must be multiplied by 10.)

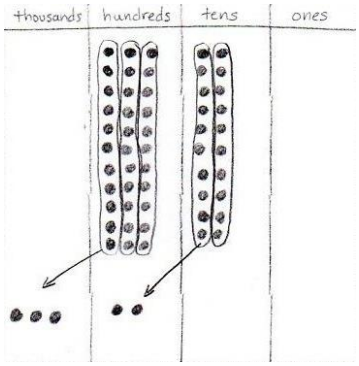
T: What is your product?

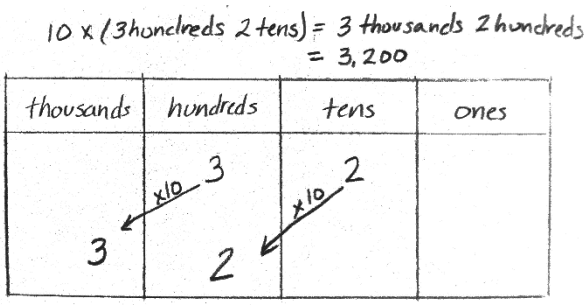
S: 3 thousands 2 hundreds.

T: (Write 10 × (3 hundreds 2 tens) = 3 thousands 2 hundreds.) How do we write this in standard form?

S: 3,200.

T: (Write 10 × (3 hundreds 2 tens) = 3 thousands 2 hundreds = 3,200.)

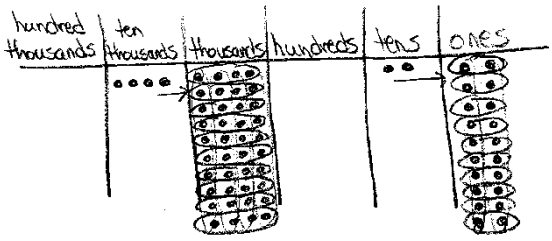


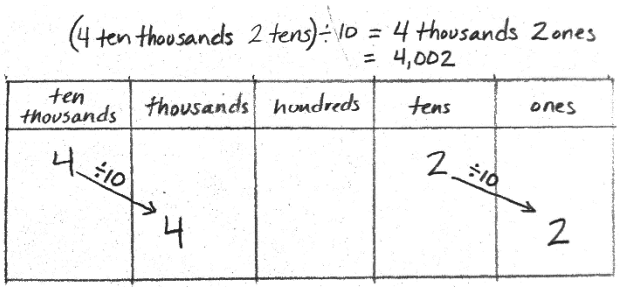


T: (Write (4 ten thousands 2 tens) ÷ 10.) In this expression we have two units. Explain how you will find your answer.

S: We can use the place value chart again and represent the unbundled units, then divide. (Represent in the place value chart and record the number sentence (4 ten thousands 2 tens) ÷ 10 = 4 thousands 2 ones = 4,002.)

T: Watch as I represent numbers in the place value chart to multiply or divide by ten instead of drawing





disks.

Repeat with 10 × (4 thousands 5 hundreds) and (7 hundreds 9 tens) ÷ 10.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (9 minutes)

Lesson Objective: Recognize a digit represents 10 times the value of what it represents in the place to its right.

Invite students to review their solutions for the Problem Set and the totality of the lesson experience. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set.

You may choose to use any combination of the questions below to lead the discussion.

- How did we use patterns to predict the increasing units on the place value chart up to **1 million**?
  Can you predict the unit that is 10 times 1 million?
  100 times 1 million?
- What happens when you multiply a number by 10?
  1 **ten thousand** is what times 10?
  1 **hundred thousand** is what times 10?
- Gail said she noticed that when you multiply a number by 10, you shift the digits one place to the left and put a zero in the ones place.
  Is she correct?
- How can you use multiplication and division to describe the relationship between units on the place value chart? Use Problem 1(a) and (c) to help explain.

- Practice reading your answers in Problem 2 out loud. What similarities did you find in saying the numbers in unit form and standard form? Differences?

- In Problem 7, did you write your equation as a multiplication or division sentence? Which way is correct?
- Which part in Problem 3 was hardest to solve?
- When we multiply 6 tens times 10, as in Problem 2, are we multiplying the 6, the tens, or both? Does the digit or the unit change?
- Is 10 times 6 tens the same as 6 times 10 tens? (Use a place value chart to model.)
- Is 10 times 10 times 6 the same as 10 tens times 6? (Use a place value chart to model 10 times 10 is the same as 1 ten times 1 ten.)
- When we multiply or divide by 10, do we change the digits or the unit? Make a few examples.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name _____ Date _____

1. As you did during the lesson, label and represent the product or quotient by drawing disks on the place value chart.

   a. 10 × 2 thousands = _____ thousands = _____

   b. 10 × 3 ten thousands = _____ ten thousands = _____

   c. 4 thousands ÷ 10 = _____ hundreds ÷ 10 = _____

2. Solve for each expression by writing the solution in unit form and in standard form.

Expression	Unit form	Standard Form
10 × 6 tens		
7 hundreds × 10		
3 thousands ÷ 10		
6 ten thousands ÷ 10		
10 × 4 thousands		

3. Solve for each expression by writing the solution in unit form and in standard form.

Expression	Unit form	Standard Form
(4 tens 3 ones) × 10		
(2 hundreds 3 tens) × 10		
(7 thousands 8 hundreds) × 10		
(6 thousands 4 tens) ÷ 10		
(4 ten thousands 3 tens) ÷ 10		

4. Explain how you solved 10 × 4 thousands. Use a place value chart to support your explanation.

   a. Write a sentence to compare the populations for each planet using the words *10 times as many*.

5. Explain how you solved (4 ten thousands 3 tens) ÷ 10. Use a place value chart to support your explanation.

6. Jacob saved 2 thousand dollar bills, 4 hundred dollar bills, and 6 ten dollar bills to buy a car. The car costs 10 times as much as he has saved. How much does the car cost?

7. Last year the apple orchard experienced a drought and didn't produce many apples. But this year, the apple orchard produced 45 thousand Granny Smith apples and 9 hundred Red Delicious apples, which is 10 times as many apples as last year. How many apples did the orchard produce last year?

8. Planet Ruba has a population of 1 million aliens. Planet Zamba has 1 hundred thousand aliens.
   a. How many more aliens does Planet Ruba have than Planet Zamba?

Name _____ Date _____

1. Fill in the blank to make a true number sentence. Use standard form.

   a. (4 ten thousands 6 hundreds) × 10 = _____

   b. (8 thousands 2 tens) ÷ 10 = _____

2. The Carson family saved up $39,580 for a new home. The cost of their dream home is 10 times as much as they have saved. How much does their dream home cost?

Name _____ Date _____

_____

1. As you did during the lesson, label and represent the product or quotient by drawing disks on the place value chart.
   a. 10 × 4 thousands = _____ thousands = _____

   b. 4 thousands ÷ 10 = _____ hundreds ÷ 10 = _____

Expression	Unit Form	Standard Form
10 × 3 tens		
5 hundreds × 10		
9 ten thousands ÷ 10		
10 × 7 thousands		

2. Solve for each expression by writing the solution in unit form and in standard form.

3. Solve for each expression by writing the solution in unit form and in standard form.

Expression	Unit Form	Standard Form

(2 tens 1 one) × 10		
(5 hundreds 5 tens) × 10		
(2 thousands 7 tens) ÷ 10		
(4 ten thousands 8 hundreds) ÷ 10		

4.

a. Emily collected $950 selling Girl Scout cookies all day Saturday. Emily's troop collected 10 times as much as she did. How much money did Emily's troop raise?

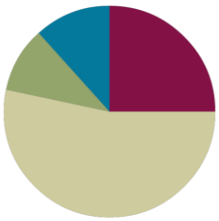
b. On Saturday, Emily made 10 times as much as on Monday. How much money did Emily collect on Monday?

Lesson 3

Objective: Name numbers within 1 million by building understanding of the place value chart and placement of commas for naming base thousand units.

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(7 minutes)
Total Time	**(60 minutes)**



A NOTE ON STANDARDS ALIGNMENT:

In this lesson, students extend pass 1 million (4.NBT standards limit to whole numbers less than or equal to 1 million) to establish a pattern of ones, tens, and hundreds within each base ten unit (thousands, millions, billions, trillions).

Calculations in following lessons are limited to less than or equal to 1 million. If students are not ready for this step, omit establishing the pattern and internalize the units of the thousands period.

[27] unlabeled millions place value chart

Fluency Practice (15 minutes)

- Sprint: Multiply by 3 **3.OA.7** (10 minutes)
- Place Value and Value **4.NBT.2** (3 minutes)
- Base Ten Units **4.NBT.1** (2 minutes)

Sprint: Multiply by 3 (10 minutes)

Materials: (S) Multiply by 3 Sprint

Note: This fluency activity reviews a foundational third grade standard that helps students learn standard **4.NBT.5**.

Place Value and Value (3 minutes)

Materials: (T) Unlabeled millions place value chart (Lesson 2 Template)

Note: Reviewing and practicing place value skills in isolation prepares students for success in multiplying different place value units during the lesson.

T: (Project the number 1,468,357 on a place value chart. Underline the 5.) Say the digit.

S: 5.

T: Say the place value of the 5.

S: Tens.

T: Say the value of 5 tens.

S: 50.

Repeat process, underlining 8, 4, 1, and 6.

Base Ten Units (2 minutes)

Note: This fluency activity bolsters students' place value proficiency while reviewing multiplication concepts learned in Lessons 1 and 2.

T: (Project 2 tens =____.) Say the number in standard form.

S: 2 tens = 20.

Repeat for possible sequence: 3 tens, 9 tens, 10 tens, 11 tens, 12 tens, 19 tens, 20 tens, 30 tens, 40 tens, 80 tens, 84 tens, and 65 tens.

Application Problem (6 minutes)

The school library has 10,600 books.
The town library has 10 times as many books.
How many books does the town library have?

Note: This Application Problem builds on the concept from the previous lesson of determining 10 times as much as a number.

Concept Development (32 minutes)

Materials: (S) Personal white board, unlabeled millions place value chart (Lesson 2 Template)

NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Scaffold partner talk with sentence frames such as:

- "I notice _____."
- "The place value headings are alike because _____."
- "The place value headings are not alike because _____."

- "The pattern I notice is _____."
- "I notice the units _____."

Note: Students will go beyond the **4.NBT** standard of using numbers less than or equal to 1 million to establish a pattern within the base ten units.

Introduction: Patterns of the base ten system.

T: In the last lesson, we extended the place value chart to 1 million. Take a minute to label the place value headings on your place value chart. (Circulate and check all headings.)

T: Excellent. Now talk with your partner about similarities and differences you see in those heading names.

S: I notice some words repeat, like ten, hundred, and thousand, but *ones* appears once. ❼ I notice the thousand unit repeats 3 times—thousands, ten thousands, hundred thousands.

T: That's right! Beginning with thousands, we start naming new place value units by how many one thousands, ten thousands, and hundred thousands we have. What do you think the next unit might be called after 1 million?

S: **Ten millions**.

T: (Extend chart to the ten millions.) And the next?

S: **Hundred millions**.

T: (Extend chart again.) That's right! Just like with thousands, we name new units here in terms of how many one millions, ten millions, and hundred millions we have. 10 hundred millions gets renamed as 1 billion. Talk with your partner about what the next two place value units should be.

S: Ten billions and hundred billions. ❼ It works just like it does for thousands and millions.

Problem 1: Placing commas in and naming numbers.

T: You've noticed a pattern: ones, tens, and hundreds; one thousands, ten thousands, and hundred thousands; one millions, ten millions, and hundred millions; and so on. We use commas to indicate this grouping of units, taken 3 at a time. For example, ten billion would be written: 10,000,000,000.

T: (Write 3608430325.) Record this number and place the commas to show our groupings of units.

S: (Record the number and place the commas.)

T: (Show 430,325 on a place value chart.) How many thousands are in this number?

S: 430.

T: 430 what?

S: 430 thousands.

T: Correct, we read this number as "four hundred thirty thousand, three hundred twenty-five."

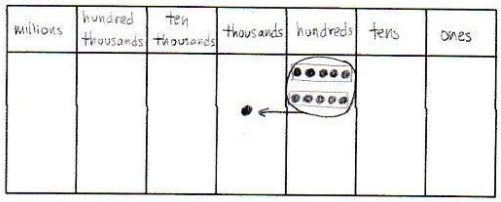
T: (Extend chart and show 608,430,325.) How many millions are there in this number?

S: 608 millions.

T: Using what you know about our pattern in naming units, talk with your partner about how to name this number.

S: Six hundred eight million, four hundred thirty thousand, three hundred twenty-five.

Problem 2: Add to make 10 of a unit and bundling up to 1 million.

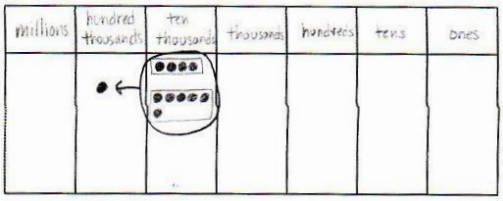


T: What would happen if we combined 2 groups of 5 hundreds? With your partner, draw place value disks to solve. Use the largest unit possible to express your answer.

S: 2 groups of 5 hundreds equals 10 hundreds. ❼
It would make 10 hundreds which can be bundled to make 1 thousand.

T: Now, solve for 5 thousands plus 5 thousands. Bundle in order to express your answer using the largest unit possible.

S: 5 thousands plus 5 thousands equals 10 thousands. We can bundle 10 thousands to make 1 ten thousand.



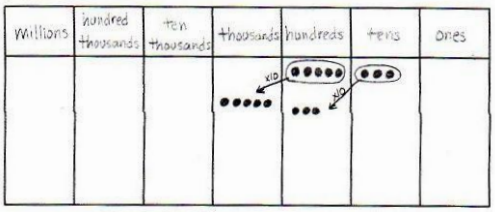
T: Solve for 4 ten thousands plus 6 ten thousands. Express your answer using the largest unit possible.

S: 4 ten thousands plus 6 ten thousands equals 10 ten thousands. We can bundle 10 ten thousands to make 1 hundred thousand.

Continue renaming problems, showing regrouping as necessary.

- 3 hundred thousands + 7 hundred thousands
- 23 thousands + 4 ten thousands
- 43 ten thousands + 11 thousands

Problem 3: 10 times as many with multiple units.



T: On your place value chart, model 5 hundreds and 3 tens with place value disks. What is 10 times 5 hundreds 3 tens?

S: (Show charts.) 5 thousands 3 hundreds.

T: Model 10 times 5 hundreds 3 tens with digits on the place value chart. Record your answer in standard form.

S: (Students show 10 times 5 hundreds is 5 thousands and 10 times 3 tens is 3 hundreds as digits.) 5,300.

T: Check your partner's work and remind him/her of the comma's role in this number.

T: (Write 10 × 1 ten thousand 5 thousands 3 hundreds 9 ones = _____.) With your partner, solve this problem and write your answer in standard form.

S: 10 × 15,309 = 153,090.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Commas are optional for 4-digit numbers, as omitting them supports visualization of the total amount of each unit. For example, in the number *3247*, 32 hundreds or 324 tens is easier to visualize when 3247 is written without a comma. In Grade 3, students understand 324 as 324 ones, 32 tens 4 ones, or 3 hundreds 2 tens 4 ones. This flexible thinking allows for seeing simplifying strategies (e.g., to solve 3247 – 623, rather than decompose 3 thousands, students might subtract 6 hundreds from 32 hundreds: 32 hundreds – 6 hundreds + 47 ones – 23 ones is 26 hundreds and 24 ones or 2624).

Student Debrief (7 minutes)

Lesson Objective: Name numbers within 1 million by building understanding of the place value chart and placement of commas for naming base thousand units.

Invite students to review their solutions for the Problem Set and the totality of the lesson experience. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1, how did you know where to place commas within a number?
- Read aloud the numbers in Problem 1(d) and (e) with your partner. What role do the commas have as you read the numbers?
- How do place value understanding and the role of commas help you to read the value in the millions period that is represented by the number of millions, **ten millions**, and **hundred millions**?
- What did you discover as you solved Problem 3? How did 3(a) help you to solve 3(b)?
- How did you use the place value chart to help you compare unlike units in Problem 5?

- When might it be useful to omit commas?
  (Please refer to the UDL box for commas to guide your discussion.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

A # Correct _____

Multiply.

1	1 x 3 =		23	10 x 3 =	
2	3 x 1 =		24	9 x 3 =	
3	2 x 3 =		25	4 x 3 =	
4	3 x 2 =		26	8 x 3 =	
5	3 x 3 =		27	5 x 3 =	
6	4 x 3 =		28	7 x 3 =	
7	3 x 4 =		29	6 x 3 =	
8	5 x 3 =		30	3 x 10 =	
9	3 x 5 =		31	3 x 5 =	
10	6 x 3 =		32	3 x 6 =	
11	3 x 6 =		33	3 x 1 =	
12	7 x 3 =		34	3 x 9 =	
13	3 x 7 =		35	3 x 4 =	
14	8 x 3 =		36	3 x 3 =	
15	3 x 8 =		37	3 x 2 =	
16	9 x 3 =		38	3 x 7 =	
17	3 x 9 =		39	3 x 8 =	
18	10 x 3 =		40	11 x 3 =	
19	3 x 10 =		41	3 x 11 =	
20	3 x 3 =		42	12 x 3 =	
21	1 x 3 =		43	3 x 13 =	
22	2 x 3 =		44	13 x 3 =	

B Improvement _____ # Correct _____

Multiply.

1	3 x 1 =		23	9 x 3 =	
2	1 x 3 =		24	3 x 3 =	
3	3 x 2 =		25	8 x 3 =	
4	2 x 3 =		26	4 x 3 =	
5	3 x 3 =		27	7 x 3 =	
6	3 x 4 =		28	5 x 3 =	
7	4 x 3 =		29	6 x 3 =	
8	3 x 5 =		30	3 x 5 =	
9	5 x 3 =		31	3 x 10 =	
10	3 x 6 =		32	3 x 1 =	
11	6 x 3 =		33	3 x 6 =	
12	3 x 7 =		34	3 x 4 =	
13	7 x 3 =		35	3 x 9 =	
14	3 x 8 =		36	3 x 2 =	
15	8 x 3 =		37	3 x 7 =	
16	3 x 9 =		38	3 x 3 =	
17	9 x 3 =		39	3 x 8 =	
18	3 x 10 =		40	11 x 3 =	
19	10 x 3 =		41	3 x 11 =	
20	1 x 3 =		42	13 x 3 =	
21	10 x 3 =		43	3 x 13 =	
22	2 x 3 =		44	12 x 3 =	

Name _____ Date _____

1. Rewrite the following numbers including commas where appropriate:

   a. 1234 _____

   b. 12345 _____

   c. 123456 _____

   d. 1234567 _____

   e. 12345678901 _____

2. Solve each expression. Record your answer in standard form.

Expression	Standard Form
5 tens + 5 tens	
3 hundreds + 7 hundreds	
400 thousands + 600 thousands	
8 thousands + 4 thousands	

3. Represent each addend with place value disks in the place value chart. Show the composition of larger units from 10 smaller units. Write the sum in standard form.

   a. 4 thousands + 11 hundreds = _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

   b. 24 ten thousands + 11 thousands = _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

4. Use digits or disks on the place value chart to represent the following equations. Write the product in standard form.

   a. 10 × 3 thousands = _____

   How many thousands are in the answer? _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

   b. (3 ten thousands 2 thousands) × 10 = _____

   How many thousands are in the answer? _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

   c. (32 thousands 1 hundred 4 ones) × 10 = _____

   How many thousands are in your answer? _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

5. Lee and Gary visited South Korea. They exchanged their dollars for South Korean bills. Lee received 15 ten thousand South Korean bills. Gary received 150 thousand bills. Use disks or numbers on a place value chart to compare Lee and Gary's money.

Name _____ Date _____ _____

1. In the spaces provided write the following units in standard form. Be sure to place commas where appropriate.

   a. 9 thousands 3 hundreds 4 ones _____

   b. 6 ten thousands 2 thousands 7 hundreds 8 tens 9 ones _____

   c. 1 hundred thousand 8 thousands 9 hundreds 5 tens 3 ones _____

2. Use digits or disks on the place value chart to write 26 thousands 13 hundreds.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

How many thousands are in the number you have written? _____

Name _____ Date _____

1. Rewrite the following numbers including commas where appropriate:

   a. 4321 _____ b. 54321 _____

   c. 224466 _____ d. 2224466 _____

   e. 10010011001 _____

2. Solve each expression. Record your answer in standard form.

Expression	Standard Form
4 tens + 6 tens	
8 hundreds + 2 hundreds	
5 thousands + 7 thousands	

3. Represent each addend with place value disks in the place value chart. Show the composition of larger units from 10 smaller units. Write the sum in standard form.

   a. 2 thousands + 12 hundreds = _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

   b. 14 ten thousands + 12 thousands = _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

4. Use digits or disks on the place value chart to represent the following equations. Write the product in standard form.

   a. 10 × 5 thousands = _____

   How many thousands are in the answer? _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

   b. (4 ten thousands 4 thousands) × 10 = _____
   How many thousands are in the answer? _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

   c. (27 thousands 3 hundreds 5 ones) × 10 = _____

   How many thousands are in your answer? _____

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

5. A large grocery store received an order of 2 thousand apples. A neighboring school received an order of 20 boxes of apples with 100 apples in each. Use disks or disks on a place value chart to compare the number of apples received by the school and the number of apples received by the grocery store.