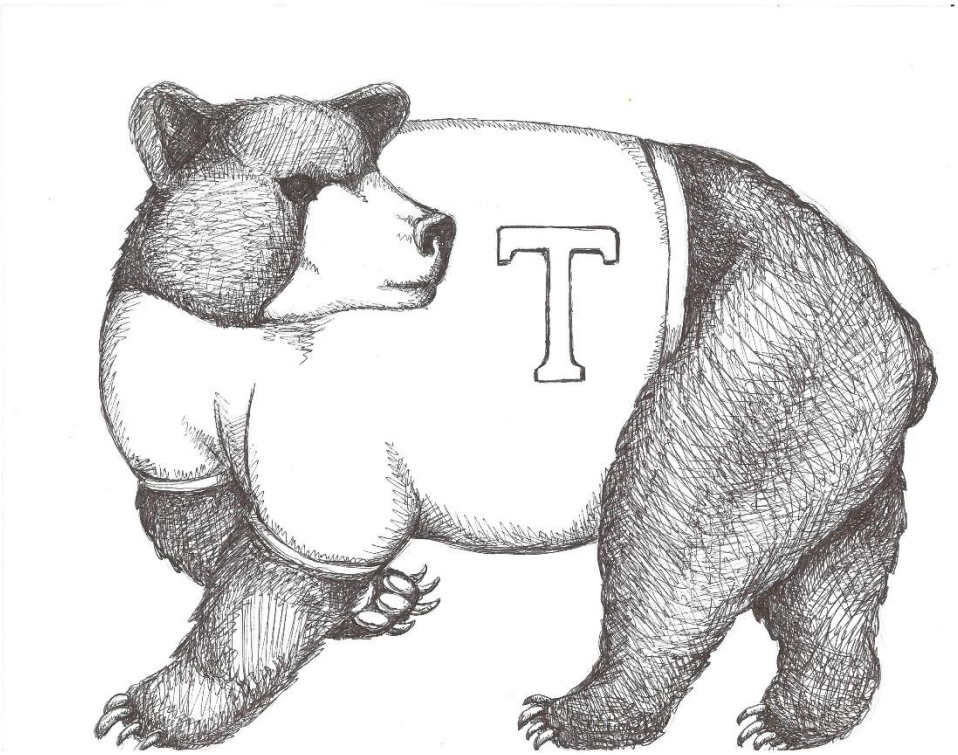
Thomaston Public Schools

158 Main Street
Thomaston, Connecticut 06787
www.thomastonschools.org – 860-283-4796



Thomaston Public Schools Curriculum
Black Rock School
Grade(s): 3 Mathematics 2015

A Nurturing Community Where Children Are Primary

Acknowledgements

Curriculum Writer(s):

Alisha DiCorpo

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

_____*Alisha DiCorpo*_____
Alisha L. DiCorpo
Director of Curriculum and Professional Development

Date of Presentation to the Board of Education: August 2015

(Math Curriculum Grade 3)

Grade 3 Mathematics

Board of Education Mission Statement:

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:

The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main Resource used when writing this curriculum:
NYS COMMON CORE MATHEMATICS CURRICULUM A Story of Units Curriculum. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. A Story of Units: A Curriculum Overview for Grades P-5 Date: 7/31/13 5 © 2013 Common Core, Inc. Some rights reserved. commoncore.org

Course Description:

Sequence of Grade 3 Units Aligned with the Standards

Unit 1: Properties of Multiplication and Division
Unit 2: Place Value and Problem Solving with Units of Measure
Unit 3: Multiplication and Division with Units of 0, 1, 6-9, and Multiples of 10
Unit 4: Multiplication and Area
Unit 5: Fractions as Numbers on the Number Line
Unit 6: Collecting and Displaying Data
Unit 7: Geometry and Measurement in Word Problems

Summary of the Year

Third Grade mathematics is about (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Key Areas of Focus for 3-5: Multiplication and division of whole numbers and fractions-concepts, skills, and problem solving

Required Fluency:

3.OA.7	Multiply and Divide within 100
3.NBT.2	Add and Subtract within 1000

Major Emphasis Clusters

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand the properties of multiplication and the relationship between multiplication and division
- Multiply and divide within 100
- Solve problems involving the four operations and identify and explain patterns in arithmetic

Number and Operations-Fractions

- Develop understanding of fractions as numbers

Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

	Grade 9 -- Algebra I	Grade 10 -- Geometry	Grade 11 -- Algebra II	Grade 12 -- Precalculus	
20 days	M1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)	M1: Congruence, Proof, and Constructions (45 days)	M1: Polynomial, Rational, and Radical Relationships (45 days)	M1: Complex Numbers and Transformations (40 days)	20 days
20 days					20 days
20 days	M2: Descriptive Statistics (25 days)	M2: Similarity, Proof, and Trigonometry (45 days)	M2: Trigonometric Functions (20 days)	M2: Vectors and Matrices (40 days)	20 days
20 days	M3: Linear and Exponential Functions		M3: Functions (45 days)		20 days
	State Examinations	State Examinations	State Examinations	State Examinations	
20 days	(35 days)			M3: Rational and Exponential Functions (25 days)	20 days
		M3: Extending to Three Dimensions (10 days)			
20 days	M4: Polynomial and Quadratic Expressions, Equations and Functions (30 days)	M4: Connecting Algebra and Geometry through Coordinates (25 days)		M4: Trigonometry (20 days)	20 days
20 days			M4: Inferences and Conclusions from Data (40 days)		20 days
20 days	M5: A Synthesis of Modeling with Equations and Functions (20 days)	M5: Circles with and Without Coordinates (25 days)		M5: Probability and Statistics (25 days)	20 days
20 days	Review and Examinations	Review and Examinations	Review and Examinations	Review and Examinations	20 days

Key:	Number and Quantity and Modeling	Geometry and Modeling	Algebra and Modeling	Statistics and Probability and Modeling	Functions and Modeling

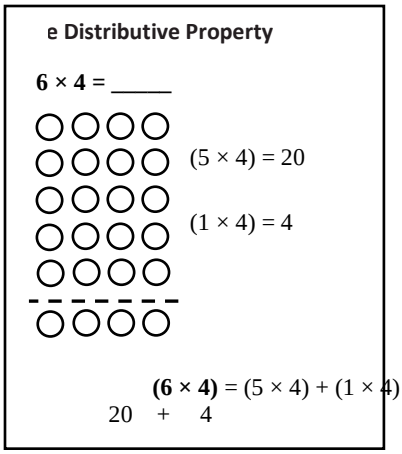
Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

OVERVIEW

This 25-day unit begins the year by building on students' fluency with addition and their knowledge of arrays. In Topic A, students initially use repeated addition to find the total from a number of equal groups (**2.OA.4**). As students notice patterns, they let go of longer addition sentences in favor of more efficient multiplication facts (**3.OA.1**). Lessons in Topic A move students' Grade 2 work with arrays and repeated addition a step further by developing skip-counting rows as a strategy for multiplication. Arrays become a cornerstone of the module. Students use the language of multiplication as they understand what factors are and differentiate between the size of groups and the number of groups within a given context. In this module, the factors 2, 3, 4, 5, and 10 provide an entry point for moving into more difficult factors in later modules.

The study of factors links Topics A and B; Topic B extends the study to division. Students understand division as an unknown factor problem and relate the meaning of unknown factors to either the number or the size of groups (**3.OA.2, 3.OA.6**). By the end of Topic B, students are aware of a fundamental connection between multiplication and division that lays the foundation for the rest of the module.

In Topic C, students use the array model and familiar skip-counting strategies to solidify their understanding of multiplication and practice related facts of 2 and 3. They become fluent enough with arithmetic patterns to "add" or "subtract" groups from known products to solve more complex multiplication problems (**3.OA.1**). They apply their skills to word problems using drawings and equations with a symbol to find the unknown factor (**3.OA.3**). This culminates in students using arrays to model the distributive property as they decompose units to multiply (**3.OA.5**).



In Topic D, students model, write, and solve partitive and measurement division problems with 2 and 3 (**3.OA.2**). Consistent skip-counting strategies and the continued use of array models are pathways for students to naturally relate multiplication and division. Modeling advances as students use tape diagrams to represent multiplication and division. A final lesson in this topic solidifies a growing understanding of the relationship between operations (**3.OA.7**).

Topic E shifts students from simple understanding to analyzing the

relationship between multiplication and division. Practice of both operations is combined—this time using units of 4—and a lesson is explicitly dedicated to modeling the connection between them (**3.OA.7**). Skip-counting, the distributive property, arrays, number bonds, and tape diagrams are tools for both operations (**3.OA.1, 3.OA.2**). A final lesson invites students to explore their work with arrays and related facts through the lens of the commutative property as it relates to multiplication (**3.OA.5**).

The Commutative Property

Topic F introduces the factors 5 and 10, familiar from skip-counting in Grade 2. Students apply the multiplication and division strategies they have used to mixed practice with all of the factors included in Module 1 (**3.OA.1, 3.OA.2, 3.OA.3**). Students model relationships between factors, analyzing the arithmetic patterns that emerge to compose and decompose numbers, as they further explore the relationship between multiplication and division (**3.OA.3, 3.OA.5, 3.OA.7**).

In the final lesson of the unit, students apply the tools, representations, and concepts they have learned to problem solving with multi-step word problems using all four operations (**3.OA.3, 3.OA.8**). They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.

The Mid-Unit Assessment follows Topic C. The End-of-Unit Assessment follows Topic F.

Math Unit 1-Grade 3
Rigorous Curriculum Design Template
Unit 1: Properties of Multiplication and Division and Solving Problems with Units of 2-5 and 10

Subject: Mathematics
Grade/Course: Grade 3
Pacing: 25 Days
Unit of Study: Unit 1: Properties of Multiplication and Division and Solving Problems with Units of 2-5 and 10

Priority Standards: Focus Grade Level Standards

Represent and solve problems involving multiplication and division.[1]

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*

Understand properties of multiplication and the relationship between multiplication

[1] Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

and division.[2]

3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*[3]

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.[4]

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.[5]

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

Foundational Standards

2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

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[2] Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

[3] The associative property is addressed in Module 3.

[4] Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

[5] In this module, problem solving is limited to factors of 2–5 and 10 and the corresponding dividends. 3.OA.9 is addressed in Module 3.

Math Practice Standards: Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students model multiplication and division using the array model. They solve two-step mixed word problems and assess the reasonableness of their solutions.

MP.2 **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships as they explore the properties of multiplication and division and the relationship between them. Students decontextualize when representing equal group situations as multiplication and when they represent division as partitioning objects into equal shares or as unknown factor problems. Students contextualize when they consider the value of units and understand the meaning of the quantities as they compute.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students represent and solve multiplication and division problems using arrays and equations. As they compare methods, they construct arguments and critique the reasoning of others. This practice is particularly exemplified in daily Application Problems and in specific lessons dedicated to problem solving in which students solve and reason with others about their work.

MP.4 **Model with mathematics.** Students represent equal groups using arrays and equations to multiply, divide, add, and subtract.

MP.7 **Look for and make use of structure.** Students notice structure when they represent quantities by using drawings and equations to represent the commutative and distributive properties. The relationship between multiplication and division also highlights structure for students as they determine the unknown whole number in a multiplication or division equation.

"Unwrapped" Standards

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)

3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*
3.OA.5	Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*[6]
3.OA.6	Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do) Depth of Knowledge Level**
Products of whole numbers	Interpret (L2)
Whole-number quotients of whole numbers	Interpret (L2) Represent and solve (L2)
Multiplication and division within 100	Understand the relationship (L2) Use (L1)
Word problems	Solve (L2)
Equal groups, arrays, and measurement quantities	
Unknown whole number in a multiplication or division equation	Determine (L2)

[6] The associative property is addressed in Module 3.

Relating three whole numbers	
Properties of operations as strategies	Apply (L4)
Division as an unknown-factor problem	Understand (L2)
Two-step word problems	Solve (L2)
Four operations	Represent (L2)
Using equations with a letter standing for the unknown	Using (L1)
Mental computation	Assess the reasonableness (L3)
Estimation strategies including rounding	

Essential Questions	**Big ideas (What do you want students to realize when they are done with this unit?)**
How are multiplication and division alike and different? How are multiplication and division related? How are subtraction and division related? How can I model multiplication by ten? How can multiplication and division be used to solve real world problems? How can multiplication be represented? How can the same array represent both multiplication and division? How can we model division? How can we write a mathematical sentence to represent division models we have made?	Multiplication and division can be represented in different ways.

Assessments

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pre-Assessments for lessons-use Exit Tickets and Sprints	**Lesson 1 Exit Ticket** **Lesson 2 Fluency : Sprints** **Lesson 2 Exit Ticket** **Lesson 3 Sprint** **Lesson 3 Exit Ticket** **Lesson 4 Sprint** **Lesson 4 Exit Ticket** **Lessons 5-8 Exit Tickets** **Lesson 9 Pattern Sheet** **Lesson 9 Exit Ticket** **Lesson 10-12 Pattern Sheet** **Lesson 10-12 Exit Ticket** **Lesson 13 Sprint** **Lesson 13 Exit Ticket** **Lesson 14 Sprint and Exit Ticket** **Lesson 15+16 Pattern Sheet and Exit Ticket** **Lesson 17 Sprint and Exit Ticket** **"A Story of Units"** **Lesson 18+19-Exit Tickets** **Lesson 20 Sprint and Exit Tickets** **Lesson 21 Pattern Sheet and Exit Ticket**	**Mid Module Assessment Task "A Story of Units" See Appendix A** **End-of-Module Assessment Task After Topic F See Appendix B** **Constructed response with rubric:** **3.OA.1** **3.OA.2** **3.OA.3** **3.OA.4** **3.OA.5** **3.OA.6** **3.OA.7** **3.OA.8** **Post Lesson Assessment-Exit Ticket**

Performance Task
To be created during the year.
Engaging Learning Experiences

To be created during the year.

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms ▪ Array[7] (arrangement of objects in rows and columns) ▪ Commutative property/commutative (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places) ▪ Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups) ▪ Distribute (with reference to the distributive property, e.g., in 12 × 3 = (10 × 3) + (2 × 3) the 3 is the multiplier for each part of the decomposition) ▪ Divide/division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., 15 ÷ 5 = 3) **Familiar Terms and Symbols[8]** **Familiar Terms and Symbols[9]** ▪ Add 1 unit, subtract 1 uni	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..."'Your solution is different from/ the same as mine because..."'My mistake was to..." Incorporate written

[7] Originally introduced in Grade 2, Module 6 but treated as new vocabulary in this module.

[8] These are terms and symbols students have seen previously. Each of the asterisked terms in this section was introduced in Grade 2, Module 8. However, given the importance of their specific definitions to this module and the amount of time elapsed between G2–M8 and G3–M7, they are bolded at first use in the lessons.

[9] These are terms and symbols students have used or seen previously.

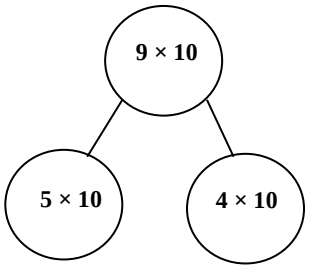
t (add or subtract a single unit of two, ten, etc.) ▪ Expression (see expanded description in box above) ▪ Number bond (illustrates part–part–whole relationship, shown at right) ▪ Ones, twos, threes, etc. (units of one, two, or three) ▪ Repeated addition (adding equal groups together, e.g., 2 + 2 + 2 + 2) ▪ Tape diagram (a method for modeling problems) ▪ Value (how much)	independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>**	reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the

	Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.

		Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Instructional Resources

Suggested Tools and Representations



- 18 counters per student
- Tape diagram (a method for modeling problems)
- Number bond (shown at right)
- Array (arrangement of objects in rows and columns)

www.learnzillion .com
www.teachingchannel.org

Number Literacy
- Multiplication Fact Dot Math Cards
Classroom Bingo using Multiplication Facts
- Students will play bingo to help memorize their facts
-Shape Bait

Interactive Online Activities
http://www.multiplication.com

Literature
*Double the Ducks (Doubling) by Stuart J. Murphy
*Too Many Kangaroo Things to Do! By Stuart J. Murphy
*The 12 Circus Rings by Seymour Chwast
*Anno's Mysterious Multiplying Jar by Masaichiro and Mitsumasa Anno
*Each Orange Has 8 Slices by Paul Giganti, Jr.

Grade 3 • Unit 2

Place Value and Problem Solving with Units of Measure

OVERVIEW

In this 25-day unit, students explore measurement using kilograms, grams, liters, milliliters, and intervals of time in minutes. Students begin by learning to tell and write time to the nearest minute using analog and digital clocks in Topic A (**3.MD.1**). They understand time as a continuous measurement through exploration with stopwatches, and use the number line, a continuous measurement model, as a tool for counting intervals of minutes within 1 hour (**3.MD.1**). Students see that an analog clock is a portion of the number line shaped into a circle. They use both the number line and clock to represent addition and subtraction problems involving intervals of minutes within 1 hour (**3.MD.1**).

Introduced in Topic B, kilograms and grams are measured using digital and spring scales. Students use manipulatives to build a kilogram and then decompose it to explore the relationship between the size and weight of kilograms and grams (**3.MD.2**). An exploratory lesson relates metric weight and liquid volume measured in liters and milliliters, highlighting the coherence of metric measurement. Students practice measuring liquid volume using the vertical number line and a graduated beaker (**3.MD.2**). Building on the estimation skills with metric length gained in Grade 2, students in Grade 3 use kilograms, grams, liters, and milliliters to estimate the weights and liquid volumes of familiar objects. Finally, they use their estimates to reason about solutions to one-step addition, subtraction, multiplication, and division word problems involving metric weight and liquid volume given in the same units (**3.MD.2**).

Now more experienced with measurement and estimation using different units and tools, students further develop their skills by learning to round in Topic C (**3.NBT.1**). They measure and then use place value understandings and the number line as tools to round two-, three-, and four-digit measurements to the nearest ten or hundred (**3.NBT.1, 3.MD.1, 3.MD.2**).

Students measure and round to solve problems in Topics D and E (**3.NBT.1, 3.MD.1, 3.MD.2**). In these topics, they use estimations to test the reasonableness of sums and differences precisely calculated using standard algorithms. From their work with metric measurement, students have a deeper understanding of the composition and decomposition of units. They demonstrate this understanding in every step of the addition and subtraction algorithms with two- and three-digit numbers, as 10 units are changed for 1 larger unit or 1 larger unit is changed for 10 smaller units (**3.NBT.2**). Both topics end in problem solving involving metric units or intervals of time. Students round to estimate, and then calculate precisely using the standard algorithm to add or subtract two- and three-digit measurements given in the same units (**3.NBT.1**, **3.NBT.2, 3.MD.1, 3.MD.2**).

Math Unit 2-Grade 3

Rigorous Curriculum Design Template

Unit 2: Place Value and Problem Solving with Units of Measure

Subject: Mathematics

Grade/Course: Grade 3

Pacing: 25 Days

Unit of Study: Unit 2: Place Value and Problem Solving with Units of Measure

Priority Standards: Foundational Standards

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

- **Use place value understanding and properties of operations to perform multi-digit arithmetic.**[10]

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

- **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by

[10] 3.NBT.3 is taught in Module 3.

representing the problem on a number line diagram.

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Math Practice Standards: MP2, MP4, MP6 and MP7

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly or quantitatively.** Students decontextualize metric measurements and time intervals in minutes as they solve problems involving addition, subtraction, and multiplication. They round to estimate, and then precisely solve problems, evaluating solutions with reference to units and with respect to real world contexts.

MP.4 **Model with mathematics** Students model measurements on the place value chart. They create drawings and diagrams and write equations to model and solve word problems involving metric units and intervals of time in minutes.

MP.6 **Attend to precision** Students round to estimate sums and differences, and then use the standard algorithms for addition and subtraction to calculate. They reason about the precision of their solutions by comparing estimations with calculations and by attending to specific units of measure.

MP.7 **Look for and make use of structure.** Students model measurements on the place value chart. Through modeling, they relate different units of measure and analyze the multiplicative relationship of the base ten system.

"Unwrapped" Standards"

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing

	the problem on a number line diagram.
3.MD.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **(Depth of Knowledge Level)**
Length of an object Inches, feet, cm and meters How much longer one object is to another Standard length unit Whole numbers to the nearest 10 or 100	Measure (L1) Select and Use appropriate tools (L1) Estimate lengths (L2) Measure to determine (L3) Expressing the length difference (L2) Use place value understanding (L2) Round (L2)

Essential Questions	**Big ideas**
Can decomposing numbers help with addition and subtraction of two-digit numbers? How does place value connect with regrouping in addition and subtraction? How are digits in a number related? How can estimation strategies help us build our addition skills? How can I decompose numbers using place	<u>Place Value and Rounding:</u> Place value is crucial when operating with numbers Estimation helps us see whether or not our answers are reasonable Using rounding is an appropriate estimations strategy for solving problems and estimating

value in multiple ways? How can I show what I know about addition, subtraction, problem solving and estimation? When would I use addition and subtraction in a real-life situation? How can I demonstrate my understanding of the measurement of time? What connections can I make between a clock and a number line? What happens when your units of measure change? What is the difference between a standard and non-standard unit of measure? What strategies could you use to figure out the mass of multiple objects? What happens to an item's measurement when units change? What are some ways I can measure liquid volume? How can estimating help me to determine liquid volume?	Rounded numbers are approximate and not exact <u>Addition and Subtraction:</u> Addition and subtraction are inverse operations; one undoes the other We can verify the results of our computation by using the inverse operation Adding zero to a number or subtracting zero from a number does not change the original amount Addition means the joining of two or more sets that may or may not be the same size. There are several types of addition problems. The counting up strategy can be used to make change Subtraction has more than one meaning. It not only means the typical "take away" operation, but also can denote finding the difference between sets.

Assessments		
Common Formative Pre-	Progress Monitoring	Common Formative Mid and or Post-

<table>
<tr><td>Assessments</td><td>Checks – “Dipsticks”</td><td>Assessments Resources</td></tr>
<tr><td>Lesson Exit Tickets for each lesson</td><td>Problem Set Data

Student Debrief</td><td>
<table>
<tr><td>Mid-Module Assessment Task</td><td>After Topic B</td><td>Constructed response with rubric</td><td>3.NBT.2
3.MD.1
3.MD.2</td></tr>
<tr><td>End-of-Module Assessment Task</td><td>After Topic E</td><td>Constructed response with rubric</td><td>3.NBT.1
3.NBT.2
3.MD.1
3.MD.2
3.OA.7
[11]</td></tr>
</table>
Exit tickets after each lesson in unit to compare with pre-assessment exit ticket.
</td></tr>
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<table>
<tr><td>Performance Task</td></tr>
<tr><td>To be created during the year.</td></tr>
<tr><td>Engaging Learning Experiences</td></tr>
<tr><td>To be created during the year.</td></tr>
</table>

[11] Although 3.OA.7 is not a focus standard in this module, it does represent the major fluency for Grade 3. Module 2 fluency instruction provides systematic practice for maintenance and growth. The fluency page on the End-of-Module Assessment directly builds on the assessment given at the end of Module 1 and leads into the assessment that will be given at the end of Module 3.

Instructional Strategies	Meeting the Needs of All Students
Marzano's Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **21st Century Skills** Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	**Scaffolds**[12] The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*." The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple

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	means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. ● Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was

	to…” ● Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with: ● Enlarge sprint print for visually impaired learners. ● Use student boards to work on one calculation at a time. ● Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** ● Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students. ● Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “_____ is ____ hundreds, ____ tens, and ____ ones. ● Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through

	the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" ● Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding.

	● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms and Symbols About (with reference to rounding and estimation, an answer that is not precise) Addend (the numbers that are added together in an addition equation, e.g., in 4 + 5, the numbers 4 and 5 are the addends) Capacity (the amount of liquid that a particular container can hold) Continuous (with reference to time as a continuous measurement) Endpoint[13] (used with rounding on the number line; the numbers that mark the beginning and end of a given interval) Gram (g, unit of measure for weight) Interval (time passed or a segment on the number line) Halfway (with reference to a number line, the midpoint between two numbers, e.g., 5 is	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression**

[13] Originally introduced in Grade 2, but treated as new vocabulary in this module.

halfway between 0 and 10) Kilogram (kg, unit of measure for mass) Liquid volume (the space a liquid takes up) Liter (L, unit of measure for liquid volume) Milliliter (mL, unit of measure for liquid volume) Plot (locate and label a point on a number line) Point (a specific location on the number line) Reasonable (with reference to how plausible an answer is, e.g., "Is your answer reasonable?") Round[14] (estimate a number to the nearest 10 or 100 using place value) Second (a unit of time) Standard algorithm (for addition and subtraction) ≈ (symbol used to show that an answer is approximate)	**Provide Multiple Means of Action and Expression** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **Provide Multiple Means of Engagement** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to

[14] Originally introduced in Grade 2, but treated as new vocabulary in this module.

Familiar Terms and Symbols[15] Analog clock (a clock that is not	to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds.	decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student

[15] These are terms and symbols students have used or seen previously.

digital) Centimeter (cm, unit of measurement) Compose (change 10 smaller units for 1 of the next larger unit on the place value chart) Divide (e.g., 4 ÷ 2 = 2) Estimate (approximation of the value of a quantity or number)	Set goals with the students regarding next steps and what to focus on next	comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Grade 3 • Unit 3

Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

OVERVIEW

This 25-day unit builds directly on students' work with multiplication and division in Unit 1. At this point, Unit 1 instruction coupled with fluency practice in Unit 2 has students well on their way to meeting the Grade 3 fluency expectation for multiplying and dividing within 100 (**3.OA.7**). Unit 3 extends the study of factors from 2, 3, 4, 5, and 10 to include all units from 0 to 10, as well as multiples of 10 within 100. Similar to the organization of Unit 1, the introduction of new factors in Unit 3 spreads across topics. This allows students to build fluency with facts involving a particular unit before moving on. The factors are sequenced to facilitate systematic instruction with increasingly sophisticated strategies and patterns.

Topic A begins by revisiting the commutative property. Students study familiar facts from Unit 1 to identify known facts using units of 6, 7, 8, and 9 (**3.OA.5, 3.OA.7**). They realize that they already know

more than half of their facts by recognizing, for example, that if they know 2 × 8, they also know 8 × 2 through commutativity. This begins a study of arithmetic patterns that becomes an increasingly prominent theme in the module (**3.OA.9**). The subsequent lesson carries this study a step further; students apply the commutative property to relate 5 × 8 and 8 × 5, and then add one more group of 8 to solve 6 × 8 and, by extension, 8 × 6. The final lesson in this topic builds fluency with familiar multiplication and division facts, preparing students for the work ahead by introducing the use of a letter to represent the unknown in various positions (**3.OA.3, 3.OA.4**).

Topic B introduces units of 6 and 7, factors that are well suited to Level 2 skip-counting strategies and to the Level 3 distributive property strategy, already familiar from Unit 1. Students learn to compose up to, then over the next ten. For example, to solve a fact using units of 7 they might count 7, 14, and then mentally add 14 + 6 + 1 to make 21. This strategy previews the associative property using addition and illuminates patterns as students apply count-bys to solve problems. In the next lesson, students apply the distributive property (familiar from Module 1) as a strategy to multiply and divide. They decompose larger unknown facts into smaller known facts to solve. For example, 48 ÷ 6 becomes (30 ÷ 6) + (18 ÷ 6), or 5 + 3 (**3.OA.5, 3.OA.7**). Topic B's final lesson emphasizes word problems, providing opportunities to analyze and model. Students apply the skill of using a letter to represent the unknown in various positions within multiplication and division problems (**3.OA.3, 3.OA.4, 3.OA.7**).

Topic C anticipates the formal introduction of the associative property with a lesson focused on making use of structure to problem solve. Students learn the conventional order for performing operations when parentheses are and are not present in an equation (**3.OA.8**). With this student knowledge in place, the associative property emerges in the next lessons as a strategy to multiply using units up to 8 (**3.OA.5**). Units of 6 and 8 are particularly useful for presenting this Level 3 strategy. Rewriting 6 as 2 × 3 or 8 as 2 × 4 makes shifts in grouping readily apparent (see example on next page) and also utilizes the familiar factors 2, 3, and 4 as students learn the new material. The following strategy may be used to solve a problem like 8 × 5:

$$8 \times 5 = (4 \times 2) \times 5$$

$$8 \times 5 = 4 \times (2 \times 5)$$

$$8 \times 5 = 4 \times 10$$

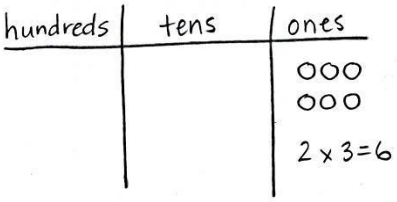
In the final lesson of Topic C, students relate division to multiplication using units up to 8. They understand division as both a quantity divided into equal groups and an unknown factor problem for which—given the large size of units—skip-counting to solve can be more efficient than dividing (**3.OA.3, 3.OA.4, 3.OA.7**).

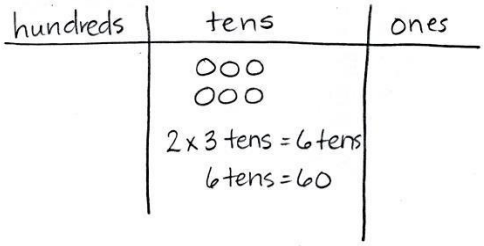
Topic D introduces units of 9 over three days, with students exploring a variety of arithmetic patterns that become engaging strategies for quickly learning facts with automaticity (**3.OA.3, 3.OA.7, 3.OA.9**). Nines are placed late in the unit so that students have enough experience with multiplication and division to recognize, analyze, and apply the rich patterns found in the manipulation of units of 9. As with other topics, the sequence ends with interpreting the unknown factor to solve multiplication and division problems (**3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7**).

In Topic E, students begin by working with facts using units of 0 and 1. From a procedural standpoint, these are simple facts that require little time for students to master; however, understanding the concept of nothing (zero) is more complex, particularly as it relates to division. This unique combination of simple and complex explains the late introduction of 0 and 1 in the sequence of factors. Students

study the results of multiplying and dividing with units of 0 and 1 to identify relationships and patterns (**3.OA.7, 3.OA.9**). The topic closes with a lesson devoted to two-step problems involving all four operations (**3.OA.8**). In this lesson, students work with equations involving unknown quantities and apply the rounding skills learned in Module 2 to make estimations that help them assess the reasonableness of their solutions (**3.OA.8**).

In Topic F, students multiply by multiples of 10 (**3.NBT.3**). To solve a fact like 2 × 30, they first model the basic fact 2 × 3 on the place value chart. Place value understanding helps them to notice that the product shifts one place value to the left when multiplied by 10: 2 × 3 tens can be found by simply locating the same basic fact in the tens column.





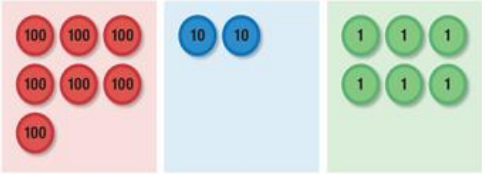
In the subsequent lesson, place value understanding becomes more abstract as students model place value strategies using the associative property (**3.NBT.3, 3.OA.5**). 2 × 30 = 2 × (3 × 10) = (2 × 3) × 10. The final lesson focuses on solving two-step word problems involving multiples of 10 and equations with unknown quantities (**3.OA.8**). As in the final lesson of Topic E, students estimate to assess the reasonableness of their solutions (**3.OA.8**).

Essential Questions	**Big ideas**
How can we model division? How can we use arrays to help develop and understanding of the commutative property? How can we write a mathematical sentence to represent division models we have made? How can multiplication and division be used to solve real world problems? How is the associative property of multiplication used in solving a problem? How is the commutative property of multiplication evident in an array model?	Multiplication and division relationships are multi-disciplinary concepts .

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Exit Tickets for Pre-Assessment of each lesson.	**Application problems** **Student Debriefs** **Problem Set Data**	**Assessment Summary** **Type** / **Administered** / **Format** / **Standards Addressed** Mid-Module Assessmen / After Topic C / Constructed response with rubric / 3.OA.3 3.OA.4 3.OA.5

		t Task | | | 3.OA.7 3.OA.9
		End-of-Module Assessment Task | After Topic F | Constructed response and timed fluency with rubric | 3.OA.3 3.OA.4 3.OA.5 3.OA.7 3.OA.8 3.OA.9 3.NBT.3
		Exit tickets as post assessments for each lesson

Performance Task
To be created during the year.
Engaging Learning Experiences
To be created during the course of the year.
Instructional Resources



Suggested Tools and Representations

Array

Number bond (model used to show part–part–whole relationships)

Place vale disks (pictured at right)

Tape diagram (a method for modeling problems)

Instructional Strategies	Meeting the Needs of All Students
Marzano's Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **21st Century Skills**	**Scaffolds**[16] The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*." The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to

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Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. ● Teach students how to ask questions (such as

	"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" ● Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with: ● Enlarge sprint print for visually impaired learners. ● Use student boards to work on one calculation at a time. ● Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** ● Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. ● Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones. ● Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting." ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are "just right" for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day's sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the

	information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” ● Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities

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New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms Multiple (specifically with reference to naming multiples of 9 and 10, e.g., 20, 30, 40, etc.) Product (the quantity resulting from multiplying two or more numbers together) **Familiar Terms and Symbols**[17] Array (a set of numbers or objects that follow a specific pattern) Commutative property (e.g., 2 × 3 = 3 × 2) Distribute (with reference to the distributive property;	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.”	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...”

[17] These are terms and symbols students have used or seen previously.

e.g., in 12 × 3 = (10 × 3) + (2 × 3), the 3 is the multiplier for each part of the decomposition) Divide, division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., 15 ÷ 5 = 3) Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups) Equation (a statement that two expressions are equal, e.g., 3 × 4 = 12) Even number (a whole number whose last digit is 0, 2, 4, 6, or 8) Expression (a number, or any combination of sums, differences, products, or divisions of numbers that evaluates to a number, e.g., 8 × 3, 15 ÷ 3) Factors (numbers that are multiplied to obtain a product) Multiply, multiplication (an operation showing how many times a number is added to itself, e.g., 5 × 3 = 15) Number bond (model used to show part–part–whole relationships) Number sentence (an equation or inequality for which both expressions are numerical and can be	Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **Provide Multiple Means of Engagement** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions.	Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-

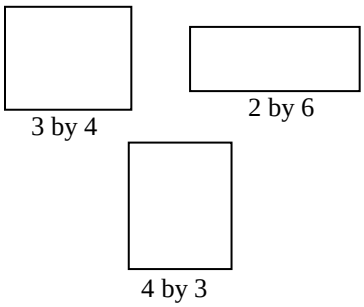
evaluated to a single number, e.g., 21 > 7 × 2, 5 ÷ 5 = 1) Odd number (a number that is not even) Ones, twos, threes, etc. (units of one, two, or three) Parentheses (the symbols ( ) used around a fact or numbers within an equation, expression, or number sentence) Quotient (the answer when one number is divided by another) Row, column (in reference to rectangular arrays) Tape diagram (a method for modeling problems) Unit (one segment of a partitioned tape diagram) Unknown (the "missing" factor or quantity in multiplication or division) Value (how much)	Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Grade 3 • Unit 4

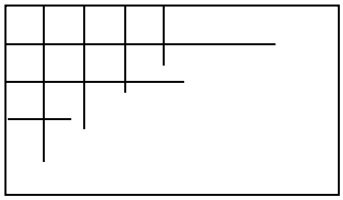
Multiplication and Area

OVERVIEW

In this 20-day unit, students explore area as an attribute of two-dimensional figures and relate it to their prior understandings of multiplication. In Grade 2, students partitioned a rectangle into rows and columns of same-sized squares and found the total number by both counting and adding equal addends represented by the rows or columns (**2.G.2**, **2.OA.4**).

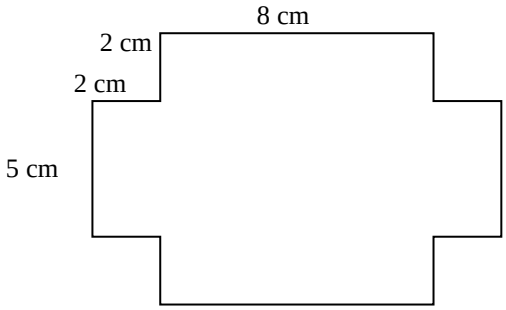


In Topic A, students begin to conceptualize area as the amount of two-dimensional surface that is contained within a plane figure. They come to understand that the space can be tiled with unit squares without gaps or overlaps (**3.MD.5**). Students decompose paper strips into square inches and square centimeters, which they use to tile 3 by 4, 4 by 3, and 2 by 6 rectangles. They compare rectangles tiled with like units and notice different side lengths but equal areas. Topic A provides students' first experience with tiling from which they learn to distinguish between length and area by placing a ruler with the same size units (inches or centimeters) next to a tiled array. They discover that the number of tiles along a side corresponds to the length of the side (**3.MD.6**).



In Topic B, students progress from using square tile manipulatives to drawing their own area models. Anticipating the final structure of an array, they complete rows and columns in figures such as the example shown to the right. Students connect their extensive work with rectangular arrays and multiplication to eventually discover the area formula for a rectangle, which is formally introduced in Grade 4 (**3.MD.7a**).

In Topic C, students manipulate rectangular arrays to concretely demonstrate the arithmetic properties in anticipation of the lessons that follow. They do this by cutting rectangular grids and rearranging the parts into new wholes using the properties to validate that area stays the same, despite the new dimensions. They apply tiling and multiplication skills to determine all whole number possibilities for the side lengths of rectangles given their areas (**3.MD.7b**).



Topic D creates an opportunity for students to solve problems involving area (**3.MD.7b**). Students decompose or compose composite regions, such as the one shown to the right—into non-overlapping rectangles, find the area of each region, and then add or subtract to determine the total area of the original shape. This leads students to find the areas of rooms in a given floor plan (**3.MD.7d**).

Math Unit 4-Grade 3

Rigorous Curriculum Design Template

Unit 4: Multiplication and Area

Subject: Mathematics

Grade/Course: Grade 3

Pacing: 20 Days

Unit of Study: Unit 4: Multiplication and Area

Priority Standards: Focus Grade Level Standards

- **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement:

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find the areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Foundational Standards

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Math Practice Standards: Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively**. **Students build toward abstraction, starting with tiling a rectangle, and then gradually move to finish incomplete grids and drawing grids of their own. Students then eventually work purely in the abstract, imaging the grid as needed.**

MP.3 **Construct viable arguments and critique the reasoning of others**. **Students explore their conjectures about area by cutting to decompose rectangles and then recomposing them in different ways to determine if different rectangles have the same area.** When solving area problems, students learn to justify their reasoning and determine whether they have found all possible solutions, when multiple solutions are possible.

MP.6 **Attend to precision**. **Students precisely label models and interpret them, recognizing that the unit impacts the amount of space a particular model represents, even though pictures may appear to show equal-sized models.** They understand why, when side lengths are multiplied, the result is given in square units.

MP.7 **Look for and make use of structure**. **Students relate previous knowledge of the commutative and distributive properties to area models.** They build from spatial structuring to understanding the number of area-units as the product of number of units in a row and number of rows.

MP.8 **Look for and express regularity in repeated reasoning.** Students use increasingly sophisticated strategies to determine area throughout the course of the module. As students analyze and compare strategies, they eventually realize that area can be found by multiplying the number in each row by the number of rows.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
Area as an attribute of plane figures Concepts of area measurement: a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.6** Areas, Counting unit squares (square cm, square m, square in, square ft, and improvised units). **3.MD.7** Area to the operations of multiplication and addition. a. Area of a rectangle with whole-number side lengths by tiling it, area is the same as would be found by multiplying the side lengths. b. Side lengths, areas of rectangles with whole-number side lengths in the context, real world and mathematical problems, and whole-number products as rectangular areas in mathematical reasoning. d. Area as additive, the areas of rectilinear figures by decomposing them into non-overlapping	Recognize (L1) Understand concepts (L2) Measure areas (L2) Counting (L2) Relate (L2) Find (L1) Show (L2) Multiply (L2) Find (L1) Solving (L3) Represent (L2)

	rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	Recognize (L1) Find (L1) Apply (L3)

Essential Questions	**Big ideas**
Do different factors with the same area cover the same amount of space? How are patterns related to multiplication? How can an addition table help you explain the Commutative Property of Multiplication? How can area be determined without counting each square? How can multiple math operations be used to solve real world problems?	Area models are related to addition and multiplication Area covers a certain amount of space using square units When finding the area of a rectangle, the dimensions represent the factors in a multiplication problem Each dimension can be considered repeated addition Multiplication can be used to find the area of rectangles with whole numbers

How do rectangle dimensions impact the area of the rectangle? How does knowing the length and width of a rectangle relate to multiplication?	Area models of rectangles and squares are directly related to the commutative property of multiplication Rearranging an area based on its dimension or factors does not change the amount of area being covered A product can have more than two factors Area in measurement is equivalent to the product in multiplication Area models can be used as a strategy for solving multiplication problems Some word problems may require two or more operations to find the solution.	
Assessments		

<table>
<tr><td>Common Formative Pre-Assessments</td><td>Progress Monitoring Checks – “Dipsticks”</td><td>Common Formative Mid and or Post-Assessments Resources</td></tr>
<tr><td>Use exit tickets for each lesson as pre-assessments of lessons.</td><td>Exit Tickets post assessment after each lesson.

Monitor in class work on hands-on assignments

Problem Set Data

Student Debrief</td><td>

Assessment Summary

<table>
<tr><th>Type</th><th>Administered</th><th>Format</th><th>Standards Addressed</th></tr>
<tr><td>Mid-Module Assessment Task</td><td>After Topic B</td><td>Constructed response with rubric</td><td>3.MD.5
3.MD.6
3.MD.7abd</td></tr>
<tr><td>End-of-Module Assessment Task</td><td>After Topic D</td><td>Constructed response with rubric</td><td>3.MD.5
3.MD.6
3.MD.7a–d</td></tr>
</table>

</td></tr>
</table>

Performance Task
To be created during the course of the year.
Engaging Learning Experiences
To be created during the course of the year.
Instructional Resources

Suggested Tools and Representations

Area model
Array
Grid paper (inch and centimeter)
Rulers (both centimeter and inch measurements)
Unit squares in both inch and centimeter lengths (e.g., square tiles used for measuring area—can be made out of paper if plastic or wood tiles are not available)

Instructional Strategies	Meeting the Needs of All Students

Marzano's Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **21st Century Skills** Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	**Scaffolds**[18] The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*." The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students

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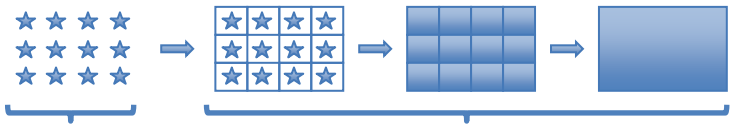


Module 1 and *Module 4*

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
Terminology **New or Recently Introduced Terms** ▪ Area (the amount of two-dimensional space in a bounded region) ▪ Area model (a model for multiplication that relates rectangular arrays to area) ▪ Square unit (a unit of area—specifically square centimeters, inches, feet, and meters) ▪ Tile (to cover a region without gaps or overlaps) ▪ Unit square (e.g., given a length unit, it is a 1 unit by 1 unit square) ▪ Whole number (an integer, i.e., a number without fractions)	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Familiar Terms and Symbols[19] ▪ Array (a set of numbers or objects that follow a specific pattern: a matrix) ▪ Commutative property (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places) ▪ Distribute (e.g., 2 × (3 + 4) = 2 × 3 + 2 × 4) ▪ Geometric shape (a two-dimensional object with a specific outline or form) ▪ Length (the straight-line distance between two points) ▪ Multiplication (e.g., 5 × 3 = 15) ▪ Rows and columns (e.g., in reference to rectangular arrays)	representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate	**Provide Multiple Means of Representation** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..."'"Your solution is different from/ the same as mine because..."'"My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **Provide Multiple**

[19] These are terms and symbols students have seen previously.

	that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **Provide Multiple Means of Engagement** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions.	**Means of Action and Expression** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through

	Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e. g., addition to multiplication), increasing

		numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B

		completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting.

Grade 3 • Unit 5

Fractions as Numbers on the Number Line

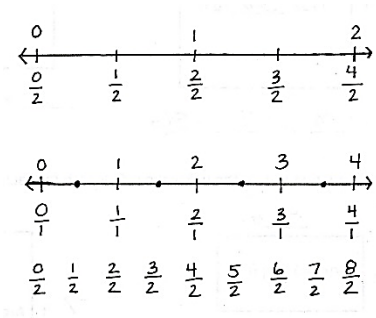
OVERVIEW

In this 35-day unit, students extend and deepen Grade 2 practice with equal shares to understanding fractions as equal partitions of a whole (**2.G.3**). Their knowledge becomes more formal as they work with area models and the number line. Throughout the unit, students have multiple experiences working with the Grade 3 specified fractional units of halves, thirds, fourths, sixths, and eighths. To build flexible thinking about fractions, students are exposed to additional fractional units such as fifths, ninths, and tenths.

Topic A opens Unit 5 with students actively partitioning different models of wholes into equal parts (e.g., concrete models and drawn pictorial area models on paper). They identify and count equal parts as *1 half, 1 fourth, 1 third, 1 sixth,* and *1 eighth* in unit form before introduction to the unit fraction $\frac{1}{?}$ (**3.NF.1**). In Topic B, students compare and make copies of unit fractions to build non-unit fractions. They understand unit fractions as the basic building blocks that compose other fractions (**3.NF.3d**), which parallels the understanding that the number 1 is the basic building block of whole numbers. In Topic C, students practice comparing unit fractions to fraction strips. They specify the whole and label fractions in relation to the number of equal parts in that whole (**3.NF.3d**).

Compare unit fractions using fraction strips.

They count,



Students transfer their work to the number line in Topic D. begin by using the interval from 0 to 1 as the whole. Continuing beyond the first interval, they partition, place, and compare fractions on the number line (**3.NF.2a**, **3.NF.2b**, **3.NF.3d**). In Topic E, they notice that some fractions with different units are placed at the exact same point on the number line, and therefore, are equal (**3.NF.3a**). For example, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are equivalent fractions (**3.NF.3b**). Students recognize that whole numbers can be written as fractions, as exemplified on the number lines to the left (**3.NF.3c**).

Topic F concludes the unit with comparing fractions that have the same numerator. As students compare fractions by reasoning about their size, they understand that fractions with the same numerator and a larger denominator are actually smaller pieces of the whole (**3.NF.3d**). Topic F leaves students with a new method for precisely partitioning a number line into unit fractions of any size without using a ruler.

Math Unit 5-Grade 3

Rigorous Curriculum Design Template

Unit 5: Fractions as Numbers on the Number Line

Subject: Mathematics

Grade/Course: Grade 3

Pacing: 35 Days

Unit of Study: Unit 5: Fractions as Numbers on the Number Line

Priority Standards: Focus Grade Level Standards

- **Develop understanding of fractions as numbers. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)**

3.NF.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

c. **3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form of 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by

reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

- **Reason with shapes and their attributes.**[20]

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Foundational Standards

2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Math Practice Standards: Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students represent fractions concretely, pictorially, and abstractly, as well as move between representations. Students also represent word problems involving fractions pictorially, and then express the answer in the context of the problem.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students reason about the area of a shaded region to determine what fraction of the whole it represents.

MP.6 **Attend to precision.** Students specify the whole amount when referring to a unit fraction and explain what is meant by *equal parts* in their own words.

MP.7 **Look for and make use of structure.** Students understand and use the unit fraction as the basic building block or structure of all fractions on the number line.

[20] 3.G.1 is addressed in Module 7.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **(Depth of Knowledge Level)**
3.NF.1 quantity formed by 1 part when a whole is partitioned into *b* equal parts; *a/b* as the quantity formed by *a* parts of size 1/*b*. **3.NF.2** as a number on the number line; fractions on a number line diagram. **3.NF.3** fractions in special cases, and fractions about their size. **3.G.2** Into parts with equal areas. as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	Understand a fraction (L1) Understand a fraction as a number on a number line (L1) Represent fractions (L2) Explain equivalence (L3) Compare fractions (L3) Reasoning (L2) Partition shapes (L3) Express the area of each part (L1)

Essential Questions	**Big ideas**
How are fractions parts of a whole? How do fractions relate to the number line?	Understand fractions as a number, whole or part, which have order and can be compared.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Exit tickets as pre-assessments	**Exit tickets to monitor progress** **Monitor progress of**	**Type** | **Administered** | **Format** | **Standards Addressed**

	in class assignments **Problem Set Data** **Student Debrief**	Mid-Module Assessment Task	After Topic C	Constructed response with rubric	3.G.2 3.NF.1 3.NF.3cd
		End-of-Module Assessment Task	After Topic F	Constructed response with rubric	3.NF.2ab 3.NF.3a–d

Performance Task
To be created during the course of the year.
Engaging Learning Experiences
To be created during the course of the year.

Instructional Resources

Fraction strips

Suggested Tools and Representations

1 m length of yarn

12″ × 1″ strips of yellow construction paper

1-liter beaker (optional)

2″ × 6″ strips of brown construction paper

200 g ball of clay or play dough

$4\frac{1}{4}$″ × 1″ paper strips

4″ × 4″ orange squares

Arrays

Clear plastic cups

Concrete fraction models (e.g., water, string, clay)

Food coloring (to color water)

Fraction strips (made from paper, used to fold and model parts of a whole. See example to the right.)

Number line

Pictorial fraction model (e.g., drawing of a circle or square)

Rectangular- and circular-shaped paper

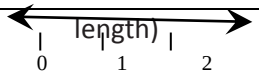
Rulers

Sets of <, >, = cards

Shapes partitioned into fractional parts

Tape diagram

Instructional Strategies	Meeting the Needs of All Students	
New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**
New or Recently Introduced Terms Copies (refers to the number of unit fractions in 1 whole) Equivalent fractions (fractions that name the same size or the same point on the number line) Fractional unit (half, third, fourth, etc.) Non-unit fraction (fraction with numerator other than 1) Unit fraction (fraction with numerator 1) Unit interval (the interval from 0 to 1, measured by	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.”	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...”

 **Familiar Terms and Symbols**[21] =, <, > (equal, less than, greater than) Array (arrangement of objects in rows and columns) Equal parts (parts with equal measurements) Equal shares (pieces of a whole that are the same size) Fraction (e.g., $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}$) Half of, one third of, one fourth of, etc. ($\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$) Halves, thirds, fourths, sixths, eighths ($\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6},$ Partition (divide a whole into equal parts) Whole (e.g., 2 halves, 3 thirds, etc.) Number Line	Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit	Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc.

[21] These are terms and symbols students have used or seen previously.

	learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-

		counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Grade 3 • Unit 6

Collecting and Displaying Data

OVERVIEW

This 10-day unit builds on Grade 2 concepts about data, graphing, and line plots. Topic A begins with a lesson in which students generate categorical data, organize it, and then represent it in a variety of forms. Drawing on Grade 2 knowledge, students might initially use tally marks, tables, or graphs with one-to-one correspondence. By the end of the lesson, they show data in tape diagrams where units are equal groups with a value greater than 1. In the next two lessons, students rotate the tape diagrams vertically so that the tapes become the units or bars of scaled graphs (**3.MD.3**). Students understand picture and bar graphs as vertical representations of tape diagrams and apply well-practiced skip-counting and multiplication strategies to analyze them. In Lesson 4, students synthesize and apply learning from Topic A to solve one- and two-step problems. Through problem solving, opportunities

naturally surface for students to make observations, analyze, and answer questions such as, "How many more?" or "How many less?" (**3.MD.3**).

In Topic B, students learn that intervals do not have to be whole numbers but can have fractional values that facilitate recording measurement data with greater precision. In Lesson 5, they generate a six-inch ruler marked in whole-inch, half-inch, and quarter-inch increments, using the Module 5 concept of partitioning a whole into parts. This creates a conceptual link between measurement and recent learning about fractions. Students then use the rulers to measure the lengths of precut straws and record their findings to generate measurement data (**3.MD.4**).

Lesson 6 reintroduces line plots as a tool for displaying measurement data. Although familiar from Grade 2, line plots in Grade 3 have the added complexity of including fractions on the number line (**2.MD.9**, **3.MD.4**). In this lesson, students interpret scales involving whole, half, and quarter units in order to analyze data. This experience lays the foundation for them to create their own line plots in Lessons 7 and 8. To draw line plots, students learn to choose appropriate intervals within which to display a particular set of data. For example, to show measurements of classmates' heights, students might notice that their data fall within the range of 45 to 55 inches and then construct a line plot with the corresponding interval.

Students end the module by applying learning from Lessons 1–8 to problem solving. They work with a mixture of scaled picture graphs, bar graphs, and line plots to problem solve using both categorical and measurement data (**3.MD.3**, **3.MD.4**).

Math Unit 6-Grade 3

Rigorous Curriculum Design Template

Unit 6: Collecting and Displaying Data

Subject: Mathematics

Grade/Course: Grade 3

Pacing: 10 Days

Unit of Study: Unit 6: Collecting and Displaying Data

Priority Standards: Focus Grade Level Standards

- **Represent and interpret data.**

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Foundational Standards

2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[22] using information presented in a bar graph.

Math Practice Standards: Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students work with data in the context of science and other content areas and interpret measurement data using line plots. Students decontextualize data to create graphs and then contextualize as they analyze their representations to solve problems.

MP.5 **Use appropriate tools strategically.** Students create and use rulers marked in inches, half inches, and quarter inches. Students plot measurement data on a line plot and reason about the appropriateness of a line plot as a tool to display fractional measurements.

MP.6 **Attend to precision.** Students generate rulers using precise measurements and then measure lengths to the nearest quarter inch to collect and record data. Students label axes on graphs to clarify the relationship between quantities and units and attend to the

[22] See Glossary, Table 1

scale on the graph to precisely interpret the quantities involved.

MP.7 **Look for and make use of structure.** Students use an auxiliary line to create equally spaced increments on a six-inch strip, which is familiar from the previous module. Students look for trends in data to help solve problems and draw conclusions about the data.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
3.MD.3 Scaled picture graph and a scaled bar graph Data set with several categories. One- and two-step "how many more" and "how many less" problems presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	Draw (L1) Solve (L2) Represent (L2)
3.MD.4 measurement data measuring lengths using rulers data line plot	Generate (L3) Measuring (L2) Show data (L2) Making (L2)
Essential Questions	**Big ideas**

How can data displayed in tables and graphs be used to inform? How can data be used to make decisions? How can data be used to describe events? How can graphs be used to organize data? How can graphs be used to compare related data? How can I analyze data and use what I've learned to answer mathematical questions about it?	Charts, tables, line plot graphs, pictographs, Venn diagrams, and bar graphs may be used to display data. One way to compare data is through the use of graphs.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Use exit tickets as pre-assessments	Monitor in class assignments over time as well as use exit tickets for post assessments of individual lessons. Problem Set Data Student Debrief	**See chart below for detailed description of mid and end of year assessment.** **Assessment Summary**

Type	Administered	Format	Standards Addressed
End-of-Module Assessment Task	After Topic B	Constructed response with rubric	3.MD.3 3.MD.4

Performance Task

To be created during the course of the year.
Engaging Learning Experiences
To be created during the course of the year.

Instructional Resources
Suggested Tools and Representations Bar graph Grid paper Line plot Picture graph Rulers (measuring in inches, half inches, and quarter inches) Sentence strips Tape diagram

Instructional Strategies	Meeting the Needs of All Students
Marzano's Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **21st Century Skills** Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	**Scaffolds**[23] The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*." The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific

[23] Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

	UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **Provide Multiple Means of Representation** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. ● Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" ● Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with: ● Enlarge sprint print for visually impaired learners. ● Use student boards to work on one calculation at a time. ● Invest in or make math picture dictionaries or word walls. **Provide Multiple Means of Action and Expression**

	● Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students. ● Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “_____ is ____ hundreds, ____ tens, and ____ ones. ● Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” ● Focus on students’ mathematical reasoning (i.e.,

	their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

New or Recently Introduced Terms Frequent (most common measurement on a line plot) Key (notation on a graph explaining the value of a unit) Measurement data (e.g., length measurements of a collection of pencils) Scaled graphs (bar or picture graph in which the scale uses units with a value greater than 1) **Familiar Terms and Symbols**[24] Bar graph (graph generated from categorical data with bars to represent a quantity) Data (information) Fraction (numerical quantity that is not a whole number, e.g., $\frac{1}{3}$) Line plot (display of measurement data on a horizontal line) Picture graph (graph generated from categorical data with graphics to represent a quantity) Scale (a number line used to indicate the various quantities represented in a bar graph) Survey (collecting data by	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..."'"Your solution is different from/ the same as mine because..."'"My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in

[24] These are terms and symbols students have seen previously.

asking a question and recording responses)	thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions.

		Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Grade 3 • Unit 7

Geometry and Measurement Word

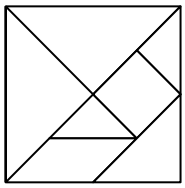
Problems

OVERVIEW

The final unit of the year offers students intensive practice with word problems, as well as hands-on investigation experiences with geometry and perimeter.

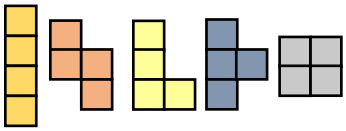
Topic A begins with solving one- and two-step word problems based on a variety of topics studied throughout the year, using all four operations (**3.OA.8**). The lessons emphasize modeling and reasoning to develop solution paths. They incorporate teacher facilitated problem solving, opportunities for students to independently make sense of problems and persevere in solving them, and time for students to share solutions and critique peer strategies.

Topic B introduces an exploration of geometry. Students build on Grade 2 ideas about polygons and their properties, specifically developing and expanding their knowledge of quadrilaterals**.** They explore the attributes of quadrilaterals and classify examples into various categories, including recognizing the characteristics of polygons (**3.G.1**). Students draw polygons based on their attributes, producing sketches from descriptions like, "This shape has two long sides that are parallel, two short sides, and no right angles."



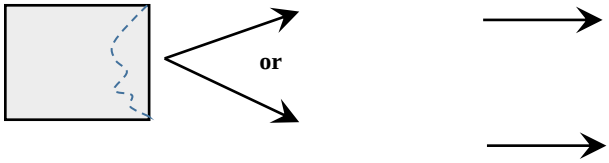
Tangram Puzzle

Students next use tangrams and tetrominoes (see examples to the right) to compose and decompose shapes. They reason about the relationships between shapes and between attributes. For example, students understand that quadrilaterals can be decomposed into triangles, and recognize that the two smallest triangles in a tangram puzzle can be put together to form a parallelogram, a square, and a medium triangle.



Tetrominoes

Students tessellate to bridge geometry experience with the study of perimeter in Topic C. They first decompose a quadrilateral and then rearrange the parts. They use the new shape to tile. Students then define perimeter in two distinct ways: (1) as the boundary of a planar region and (2) as the length of the boundary curve. Students see varied examples from the tiles used to tessellate.



Cut on the line. Then slide the piece to the opposite side or rotate it to an adjacent side to make a new shape.

As they learn about perimeter as an attribute of plane figures, students apply their knowledge to real world situations through problem solving (**3.MD.8**). They measure side lengths of shapes in whole number units to determine perimeter and solve problems where side lengths are given. They use string

and rulers to measure the length around circles of different sizes. This variation prompts students to think more flexibly about perimeter, and to understand that it can be the boundary of any shape and that its measurements are not limited to whole numbers. The topic ends with problems in which some measurements around the perimeter of a polygon are missing but can be determined by reasoning. Students consider the efficiency of their strategies and identify tools for solving; for example, they use multiplication as a tool when measurements are repeated.

Topic D utilizes the line plot, familiar from Unit 6, to help students draw conclusions about perimeter and area measurements (**3.MD.4**). Early in the topic, students find different possible perimeters or areas for rectangles based on information given about the rectangles. For example, using knowledge of factors from experience with multiplication, students determine the following:

- Different perimeters of rectangles comprised of a given number of unit squares (**3.MD.8**).
    - For example, given a rectangle composed of 24 unit squares, students find four possible perimeters: 50, 28, 22, and 20 length units.
- Different areas of rectangles comprised of unit squares with a given perimeter.
    - For example, students use unit squares to build rectangles with a perimeter of 12 units and determine that they can do so using 5, 8, or 9 unit squares.

(Rectangles are formed with unit squares, and as a result they have whole number side lengths.)

Students then draw their rectangles on grid paper and reason about their findings, noticing, for example, that for rectangles of a given area, those with side lengths that are equal or almost equal (more square-like) have smaller perimeters than those whose side lengths are very different (a long and narrow shape). They use line plots to show the number of rectangles they were able to construct for each set of given information. The line plots are a tool that students use to help them reason and draw conclusions about their data.

As they move through the lessons in this topic, students notice and compare differences in the strategies for finding area when given a perimeter and for finding perimeter given an area. By the end of the topic they are able to conclude that there is no direct relationship between area and perimeter, meaning that if an area is given there is no way of knowing a shape's corresponding perimeter.

In Topic E, students solve problems involving area and perimeter. After an initial lesson problem solving with perimeter, students apply this knowledge to create a robot composed of rectangles. Given specific perimeter measurements, they reason about the different side lengths that may be produced. Students compare and analyze their work, discussing how different choices for side lengths can affect area while conforming to the criteria for perimeter. Students synthesize their learning in the final lessons through solving word problems involving area and perimeter using all four operations (**3.OA.8**).

Topic F concludes the school year with a set of engaging lessons that briefly review the fundamental Grade 3 concepts of fractions, multiplication, and division. This topic comes after the End-of-Unit Assessment. It begins with a pair of lessons on fractions, engaging students in analyzing and creating unusual representations of one-half such as those shown to the right. Students analyze and discuss these representations, using their knowledge of fractions to justify their constructions and critique the work of others to make adjustments as necessary. The final lessons in this topic are fluency based and engage students in games that provide practice to solidify their automaticity with Grade 3 skills. Using simple origami

techniques they create booklets of these games. The booklets go home and become summer practice.

Math Unit 7-Grade 3

Rigorous Curriculum Design Template

Unit 7: Geometry and Measurement Word Problems

Subject: Mathematics

Grade/Course: Grade 3

Pacing: 40 Days

Unit of Study: Unit 7: Geometry and Measurement Word Problems

Priority Standards: Focus Grade Level Standards

- **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[25]

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order [Order of Operations].)

- **Represent and interpret data.**[26]

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

- **Reason with shapes and their attributes.**[27]

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[25] 3.OA.9 is addressed in Module 3.

[26] 3.MD.3 is addressed in Module 6.

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

[27] 3.G.2 is addressed in Module 5.

Foundational Standards

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement:

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find the areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Math Practice Standards: Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** This module concentrates on word problems, with an emphasis on modeling and reasoning to develop solution paths

for complex problems. Students have the opportunity to work independently and in small groups to develop the solutions to two-step problems involving all four operations. Additionally, students make conjectures about the properties of polygons, test their thinking, and refine their ideas as they make new discoveries.

MP.3 **Construct viable arguments and critique the reasoning of others.** The focus on problem solving in Module 7 provides opportunities for students to present their strategies, engage in peer critique, and discuss how to improve their solution pathways. Two lessons explicitly focus on these skills. In addition to engaging in this practice through word problems, students also justify why certain shapes belong in certain categories based on their shared attributes.

MP.5 **Use appropriate tools strategically.** When solving perimeter problems, students recognize that using multiplication strategies, when appropriate, is more efficient than addition.

MP.6 **Attend to precision.** Students learn to precisely define terms based on their observations of properties of quadrilaterals. They accurately draw shapes using descriptions of properties and straight-edge tools.

"Unwrapped" Standards	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
Solve word problems using the four operations. Problems using equations with a letter standing for the unknown quantity. Reasonableness of answers using mental computation and estimation strategies including rounding Measurement data by measuring lengths using rulers marked with halves and fourths of an inch. data by making a line plot, Real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown	Solve (L2) Represent using equations (L2) Assess reasonableness (L3) Generate (L3) Show (L2) Solve (L3)

side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. Shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	Understand (L1) Recognize (L1)

Essential Questions	**Big ideas**
Can all shapes be split into halves, thirds, fourths, sixths and eighths? Prove it. How can I demonstrate my understanding of the measurement of area and perimeter? How does measurement shape our world?	BIG IDEAS: Attributes, Units and Measurement Sense 1. Objects and events have a variety of attributes that can be measured. 2. Measuring an attribute involves finding the number of non-standard or standard units that are needed to match, cover, or fill the object being measured. 3. Measuring sense involves an understanding of appropriate measurement units in various situations, of the "how muchness" of measurement units, of measurement processes, of the use of measurement tools, and of estimation in measurement. Measurement Relationships 4. Objects can be compared and ordered according to measurable attributes. 5. Relationships exist between measurement units. Geometry and Spatial Sense 6. The location of an object can be described in terms of its spatial relationship to another object in terms of its position on a grid.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks" **Use exit slip and in**	Common Formative Mid and or Post-Assessments Resources

<table>
<tr><td></td><td>class work to gauge readiness for the next concept</td><td></td></tr>
<tr><td>Use exit slips part of each lesson for pre-assessment</td><td>Problem Set Data

Student Debrief</td><td>

Assessment Summary

<table>
<tr><th>Type</th><th>Administered</th><th>Format</th><th>Standards Addressed</th></tr>
<tr><td>Mid-Module Assessment Task</td><td>After Topic C</td><td>Constructed response with rubric</td><td>3.OA.8
3.MD.8
3.G.1</td></tr>
<tr><td>End-of-Module Assessment Task</td><td>After Topic E</td><td>Constructed response with rubric</td><td>3.OA.8
3.MD.4
3.MD.8
3.G.1</td></tr>
</table>

</td></tr>
</table>

Performance Tasks
City Farmers Performance Task http://schools.nyc.gov/NR/rdonlyres/CD824F33-84DA-4D5F-8D4A-B450EA8C8000/0/NYCDOE_G3_Math_CityFarmers_Final.pdf

Engaging Learning Experiences

Task Description:

Students extend their knowledge about area defined as counting the same size squares in a rectilinear array partitioned into rows and columns. In this unit, students are introduced to the concept of unit-square as an area of measurement. Students then explore the concept of area using operations of both multiplication and addition in real-world problems and hands-on gardening activities.

Instructional Resources

Suggested Tools and Representations

- Cardstock (for making student copies of templates)
- Grid paper
- Pattern Blocks
- Rulers (measuring to the nearest quarter inch, constructed by students in Module 6)

- String
- Tangrams (see example illustrated in overview narrative)
- Tetrominoes (see example illustrated in overview narrative)

Instructional Strategies	Meeting the Needs of All Students
Marzano's Effective Teaching Strategies Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback **21st Century Skills** Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information	Scaffolds[28] The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*." The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access

[28] Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website, www.p12.nysed.gov/specialed/aim, for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

	for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as

	money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. ● Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" ● Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with: ● Enlarge sprint print for visually impaired learners. ● Use student boards to work on one calculation at a time. ● Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** ● Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. ● Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.

	● Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” ● Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>**

	● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms Attribute (any characteristic of a shape, including properties and other defining characteristics, e.g., straight sides, and non-defining characteristics, e.g., blue) Diagonal (e.g., the line drawn between opposite corners of a quadrilateral) Perimeter (boundary or length of the boundary of a two-dimensional shape) Property (e.g., having all sides equal in length) Regular polygon (polygon whose side lengths and interior angles are all equal) Tessellate (to tile a plane without gaps or overlaps) Tetrominoes (four squares arranged to form a shape so that every square	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression**	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..."'"Your solution is different from/ the same as mine because..."'"My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** Encourage students to explain their reasoning both orally and in

shares at least one side with another square) **Familiar Terms and Symbols**[29] Area (the measurement of two-dimensional space in a bounded region) Compose (put two or more objects or numbers together) Decompose (break an object or number into smaller parts) Heptagon (flat figure enclosed by seven straight sides and seven angles) Hexagon (flat figure enclosed by six straight sides and six angles) Octagon (flat figure enclosed by eight straight sides and eight angles) Parallel (lines that do not intersect, even when extended in both directions)* Parallelogram (a quadrilateral with both pairs of opposite sides parallel) Pentagon (flat figure enclosed by five straight sides and five angles) Polygon (a closed figure with three or more straight sides, e.g., triangle, quadrilateral, pentagon, hexagon)* Quadrilaterals (a four-sided	First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order	writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.

[29] These are terms and symbols students have seen previously. Each of the asterisked terms in this section was introduced in Grade 2, Module 8. However, given the importance of their specific definitions to this module and the amount of time elapsed between G2–M8 and G3–M7, they are bolded at first use in the lessons.

polygon, e.g., square, rhombus, rectangle, parallelogram, trapezoid)* Rectangle (flat figure enclosed by four straight sides, having four right angles) Rhombus (flat figure enclosed by four straight sides of the same length) Right angle (e.g., a square corner)* Square (rectangle with four sides of the same length) Tangram (special set of puzzle pieces with five triangles and two quadrilaterals that compose a square) Trapezoid (quadrilateral with at least one pair of parallel sides)* Triangle (flat figure enclosed by three straight sides and three angles)	to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Lesson Plans

In 2014, the LearnZillion Dream Team created hundreds of task-based lesson plans. Digital, interactive, and ready to use, our math lesson plans engage students in the productive struggle that leads to higher engagement and deeper understanding of the Common Core standards.

To help with Focus, one of the three major shifts in the standards, we have flagged the "major work" of each grade.

Math Lesson Plans: 3rd Grade

This is our complete collection of lesson plans for 3rd grade.

Appendix A

Grade 3 Mid Module Assessment

Name _____ Date _____

1. Mrs. Tran plants 2 rows of 5 carrots in her garden.
   a. Draw an array that represents Mrs. Tran's carrots. Use an X to show each carrot.

   b. Mrs. Tran adds 3 more rows of 5 carrots to her garden.
      - Use circles to show her new carrots on the array in Part (a).
      - Fill in the blanks below to show how she added the five rows.

      _____ fives + _____ fives = _____ fives

      - Write a sentence to explain your thinking.

   c. Find the total number of carrots Mrs. Tran planted.

   d. Write a multiplication sentence to describe the array representing the total number of carrots
      Mrs. Tran planted.

2. Mrs. Tran picks 15 tomatoes from her garden. She puts 5 tomatoes in each bag.
   a. Draw Mrs. Tran's bags of tomatoes.

   b. Write a multiplication number sentence that describes your drawing in Part (a).

3. Mrs. Tran plants 12 sunflowers in her garden. She plants them in 3 rows.
   a. Fill in the blanks below to make a true division sentence. What does the answer represent?

      _____ ÷ _____ = _____

   b. Mrs. Tran adds 2 more identical rows of sunflowers to her 3 original rows. Draw an array to show how many flowers she has now.

   c. Mrs. Tran figured out how many flowers she planted. Her work is shown in the box below. Would Mrs. Tran get the same result if she multiplied 5 × 4? Explain why or why not.

(3 × 4) + (2 × 4) = 12 + 8

Appendix B End of Module Assessment

Name _____ Date _____

1. Melanie works in a bakery. She bakes different types of breads. She bakes 18 biscuits on a pan. 3 rows fit on the pan. Draw an array to show the total number of biscuits.

   a. Fill in the missing factor. Write a sentence telling what it represents.

      $3 \times$ _____ $= 18$

   b. Write a related division sentence to find the number of biscuits in each row.

2. Melanie packs the 18 biscuits into bags. She puts two biscuits in each bag.

   a. Draw a picture to show how many bags of biscuits Melanie packs. How many bags of biscuits does she pack?

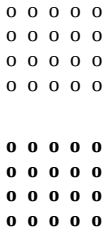
   b. Melanie bakes 18 rolls and packs them into bags of 9. Draw a picture to show how many bags of rolls Melanie packs. How many bags of rolls does she pack?

   c. Draw an array to represent her biscuits. Draw a second array to represent her rolls. Explain the relationship between the 2 arrays using number sentences and words.

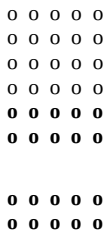
3. Melanie bakes cupcakes for a birthday party. They are shown to the right. Twenty are vanilla, and 20 are chocolate. This shows how she calculated the total number of cupcakes:

   $(4 \times 5) + (4 \times 5) = 8 \times 5$

   a. Use Melanie's method to find the total. Explain each step with words.



   b. Melanie burns 2 rows of five cupcakes. Fill in the unknowns below to describe how many are burnt and how many are not burnt.



$$8 \times 5 = \_\_\_\_\_ \times 5 + \_\_\_\_\_ \times 5$$

4. Melanie decides to bake blueberry muffins next. Her recipe calls for 5 blueberries per muffin. She makes 10 muffins.

   a. Draw a picture and write a multiplication sentence to find the total number of blueberries she uses for 10 muffins.

   b. Melanie uses the equation $10 = \_\_\_\_\_ \div 5$ to figure out how many blueberries she needs. Is her method correct? Why or why not?

   c. If she has a total of 90 blueberries, how many are left after she makes the 10 muffins?

   d. Melanie boxes the 10 muffins. Each box fits 2. Draw a picture and write a number sentence to show how many boxes she fills.

5. Complete as many problems as you can in 100 seconds. Your teacher will time you and tell you when to stop.

4 x 1 = _____	3 ÷ 1 = _____	10 x _____ = 20	2 x 3 = _____	10 ÷ 5 = _____
4 ÷ 2 = _____	2 x _____ = 4	15 ÷ 5 = _____	10 x 3 = _____	4 x _____ = 12
3 x 3 = _____	5 x _____ = 15	16 ÷ 4 = _____	2 x _____ = 8	10 x 4 = _____
2 x 4 = _____	12 ÷ 4 = _____	4 x _____ = 20	5 x 5 = _____	50 ÷ 10 = _____
15 ÷ 3 = _____	2 x _____ = 10	24 ÷ 4 = _____	10 x 6 = _____	5 x _____ = 30
2 x 6 = _____	4 x _____ = 24	35 ÷ 5 = _____	3 x _____ = 21	10 x 7 = _____
4 x 7 = _____	14 ÷ 2 = _____	3 x _____ = 24	5 x 8 = _____	80 ÷ 10 = _____
32 ÷ 4 = _____	10 x _____ = 80	27 ÷ 3 = _____	2 x 9 = _____	5 x _____ = 45

Evaluating Student Learning Outcomes

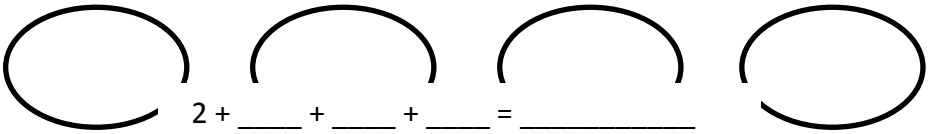
A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency.* In this chart, this progress is presented from left (Step 1) to right (Step 4) for Problems 1–4. The learning goal for each student is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the student CAN do now, and what they need to work on next. Problem 5 is scored differently since it is a timed assessment of fluency. Students complete as many problems as they can in 2 minutes. Although this page of the assessment contains 40 questions, answering 30 correct within the time limit is considered passing.

Appendix C Exit Ticket Unit 1 Lesson 1:

Exit Ticket (3 minutes)

Name _____ Date _____

1. The picture below shows 4 groups of 2 slices of watermelon. Fill in the blanks to make true repeated addition and multiplication sentences that represent the picture.



2 + ____ + ____ + ____ = _____

4 × _____ = _____

2. Draw a picture to show 3 + 3 + 3 = 9. Then, write a multiplication sentence to represent the picture.

Appendix C: Lesson Plan Examples

Module 1 Lesson 1

Lesson 1

Objective: Understand *equal groups of* as multiplication.

Suggested Lesson Structure

Fluency Practice	(5 minutes)
Application Problem	(10 minutes)
Concept Development	(35 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

NOTES ON FLUENCY PRACTICE:

Think of fluency as having three goals:

1. Maintenance (staying sharp on previously learned skills).
2. Preparation (targeted practice for the current lesson).
3. Anticipation (skills that ensure that students will be ready for the in-depth work of upcoming lessons).

Fluency Practice (5 minutes)

- Group Counting **3.OA.1** (5 minutes)

Group Counting (5 minutes)

Note: Basic skip-counting skills from Grade 2 shift focus in this Grade 3 activity. Group counting lays a foundation for interpreting multiplication as repeated addition. When

students count groups in this activity, they add and subtract groups of 2 when counting up and down.

T: Let's count to 20 forward and backward. Watch my fingers to know whether to count up or down. A closed hand means stop. (Show signals as you explain.)

T: (Rhythmically point up until a change is desired. Show a closed hand, then point down.)

S: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0.

T: Let's count to 20 forward and backward again. This time whisper every other number. Say the other numbers in a regular voice.

S: (Whisper) 1, (speak) 2, (whisper) 3, (speak) 4, (whisper) 5, (speak) 6, etc.

T: Let's count to 20 forward and backward again. This time, hum every other number instead of whispering. As you hum, think of the number.

S: (Hum), 2, (hum), 4, (hum), 6, etc.

T: Let's count to 20 forward and backward again. This time, think every other number instead of humming.

S: (Think), 2, (think), 4, (think), 6, etc.

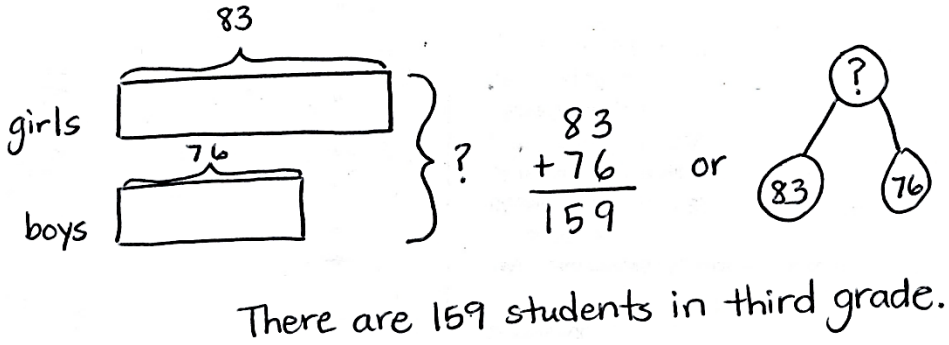
T: What did we just count by? Turn and talk to your partner.

S: Twos.

T: Let's count by twos. (Direct students to count forward to and backward from 20, changing directions at times.)

Application Problem (10 minutes)

There are 83 girls and 76 boys in the third grade. How many total students are in the third grade?



Note: Students may choose to use a tape diagram or a number bond to model the problem. They are also likely to solve today's Application Problem in less than 10 minutes. Ten minutes have been allotted in order for you to review the RDW (Read, Draw, Write) procedure for problem solving.

Directions on the Read, Draw, Write (RDW) process: Read the problem, draw and label, write an equation, and write a word sentence. The more students participate in reasoning through problems with a systematic approach, the more they internalize those behaviors and thought processes.

(Excerpted from "How to Implement *A Story of Units*." A more complete explanation can also be found in the Grade 3 Module 1 Overview.)

Concept Development (35 minutes)

Materials: (S) 12 counters, personal white board

Problem 1: Skip-count to find the total number of objects.

T: (Select 10 students to come to the front.) At the signal, say how many arms you each have. (Signal.)

S: 2 arms!

T: Since we each represent a group of 2 arms, let's skip-count our volunteers by twos to find how many arms they have all together. To keep track of our count, the students will raise up their arms when we count them.

S: (Count 2, 4, 6, …20.)

T: How many raised arms do we have in all?

S: 20.

T: Arms down. How many twos did we count to find the total? Turn and whisper to your partner.

S: 10 twos.

T: What did you count to find the number of twos?

S: I counted the number of volunteers because each person has a group of two arms.

T: Skip-count to find the total number of arms.

S: (Say 2, 4, 6, ….)

T: (As they count, write 2 + 2 + 2….)

T: Look at our addition sentence. Show thumbs up if you see the correct number of twos.

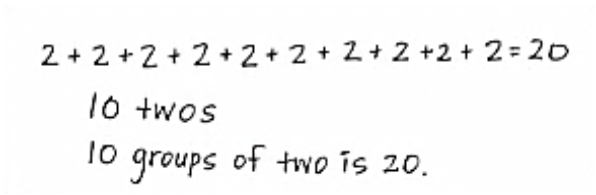
S: (Show thumbs up.)

T: (Under the addition sentence, write 10 twos.) Clap 3 times if you agree that 10 groups of two is 20.

S: (Clap 3 times.)

T: (Write *10 groups of two is 20* under the other number sentences.)

Sample teacher board:



Problem 2: Understand the relationship between repeated addition, counting groups in unit form,

and multiplication sentences.

Seat students at tables with personal white boards and 12 counters each.

T: You have 12 counters. Use your counters to make **equal groups** of two. How many counters will you put in each group? Show with your fingers.

S: (Hold up 2 fingers and make groups of two.)

T: How many equal groups of two did you make? Tell at the signal. (Signal.)

S: 6 groups.

T: 6 equal groups of how many counters?

S: 6 equal groups of 2 counters.

T: 6 equal groups of 2 counters equal how many counters all together?

S: 12 counters.

T: Write an addition sentence to show your groups on your personal white board.

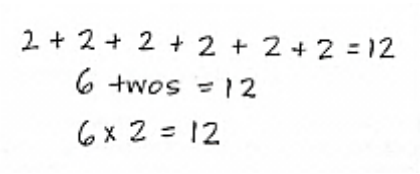
S: (Write 2 + 2 + 2 + 2 + 2 + 2 = 12.)

T: (Record the addition sentence on the board.) In unit form, how many twos did we add to make 12?

S: 6 twos.

T: (Record 6 twos = 12 under the addition sentence.)
6 × 2 is another way to write 2 + 2 + 2 + 2 + 2 + 2 or 6 twos. (Record 6 × 2 = 12 under 6 twos = 12 on the board.) These number sentences are all saying the same thing.

Sample teacher board:



T: Turn and talk to your partner. How do you think 6 × 2 = 12 relates to the other number sentences?

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

It may be necessary to explicitly connect *times* and the symbol ×. Have students analyze the model. "How many times do you see a group of two?" Have them count the groups, write the number sentence, and say the words together.

- 6 groups of two equal 12.
- 6 times 2 equals 12.

S: They all have twos in them, and the answer is 12.

❼

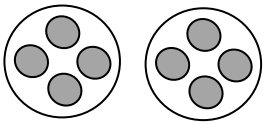
I think the 6 shows how many twos there are. ❼ You have to count two 6 times because there are 6 groups of them. That's how you get 6 times 2. ❼ 6 × 2 might be an easier way to write a long addition sentence.

T: Ways that are easier and faster are efficient. When we have equal groups, **multiplication** is a more efficient way to find the total than repeated addition.

Repeat the process with 4 threes, 3 fours, and 2 sixes to get students comfortable with the relationship between repeated addition, counting groups in unit form, and multiplication sentences.

Problem 3: Write multiplication sentences from equal groups.

Draw or project the picture to the right.



T: These are equal groups. Turn and tell your partner why they are equal.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some students may need more scaffolding to realize that multiplication cannot be used to find totals with groups that are not equal. Use the following questions to scaffold.

- Does the drawing show 3 fours?
- Does 3 times 4 represent this drawing?
- How might we redraw the picture to make it show 3 × 4?

S: There are the same number of grey circles in each group. ❼ All of the grey circles are the same size and shape, and there are 4 in each group.

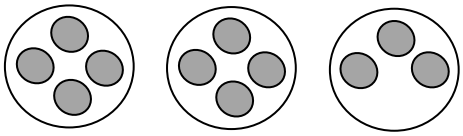
T: Work with your partner to write a repeated addition and a multiplication sentence for this picture.

S: (Write 4 + 4 = 8 and either 2 × 4 = 8 or 4 × 2 = 8.)

T: (Project or draw the following image.) Look at my

new drawing and the multiplication sentence I wrote to represent it. Check my work by writing an addition sentence and counting to find the total number of objects.

MP.



$3 \times 4 = 12$

S: (Write 4 + 4 + 3 = 11.)

T: Use your addition sentence as you talk to your partner about why you agree or disagree with my work.

S: I disagree because my addition sentence equals 11, not 12. ❼ It's because that last group doesn't have 4 circles. ❼ You can do multiplication when the groups are equal. ❼ Here, the groups aren't equal, so the drawing doesn't show 3 × 4.

T: I hear most students disagreeing because my groups are not equal. True, to **multiply** you must have equal groups.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

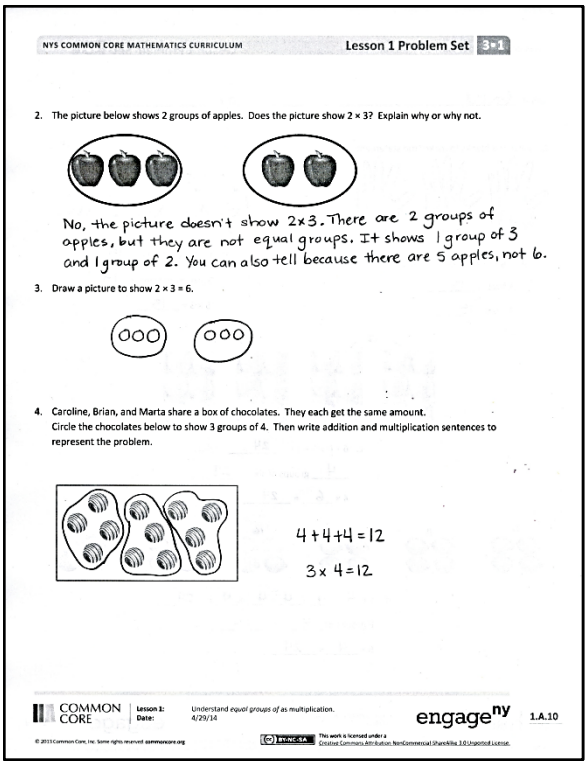
For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the careful sequencing of the Problem Set guide your selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Assign incomplete problems for homework or at another time during the day.

Student Debrief (10 minutes)

Lesson Objective: Understand *equal groups of* as multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 1 Problem Set 3•1

2. The picture below shows 2 groups of apples. Does the picture show 2 × 3? Explain why or why not.

No, the picture doesn't show 2×3. There are 2 groups of apples, but they are not equal groups. It shows 1 group of 3 and 1 group of 2. You can also tell because there are 5 apples, not 6.

3. Draw a picture to show 2 × 3 = 6.

4. Caroline, Brian, and Marta share a box of chocolates. They each get the same amount. Circle the chocolates below to show 3 groups of 4. Then write addition and multiplication sentences to represent the problem.

4 + 4 + 4 = 12

3 × 4 = 12

COMMON CORE | Lesson 1: Understand *equal groups of* as multiplication. | Date: 4/29/14 | engage^ny | 1.A.10

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You may choose to use any combination of the questions below to lead the discussion.

- On the first page, what did you notice about the answers to your problems?
- Discuss the relationship between repeated addition and the unit form *2 groups of three* or *3 groups of two,* depending on the drawing.
- Discuss the relationship between repeated addition, unit form, and the multiplication sentence $3 \times 2 = 6$.
- Review the new vocabulary presented in the lesson: **equal groups**, **multiplication**, and **multiply**.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

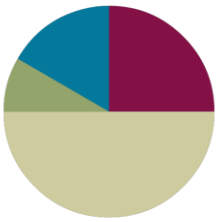
Module 1 Lesson 2

Lesson 2

Objective: Relate multiplication to the array model.

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Application Problem	(5 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (15 minutes)

- Sprint: Add and Subtract by 2 **3.OA.1** (9 minutes)
- Group Counting **3.OA.1** (3 minutes)
- Add Equal Groups **3.OA.1** (3 minutes)

Sprint: Add and Subtract by 2 (9 minutes)

Materials: (S) Add and Subtract by 2 Sprint

Note: This Sprint supports group counting skills that are foundational to interpreting multiplication as repeated addition.

Directions for Administration of Sprints

A Sprint has two parts, A and B, with closely related problems on each. Each part is organized into four quadrants that move from simple to complex. This builds a challenge into each Sprint for every learner. Before the lesson, print Sprint A and Sprint B on two separate sheets of paper. Students complete the two parts of the Sprint in quick succession with the goal of improving for the second part, even if only by one more. With practice, the following routine takes about 9 minutes.

Sprint A

Place Sprint A face down on student desks, and instruct students not to look at the problems until a signal is given.

T: You will have 60 seconds to do as many problems as you can. I do not expect you to finish all of them, just as many as you can, trying for your personal best.

T: Take your mark! Get set! THINK!

Students turn papers over and work furiously to finish as many problems as they can in 60 seconds. Time precisely.

T: Stop! Circle the last problem you completed. I will read just the answers. If you got the answer right, call out "Yes!" If you made a mistake, circle it. Ready?

Repeat to the end of Sprint A or until no student has a correct answer.

T: Now, at the top of the page, write the number of problems you got correct. This is your personal goal for Sprint B.

T: How many of you got one right? (All hands should go up.)

T: Keep your hand up until I say a number that is one more than the number you got right. So, if you got 14 right, when I say 15, your hand goes down. Ready?

T: (Continue quickly.) How many got two right? Three? Four? Five? (Continue until all hands are down.)

If the class needs more practice with Sprint A, continue with the optional routine presented below.

T: Take one minute to do more problems on this half of the Sprint.

As students work, the student who scored highest on Sprint A might pass out Sprint B.

T: Stop! I will read just the answers. If you got it right, call out "Yes!" If you made a mistake, circle it. Ready?

Read the answers to the first half again as students stand.

Movement: To keep the energy and fun going, do a stretch or a movement game in between Sprints.

Sprint B

Place Sprint B face down on student desks, and instruct students not to look at the problems until a signal is given. Repeat the procedure for Sprint A up through the show of hands for how many correct answers.

T: Stand up if you got more correct on the second Sprint than on the first.

S: (Stand.)

T: Keep standing until I say the number that tells how many more you got right on Sprint B. If you got three more right on Sprint B than on Sprint A, when I say *three*, you sit down. Ready?

Call out numbers, starting with one. Students sit as the number by which they improved is called. Students may take Sprints home.

Group Counting (3 minutes)

Note: Basic skip-counting skills from Grade 2 shift focus in this Grade 3 activity. Group counting lays a foundation for interpreting multiplication as repeated addition. When students count groups in this activity, they add and subtract groups of three when counting up and down.

T: Let's count to 18 forward and backward. I want you to whisper, whisper, and then speak

numbers.

T: Watch my fingers to know whether to count up or down. A closed hand means stop. (Show signals as you explain.)

T: (Rhythmically point up until a change is desired. Show a closed hand then point down.)

S: (Whisper) 1, (whisper) 2, (speak) 3, etc.

T: Let's count to 18 forward and backward again. This time, think every number instead of whispering.

S: (Think), (think), 3, (think), (think), 6, (think), (think), 9, etc.

T: What did we just count by? Turn and talk to your partner.

S: Threes.

T: Let's count by threes. (Direct students to count forward and backward to 18, periodically changing directions. Emphasize the 9 to 12 transition.)

Add Equal Groups (3 minutes)

Materials: (S) Personal white board

Note: This activity reviews Lesson 1. Students directly relate repeated addition to multiplication. They interpret products as the number of equal groups times the number of objects in each group.

T: (Project a picture array with 3 groups of 2 circled.) How many groups are circled?

S: 3.

T: How many are in each group?

S: 2.

T: Write this as an addition sentence.

S: (Write 2 + 2 + 2 = 6.)

T: Write a multiplication sentence for 3 twos equals 6.

S: (Write 3 × 2 = 6.)

Continue with possible sequence: 3 groups of 5, 5 groups of 10, and 3 groups of 4.

Application Problem (5 minutes)

Jordan uses 3 lemons to make 1 pitcher of lemonade. He makes 4 pitchers. How many lemons does he use all together? Use the RDW process to show your solution.

$4 \times 3 = 12$

Jordan uses 12 lemons all together.

Note: Present the image of 4 groups of 3 lemons with the word problem as a scaffold. This problem reviews multiplying equal groups from Lesson 1. It also leads into today's Concept Development in which students relate multiplication to the array model.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

The words *array* and *row* were introduced in Grade 2, Module 6 but treated as new vocabulary in this lesson.

When reviewing the concept, have students trace a row on the array with a finger while saying the word *row*. Provide a real world example by having students count the rows on various cupcake pans (mini and regular size) before using the template.

Concept Development (30 minutes)

Materials: (S) Personal white board with threes array (Template) inserted (pictured below), lemons image from Application Problem, 1 sheet of blank paper

Problem 1: Relate equal groups to arrays.

Note: Students' templates should be vertical rather than horizontal, as shown below.

T: Look back at Jordan's lemons. Compare the way his lemons are organized with the groups of 3 circles on your template.

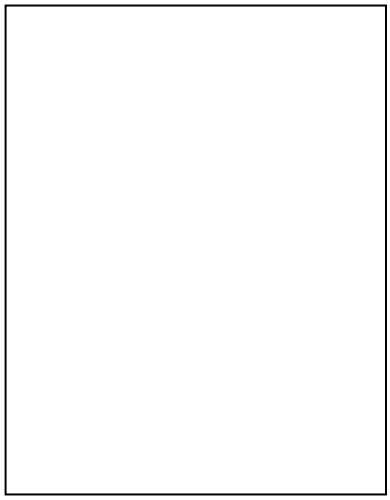
S: The lemons are touching each other, but the circles have space between them. ❼ Each line on the template shows three, like each group of lemons. ❼ The template is organized with everything in straight lines.

T: Many students are noticing straight lines on the template. Let's call a straight line going across a **row**. Use your blank paper to cover all but the top row.

S: (Cover all but the top row.)

T: Uncover 1 row at a time in the picture. As you uncover each row, write the new total number of

Threes array template (with student work)



circles to the right of it.

S: (Skip-count by three using the threes array template.)

T: At the signal, say the total number of circles you counted. (Signal.)

S: 30 circles!

T: Take 10 seconds to find how many rows of 3 you counted. At the signal say how many. (Signal.)

S: 10 rows!

T: True or false: 10 rows of 3 circles equals 30 circles?

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

When presenting the concept of *array*, it may be beneficial to ask students to turn and talk, describing or defining an array for their partner.

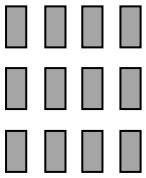
S: True!

T: (Write 10 × 3 = 30 on the board.) Use the picture on your template to talk with your partner about why this equation is true.

S: Yesterday, we learned that we can multiply equal groups. ❼ We skip-counted 10 rows of 3 circles each and the total is 30. ❼ It means 10 groups of 3. When you add 10 threes, you get 30! ❼ Yeah, but writing 10 × 3 is a lot easier than writing out 3 + 3 + 3 + 3….

T: We call this type of organized picture an **array**.

T: (Project or draw the image at right.) Take a look at this array. At the signal, tell how many rectangles are in the top row. (Signal.)



S: 4 rectangles.

T: The size of 1 row is 4 rectangles. Each row of 4 can also be called a group of 4. At the signal, tell how many groups of four are in the array. (Signal.)

S: 3 groups of four.

T: To write this as an equation, we first write the **number of groups**. How many groups?

S: 3 groups!

T: (Write 3 × ____ = ____.) Next, we write the **size of the group**. How many rectangles are in each group?

S: 4 rectangles!

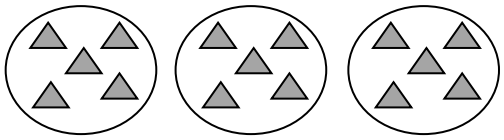
T: (Fill in the equation to read 3 × 4 = ____.) Skip-count to find the total number of rectangles in the array.

S: 4, 8, 12!

T: (Fill in the equation to read 3 × 4 = 12.) We just found the answer to the multiplication equation that represents the array.

Show an array of 2 rows of 6 and repeat the process.

Problem 2: Redraw equal groups as arrays.



T: (Project or draw the image at right.) The drawing shows 3 equal groups of 5. On your personal white board, re-draw the picture as an array with 3 rows of 5.

S: (Draw 3 rows of 5.)

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Provide a challenge in this part of the lesson by giving an equation (e.g., 5 × 4 = _____) and no picture. Have students draw both the equal groups and the array to represent the equation. Then, they skip-count to find the total.

T: Write a multiplication expression to describe your array. Remember, an expression is different from an equation because it doesn't have an equal sign.

S: (Write 3 × 5.)

T: Skip-count to find the answer to the expression.

S: 5, 10, 15.

T: With your partner, compare my drawing with your array. Which is easier to count? Why?

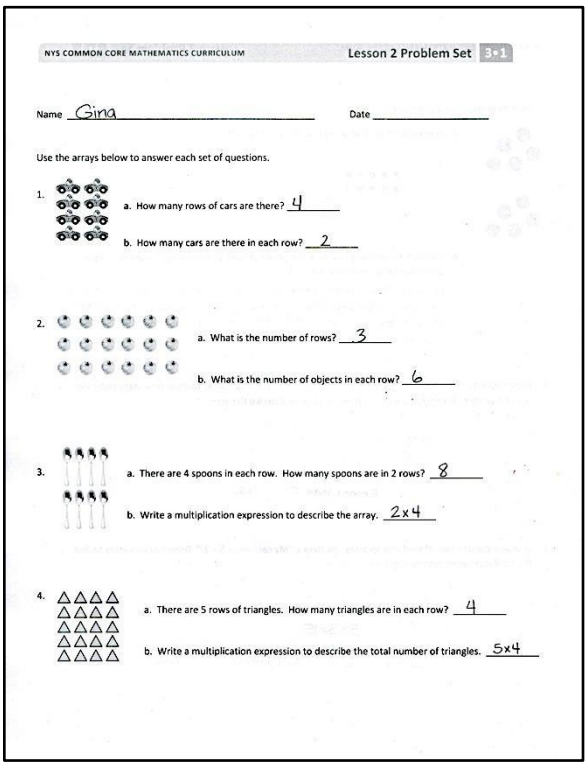
S: (Discuss.)

Show 6 groups of 2 and repeat the process.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Directions on this Problem Set include the words *expression* and *equation*. Remind students that while an answer is not required with an expression, it should be included with an equation.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 Problem Set 3•1

Name Gina Date

Use the arrays below to answer each set of questions.

1. a. How many rows of cars are there? 4

b. How many cars are there in each row? 2

2. a. What is the number of rows? 3

b. What is the number of objects in each row? 6

3. a. There are 4 spoons in each row. How many spoons are in 2 rows? 8

b. Write a multiplication expression to describe the array. 2x4

4. a. There are 5 rows of triangles. How many triangles are in each row? 4

b. Write a multiplication expression to describe the total number of triangles. 5x4

Student Debrief (10 minutes)

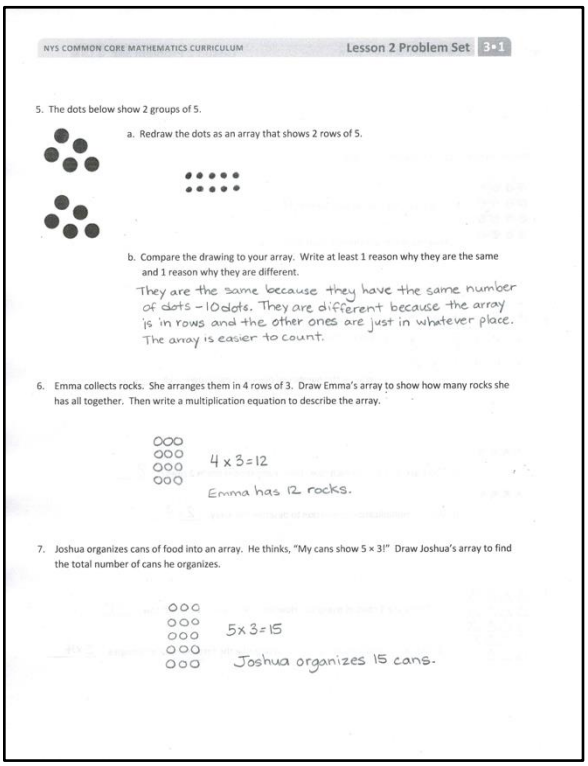
Lesson Objective: Relate multiplication to the array model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problems 5 and 6, how do the arrays represent equal groups?
- Compare Problems 6 and 7. (Arrays have the same number in each group but a different number of groups.)
- Compare equal groups in scattered configurations and arrays.
- Review new vocabulary: **row**, **array**, **number of groups**, and **size of groups**.
- Prompt students to notice arrays around the room and possibly think of arrays in real world situations.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 Problem Set 3•1

5. The dots below show 2 groups of 5.

a. Redraw the dots as an array that shows 2 rows of 5.

b. Compare the drawing to your array. Write at least 1 reason why they are the same and 1 reason why they are different.

They are the same because they have the same number of dots - 10 dots. They are different because the array is in rows and the other ones are just in whatever place. The array is easier to count.

6. Emma collects rocks. She arranges them in 4 rows of 3. Draw Emma's array to show how many rocks she has all together. Then write a multiplication equation to describe the array.

4 x 3 = 12

Emma has 12 rocks.

7. Joshua organizes cans of food into an array. He thinks, "My cans show 5 × 3!" Draw Joshua's array to find the total number of cans he organizes.

5 x 3 = 15

Joshua organizes 15 cans.

Exit Ticket (3 minutes)

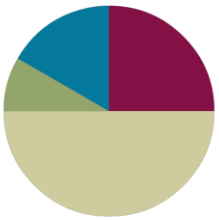
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Lesson 3

Objective: Interpret the meaning of factors—the size of the group or the number of groups.

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Application Problem	(5 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (15 minutes)

- Sprint: Add Equal Groups **3.OA.1** (9 minutes)
- Group Counting **3.OA.1** (3 minutes)
- Add to Multiply **3.OA.1** (3 minutes)

Sprint: Add Equal Groups (9 minutes)

Materials: (S) Add Equal Groups Sprint

Note: This Sprint reviews Lesson 1. See Lesson 2 for the directions for administering a Sprint.

Group Counting (3 minutes)

Note: Basic skip-counting skills from Grade 2 shift focus in this Grade 3 activity. Group counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity anticipates work with those factors in Topic B.

T: Let's count by twos. (Direct students to count forward and backward to 20, periodically changing directions.)

T: Let's count by threes. (Direct students to count forward and backward to 21, periodically

changing directions. Emphasize the 9 to 12 and 18 to 21 transitions.)

Add to Multiply (3 minutes)

Materials: (S) Personal white board

Note: This activity reviews Lesson 2. Students directly relate repeated addition to multiplication. They interpret products using the array.

T: (Project a picture with 3 groups of 5 circled.) How many groups are circled?

S: 3.

T: How many are in each group?

S: 5.

T: Write it as an addition sentence.

S: (Write 5 + 5 + 5 = 15.)

T: Write a multiplication sentence representing *3 fives equals 15*.

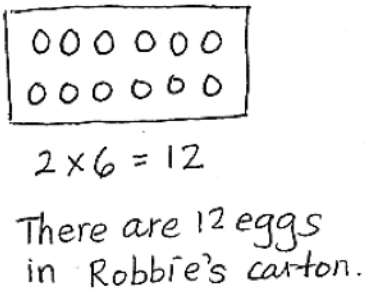
S: 3 × 5 = 15.

Continue with possible sequence: 3 groups of 10, 3 groups of 4, and 7 groups of 2.

Application Problem (5 minutes)

Robbie sees that a carton of eggs shows an array with 2 rows of 6 eggs. What is the total number of eggs in the carton? Use the RDW process to show your solution.

Note: This problem reviews writing multiplication sentences from arrays learned in Lesson 2. The egg carton provides a natural array for students to see 2 rows of 6.



NOTES ON OPENING ACTIVITY:

Adjust the directions for the opening activity depending on the total number of students in your class. Avoid having students make four groups. Instead of using corners in the classroom, use an adult, teddy bear, etc., for students to stand near when forming equal groups.

Concept Development (30 minutes)

Materials: (S) Personal white board

The following opening activity should take about 5 minutes.

T: Here are the rules for our opening activity.

1. **Divide** yourselves into 4 equal groups.
2. Each group will stand in a corner of the room.
3. Divide silently. You can use body movements to gesture, but no words.

T: Show thumbs up when your group is ready. Be sure to look around the room to double check that all 4 groups are equal before showing you're ready.

S: (Move around the room silently until there are 4 equal groups, 1 in each corner.)

NOTES ON NUMBER BONDS:

The number bond is a pictorial representation of part–part–whole relationships and shows that within a part–whole relationship, smaller numbers (the parts) make up larger numbers (the whole). (Excerpted from "How to Implement *A Story of Units.*")

T: At the signal tell how many equal groups we've made. (Signal.)

S: 4 equal groups.

T: (Write 4 × ___ = ___.) At the signal tell the size of each group. (Signal.)

S: (Respond depending on class numbers.)

T: (Fill in the equation on the board.) These numbers—the number of groups and the number in each group—are called **factors**.

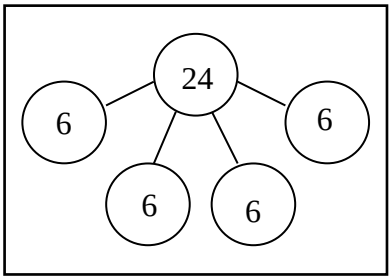
Students transition back to their seats.

T: Use the multiplication equation on the board to draw an array. Make sure that your board is vertical.

S: (Draw a 4 × ____ array.)

T: Let's draw a number bond for our equation.

Sample number bond (class of 24):



Draw a circle with our class total.

S: (Draw.)

T: Draw parts coming from the total. Make 1 part to represent each row in our array.

S: (Draw 4 circles coming from the total.)

T: Show the size of 1 row with your fingers.

S: (Show fingers.)

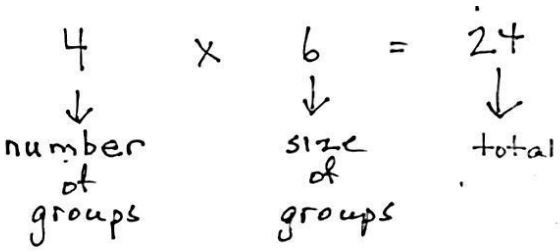
T: Write the factor representing the size of the group inside the circles.

S: (Write 6 inside each circle.)

T: Look back at the equation. How is the factor 4 represented in the number bond?

S: It's in the number of parts. ❼ Groups are like parts. ❼ In the number bond the part circles actually represent equal groups, so there are 4. The number inside is the size of the group.

T: Here is an analysis of our equation.



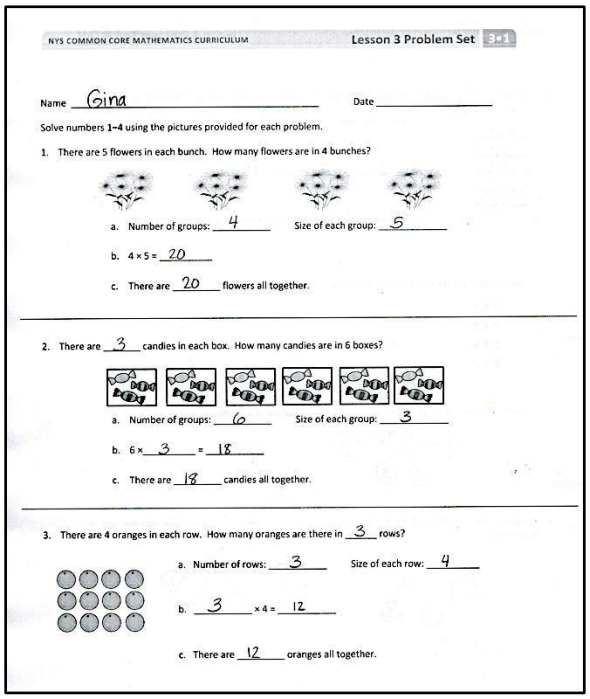
Another option is to have students compare how the number bond can represent multiplication and addition to distinguish the importance of equal groups in multiplication.

As time allows, continue with the following possible suggestions:

- 2 groups of 8
- 3 rows of 5
- Number bond showing 6 groups of 3
- The equation 5 × 4 = 20

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Problem Set 3•1

Name Gina Date

Solve numbers 1–4 using the pictures provided for each problem.

1. There are 5 flowers in each bunch. How many flowers are in 4 bunches?

a. Number of groups: 4 Size of each group: 5

b. 4 × 5 = 20

c. There are 20 flowers all together.

2. There are 3 candies in each box. How many candies are in 6 boxes?

a. Number of groups: 6 Size of each group: 3

b. 6 × 3 = 18

c. There are 18 candies all together.

3. There are 4 oranges in each row. How many oranges are there in 3 rows?

a. Number of rows: 3 Size of each row: 4

b. 3 × 4 = 12

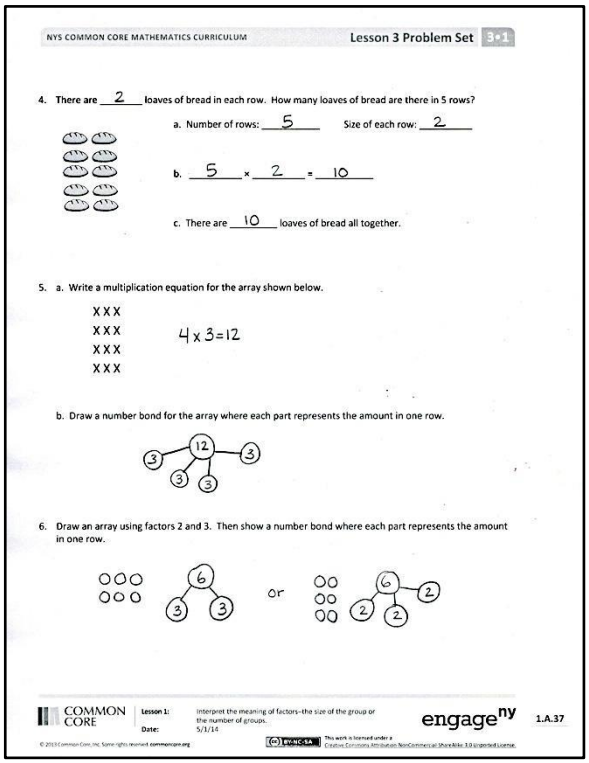
c. There are 12 oranges all together.

should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Interpret the meaning of factors—the size of the group or the number of groups.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Problem Set 3•1

4. There are 2 loaves of bread in each row. How many loaves of bread are there in 5 rows?

a. Number of rows: 5 Size of each row: 2

b. 5 × 2 = 10

c. There are 10 loaves of bread all together.

5. a. Write a multiplication equation for the array shown below.

XXX
XXX
XXX
XXX

4 x 3=12

b. Draw a number bond for the array where each part represents the amount in one row.

6. Draw an array using factors 2 and 3. Then show a number bond where each part represents the amount in one row.

or

COMMON CORE Lesson 1: Interpret the meaning of factors–the size of the group or the number of groups. Date: 5/1/14 engage^ny 1.A.37

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You may choose to use any combination of the questions below to lead the discussion.

- Why do you think I started the lesson by asking you to **divide** yourselves into equal groups in the corners of the room?
- Identify the **factors** and their meanings from each image in Problems 1–5.
- In Problem 6, discuss the two ways to draw the array and number bond with factors 2 and 3.
- Module 1 introduces many new vocabulary words: *row, array, multiply, multiplication, number of groups, size of groups, divide, factor,* etc. Consider having students make a vocabulary page in their math journals.
- Relate factors to their meaning: the size of the group or the number of groups. Have students share the definition in pairs. Then, ask students to write the word and a definition or example next to it in their journals.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.