Thomaston Public Schools

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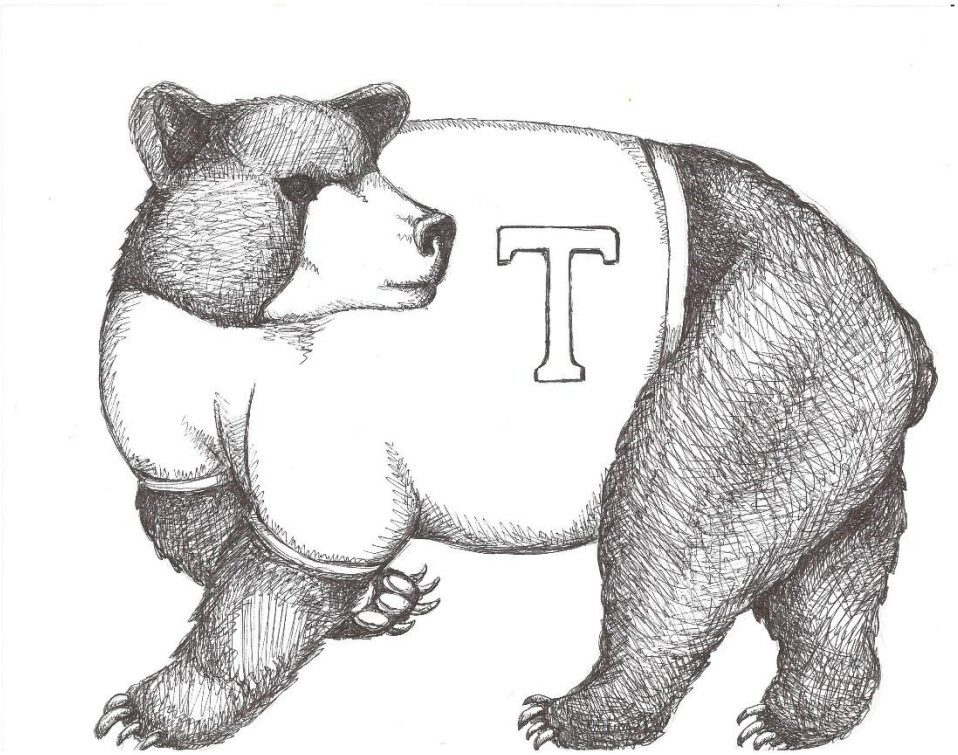

Thomaston Public Schools Curriculum

Black Rock School
Grade: 2 Mathematics 2015

A Nurturing Community Where Children Are Primary

Acknowledgements

Curriculum Writer(s):
_____Sue Dalka

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

_____*Alisha DiCorpo*_____
Alisha L. DiCorpo
Director of Curriculum and Professional Development

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(Math Curriculum Grade 2)

Grade 2 Mathematics

Board of Education Mission Statement:

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:

The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main resource used when writing this curriculum:

NYS COMMON CORE MATHEMATICS CURRICULUM A Story of Units Curriculum. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. A Story of Units: A Curriculum Overview for Grades P-5 Date: 7/31/13 5 © 2013 Common Core, Inc. Some rights reserved. commoncore.org

Course Description:

Sequence of Grade 2 Units (Modules) Aligned with the Standards
Unit 1: Sums and Differences to 20
Unit 2: Addition and Subtraction of Length Units
Unit 3: Place Value, Counting, and Comparison of Numbers to 1000
Unit 4: Addition and Subtraction Within 200 with Word Problems to 100
Unit 5: Addition and Subtraction Within 1000 with Word Problems to 100
Unit 6: Foundations of Multiplication and Division
Unit 7: Problem Solving with Length, Money, and Data
Unit 8: Time, Shapes, and Fractions as Equal Parts of Shapes

Summary of Year:
Second Grade mathematics is about (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Key Areas of Focus for K-2:
Addition and subtraction—concepts, skills, and problem solving

Required Fluency:
2.OA.2 Add and subtract within 20.
2.NBT.5 Add and subtract within 100.

CCS Major Emphasis Clusters :
Operations and Algebraic Thinking
- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.

Rationale for Module Sequence in Grade 2:

From Grade 1, students have fluency of addition and subtraction within 10 and extensive experience working with numbers to 100. Module 1 of Grade 2 establishes a motivating, differentiated fluency program in the first few weeks that will provide each student with enough practice to achieve mastery of the new required fluencies (i.e., adding and subtracting within 20 and within 100) by the end of the year. Students learn to represent and solve word problems using addition and subtraction: a practice that will also continue throughout the year.

In Module 2, students learn to measure and estimate using standard units for length and solve measurement word problems involving addition and subtraction of length. A major objective is for students to use measurement tools with the understanding that linear measure involves an iteration of units and that the smaller a unit, the more iterations are necessary to cover a given length. Students work exclusively with metric units, i.e. centimeters and meters, in this module to support upcoming work with place value concepts in Module 3. Units also play a central role in the addition and subtraction algorithms of Modules 4 and 5. An underlying goal for this module is for students to learn the meaning of a "unit" in a different context, that of length. This understanding serves as the foundation of arithmetic, measurement, and geometry in elementary school. All arithmetic algorithms are manipulations of place value units: ones, tens, hundreds, etc.

In Module 3, students extend their understanding of base-ten notation and apply their understanding of place value to count and compare numbers to 1000. In Grade 2 the place value units move from a proportional model to a non-proportional number disk model (see picture). The place value table with number disks can be used through Grade 5 for modeling very large numbers and decimals, thus providing students greater facility with and understanding of mental math and algorithms.

In Module 4, students apply their work with place value units to add and subtract within 200 moving from concrete to pictorial to abstract. This work deepens their understanding of base-ten, place value, and the properties of operations. It also challenges them to apply their knowledge to one-step and two-step word problems. During this module, students also continue to develop one of the required fluencies of the grade: addition and subtraction within 100.

Module 5 builds upon the work of Module 4. Students again use place value strategies, manipulatives, and math drawings to extend their conceptual understanding of the addition and subtraction algorithms to numbers within 1000. They maintain addition and subtraction fluency within 100 through daily application work to solve one- and two-step word problems of all types. A key component of Modules 4 and 5 is that students use place value reasoning to explain why their addition and subtraction strategies work.

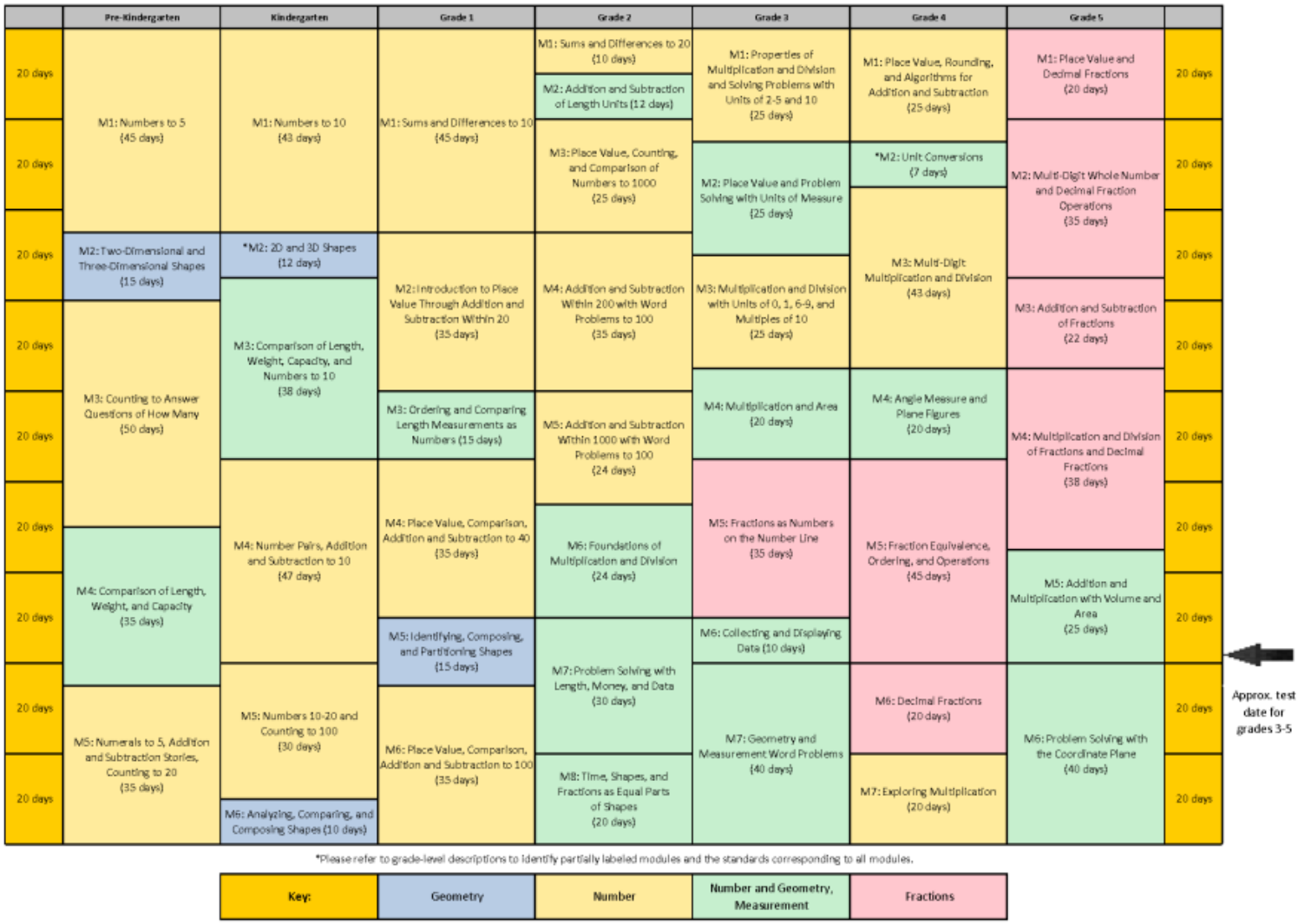
In Module 6, students extend their understanding of a unit to build the foundation for multiplication and division wherein any number, not just powers of ten, can be a unit. Making equal groups of "four apples each" establishes the unit "four apples" (or just four) that can then be counted: 1 four, 2 fours, 3 fours, etc. Relating the new unit to the one used to create it lays the foundation for multiplication: 3 groups of 4 apples equal 12 apples (or 3 fours is 12).

Module 7 provides another opportunity for students to practice their algorithms and problem-solving skills with perhaps the most well-known, interesting units of all: dollars, dimes, and pennies. Measuring and estimating length is revisited in this module in the context of units from both the customary system (e.g., inches and feet)

and the metric system (e.g., centimeters and meters). As they study money and length, students represent data given by measurement and money data using picture graphs, bar graphs, and line plots. Students finish Grade 2 by describing and analyzing shapes in terms of their sides and angles.

In Module 8, students investigate, describe, and reason about the composition and decomposition of shapes to form other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

K-5 Pacing Guide:



Grade 2 • Unit 1 (Module 1)

Sums and Differences to 20

OVERVIEW

Module 1 sets the foundation for students to master sums and differences to 20 (**2.OA.2**). Students subsequently apply these skills to fluently add one-digit to two-digit numbers at least through 100 using place value understanding,

properties of operations, and the relationship between addition and subtraction (**2.NBT.5**). In Grade 1, students worked extensively with numbers to 10 and developed Level 2 and Level 3 mental strategies to add and subtract within 20 (**1.OA.1**) and 100 (**1.NBT.4–6**).

For example, to solve 12 + 3 students might make an equivalent but easier problem by decomposing 12 as 10 and 2 and composing 2 with 3 to make 5. Students can use this knowledge to solve related problems such as 92 + 3. They also apply this skill using smaller numbers to subtract problems with larger numbers: 12 – 8 = 10 – 8 + 2 = 2 + 2, just as 72 – 8 = 70 – 8 + 2 = 62 + 2.

Daily fluency activities provide sustained practice to help students attain fluency within 20. This fluency is essential to the work of later modules and future grade levels, where students must efficiently recompose place value units to work adeptly with the four operations. Activities such as Say Ten counting, Take from Ten, and the use of ten-frame and Hide Zero cards solidify student fluency. Because the amount of practice required by each student to achieve mastery will vary, a motivating, differentiated fluency program needs to be established in these first weeks to set the tone for the rest of the year.

Throughout the module, students will represent and solve one-step word problems through the daily Application Problem (**2.OA.1**). Note that one-step problems may have multiple parts that are separated by bullets or letters. Each part requires only one operation. These multi-part problems serve as a stepping-stone toward multi-step problems. Application Problems can precede a lesson to act as the lead-in to a concept, allowing students to discover through problem-solving the logic and usefulness of a strategy before that strategy is formally presented. Or, they can follow the Concept Development so that students connect and apply their learning to real-world situations. This latter structure can also serve as a bridge between teacher-directed work and students solving problems independently on Problem Sets and at home. In either case, problem-solving begins as a guided activity, with the goal being to move students to independent problem-solving, wherein they reason through the relationships of the problem and choose an appropriate strategy to solve. In Module 1, Application Problems follow Concept Development.

Topic A reactivates students' Kindergarten and Grade 1 learning as they practice prerequisite skills for Level 3 decomposition and composition methods: partners to 10 and decompositions for all numbers within 10. Students move briskly from concrete to pictorial to abstract as they remember their make ten facts. They use ten-frame cards to visualize 10, and they write the number bonds of 10 from memory. They use those facts to see relationships in larger numbers (e.g., 28 needs how many to make 30?). The number bond is also used to represent related facts within 10.

Topic B also moves from concrete to pictorial to abstract, as students use decomposing strategies to add and subtract within 20. By the end of Grade 1 Module 2, students learned to form ten as a unit. Hence, the phrase *make ten* now transitions to *make a ten.* Students use the ten-structure to reason about making a ten to add to the teens, and they use this pattern and math drawings to solve related problem sets (e.g., 9 + 4, 9 + 5, 9 + 6). Students reason about the relationship between problems such as 19 + 5 and 20 + 4 to 9 + 5 and 10 + 4. They use place value understanding to add and subtract within 20 by adding to and subtracting from the ones. The topic ends with a lesson in which students subtract from 10. The goal in making a 10 and taking from 10 is for students to master mental math.

Topic C calls on students to review strategies to add and subtract within 100 (**1.NBT.4–6**) to set the foundation for Grade 2's work towards mastery of fluency with the same set of problems (**2.NBT.5**). They use basic facts and place value understanding to add and subtract within multiples of 10 without crossing the multiple (e.g., 7 – 5 = 2, so 47 – 5 = 42). This segues into the use of basic facts and properties of addition to cross multiples of 10 (e.g., 26 + 9 = 20 + 6 + 4 + 5). In

the final lesson, students decompose to make a ten and then subtract from numbers that have both tens and ones.

Math Unit 1

Rigorous Curriculum Design Template

Unit 1: Sums and Differences to 20

Subject: Mathematics

Grade/Course: Grade 2

Pacing: 10 Days

Unit of Study: Unit 1: Sums and Differences to 20

Priority Standards:

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See CCS Glossary, Table 1.)

Add and subtract within 20.

2.OA.2 Fluently add and subtract within 20 using mental strategies. (See standard 1.OA.6 for a list of mental strategies.) By end of Grade 2, know from memory all sums of two one-digit numbers.

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Foundational Standards:

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 = 1 = 12 + 1 = 13).

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Math Practice Standards:

MP.1 **Make sense of problems and persevere in solving them.** Students make math drawings and use recomposing strategies to reason through the relationships in word problems. They write equations and word sentences to explain their solutions.

MP.2 **Reason abstractly and quantitatively.** Students decompose numbers and use the associative property to create equivalent but easier problems, e.g., 25 + 6 = 20 + 5 + 5 + 1. They reason abstractly when they relate subtraction to addition and change 13 – 8 = ___ into an unknown addend, 8 + ___ = 13, to solve.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students explain their reasoning to prove that 9 + 5 = 10 + 4. They communicate how simpler problems embedded within more complex problems enable them to solve mentally, e.g., 8 + 3 = 11, so 68 + 3 = 71.

MP.7 **Look for and make use of structure.** Students use the structure of ten to add and subtract within 20, and later, within 100, e.g., 12 – 8 = 10 – 8 + 2 = 2 + 2, and 92 + 3 = 90 + 2 + 3 = 90 + 5.

"Unwrapped" Standards	
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See CCS Glossary, Table 1- Attached Below) Fluently add and subtract within 20 using mental strategies. (See standard 1.OA.6 for a list of mental strategies- Attached.) By end of Grade 2, know from memory all sums of two one-digit numbers. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	
Concepts - What Students Need to Know (context)	**Skills - What Students Need to Be Able to Do (Depth of Knowledge Level)**
addition and subtraction (within 100)	Use (DOK-1)
One- and Two-step word problems (with unknowns in all positions- see table 1 below)	Solve (DOK- 2)
Using mental strategies (within 20)	Fluently add and subtract (DOK - 2)
All sums of two one-digit numbers	Know (DOK- 1)
Using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (Within 100)	Fluently add and subtract (DOK- 2)

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Table 1. Common addition and subtraction situations.[a]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 − 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 − ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? − 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[b]
Put Together/ Take Apart[c]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 − 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[d]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 − 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 − 3 = ?, ? + 3 = 5

[a]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[b]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[c]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Essential Questions	Big ideas
● What does fluency mean? ● How can you represent a word problem?	● Number relationships help build fluency ● Symbols, objects, and equations can be used to interpret word problems

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Lesson Exit tickets for each lesson	Application Problem Student Debriefs Problem Set Data	Exit Ticket Data End-of Module Assessment *See Table Below

*Assessment Summary

Type	Administered	Format	Standards Addressed
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	2.OA.1 2.OA.2 2.NBT.5

Performance Task (*To be completed by grade level team)
Overview:
Engaging Learning Experiences

Task 1:

Task 2:

Task 3:

Task 4:

Instructional Resources

Useful Websites:
Engage NY K-5 Curriculum overview and guiding documents:
https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents

Engage NY Grade 2 Resources:
https://www.engageny.org/resource/grade-2-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 2nd Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/2nd.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Books:
Mission: Addition by Loreen Leedy
The Hershey's Kisses Subtraction Book by Jerry Pallotta

Suggested Tools and Representations:

Dice
Hide Zero cards (Lesson 2 Template 1)
Linking cubes
Personal white boards
Rekenrek
Ten-frame cards, 1 set per student (Lesson 1 Template 1)

1 each of 1–4 and 6–9 2 fives 10 tens Blank frame Ten-frame cards large set for teacher Ten-strip (Lesson 4 Template) Two-sided counters for each student (e.g., large white beans spray painted red on one side)

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> ● Critical thinking and problem solving ● Collaboration and leadership ● Agility and adaptability ● Initiative and entrepreneurialism ● Effective oral and written communication ● Accessing and analyzing information ● Curiosity and imagination **<u>Marzano's Nine Instructional Strategies for Effective Teaching and Learning</u>** 1. **Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way 2. **Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words 3. **Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. 4. **Homework and Practice**: provides opportunities to	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

Familiar Terms and Symbols	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.
● Expression (e.g., 2 + 1, 13 – 6) ● Make ten and subtract from ten (e.g., 8 + 3 = 8 + 2 + 1 and 15 – 7 = 10 – 7 + 5 = 3 + 5) ● Number bond (e.g., 5 + 1 = 6, 1 + 5 = 6, 6 – 1 = 5, 6 – 5 = 1) ● Say Ten counting (e.g., 11 is "1 ten 1," 12 is "1 ten 2," 20 is "2 tens," 27 is "2 tens 7," 35 is "3 tens 5," 100 is "1 hundred," 146 is "1 hundred 4 tens 6") ● Ten plus (e.g., 10 + 3 = 13, 30 + 5 = 35, 70 + 8 = 78)	**<u>Provide Multiple Means of Representation</u>** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. "I do, we do, you do." ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' ● Encourage students to explain	**<u>Provide Multiple Means of Representation</u>** ● Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. ● Model and post conversation "starters," such as: "I agree because..." "Can you ● explain how you solved it?""I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word

	their thinking and strategy for the solution. ● Choose numbers and tasks that are “just right” for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom’s Taxonomy with questions such as: “What would happen if...?”“Can you propose an alternative...?”“How would you evaluate...?”“What choice would you have made...?” Ask “Why?” and “What if?” questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

OVERVIEW

In this 12-day Grade 2 module, students engage in activities designed to deepen their conceptual understanding of measurement and to relate addition and subtraction to length. Their work in Module 2 is exclusively with metric units in order to support place value concepts. Customary units are introduced in Module 7.

Topic A opens with students exploring concepts related to the centimeter ruler. In the first lesson, they are guided to connect measurement with physical units as they find the total number of unit lengths by laying multiple copies of centimeter cubes (physical units) end-to-end along various objects. Through this, students discover that to get an accurate measurement, there must be no gaps or overlaps between consecutive length units.

Next, students measure by iterating with one physical unit, using the *mark and advance* technique, also known as *mark and move forward*. Students then repeat the process by laying both multiple copies and a single cube along a centimeter ruler. This helps students create a mental benchmark for the centimeter. It also helps them realize that the distance between 0 and 1 on the ruler indicates the amount of space already covered. Hence 0, not 1, marks the beginning of the total length. Students use this understanding to create their own centimeter rulers using a centimeter cube and the mark and advance technique. Topic A ends with students using their unit rulers to measure lengths (**2.MD.1**), thereby connecting measurement with a ruler.

Students build skill in measuring using centimeter rulers and meter sticks in Topic B. They learn to see that a length unit is not a cube, or a portion of a ruler (which has width), but is a segment of a line. By measuring a variety of objects, students build a bank of known measurements or benchmark lengths, such as a doorknob being a meter from the floor, or the width of a finger being a centimeter. Then, students learn to estimate length using knowledge of previously measured objects and benchmarks. This enables students to internalize the mental rulers of a centimeter or meter, empowering them to mentally iterate units relevant to measuring a given length (**2.MD.3**). The knowledge and experience signal that students are determining which tool is appropriate to make certain measurements (**2.MD.1**).

In Topic C, students measure and compare to determine how much longer one object is than another (**2.MD.4**). They also measure objects twice using different length units, both standard and non-standard, thereby developing their understanding of how the total measurement relates to the size of the length unit (**2.MD.2**). Repeated experience and explicit comparisons help students recognize that the smaller the length unit, the larger the number of units, and the larger the length unit, the smaller the number of units.

The module culminates as students relate addition and subtraction to length. They apply their conceptual understanding to choose appropriate tools and strategies, such as the ruler as a number line, benchmarks for estimation, and tape diagrams for comparison, to solve word problems (**2.MD.5**, **2.MD.6**). The problems progress from concrete (i.e., measuring objects and using the ruler as a number line to add and subtract) to abstract (i.e., representing lengths with tape diagrams to solve *start unknown* and two-step problems).

Math Unit 2

Rigorous Curriculum Design Template

Unit : Addition and Subtraction of Length Units

Subject: Mathematics

Grade/Course: Grade 2

Pacing: 12 Days

Unit of Study: Unit 2: Addition and Subtraction of Length Units

Priority Standards:

Measure and estimate lengths in standard units.

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.

Foundational Standards:

1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlap*

Math Practice Standards:

MP.2 **Reason abstractly and quantitatively.** Students reason quantitatively when they measure and compare lengths. They reason abstractly when they use estimation strategies such as benchmarks and mental rulers and when they relate number line diagrams to measurement models.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students reason to solve word problems involving length measurement using tape diagrams and analyze the reasonableness of the work of their peers.

MP.5 **Use appropriate tools strategically.** Students consider the object being measured and choose the appropriate measurement tool. They use the tape diagram as a tool to solve word problems.

MP.6 **Attend to precision.** Students accurately measure by laying physical units end-to-end with no gaps and by using a measurement tool. They correctly align the zero-point on a ruler as the beginning of the total length. They attend to precision when they verbally and in writing specify the length unit, when they use a ruler to measure or draw a straight line of a given length, and when they verify estimates by measuring.

“Unwrapped” Standards *Most Important to this Unit	
Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.	
Concepts - What Students Need to Know (context)	**Skills - What Students Need to Be Able to Do (Depth of Knowledge Level)**
The length of an object	Measure (DOK - 1)
appropriate tools	Select (DOK - 2)
appropriate tools	Use (DOK - 1)
How two measurements relate to the size of the unit chosen (using units of different lengths for the two measurements)	Describe (DOK - 3)
whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …,	Represent (DOK - 2)
whole-number sums and differences within 100 on a number line diagram.	Represent (DOK - 2)

Essential Questions	**Big ideas**
● Why do you measure? ● How do you measure an object? ● Why do we use standard units of measurement? ● How does the size of a unit relate to a measurement?	● Measurement describes attributes of objects ● Standard units allow people to compare and understand measurement ● The size of a unit should be chosen deliberately

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Lesson Exit tickets for each lesson	Application Problem Student Debriefs Problem Set Data	Exit Ticket Data End-of Module Assessment *See Table Below

*Assessment Summary

Type	Administered	Format	Standards Addressed
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	2.MD.1 2.MD.2 2.MD.3 2.MD.4 2.MD.5 2.MD.6

Performance Task (*To be completed by grade level team)
Overview:
Engaging Learning Experiences
Task 1: Task 2: Task 3: Task 4:

Instructional Resources
Useful Websites: Engage NY K-5 Curriculum overview and guiding documents:

https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents

Engage NY Grade 2 Resources:
https://www.engageny.org/resource/grade-2-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 2nd Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/2nd.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Books:
Inch by Inch by Leo Lionni
How Big is a Foot? by Rolph Myller
Twelve Snails to One LIzard by Susan Hightower
Measuring Penny by Loreen Leedy
Millions to Measure by David Schwartz

Suggested Tools and Representations

- Centimeter cubes
- Centimeter rulers
- Large and small paper clips
- Meter stick
- Paper meter strips (Lesson 6 Template)
- Personal white boards
- Tape diagram

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way

2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words

3. Reinforcing Effort and Providing Recognition: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.

4. Homework and Practice: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.

5. Nonlinguistic Representations: has recently been proven to stimulate and increase brain activity.

6. Cooperative Learning: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.

7. Setting Objectives and Providing Feedback: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students.

Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations.

Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.

It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

We encourage teachers to pay particular attention to the manner in which knowledge is sequenced in A Story of Units and to capitalize on that sequence when working with special student populations. Most lessons contain a suggested teaching sequence that moves from simple to complex, starting, for example, with an introductory problem for a math topic and building up inductively to the general case encompassing multifaceted ideas. By breaking down problems from simple to complex, teachers can locate specific steps that students are struggling with or stretch out problems for students who desire a challenge.

Throughout A Story of Units, teachers are encouraged to give classwork utilizing a "time frame" rather than a "task frame." Within a given time frame, all students are expected to do their personal best, working at their maximum potential. "Students, you have 10 minutes to work independently." Bonus questions are always ready for accelerated students. The teacher circulates

much positive feedback, however, the method in which you give that feedback should be varied. **8. Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. **9. Cues, Questions, and Advanced Organizers**: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	and monitors the work, error-correcting effectively and wisely. Some students will complete more work than others. Neither above nor below grade level students are overly praised or penalized. Personal success is what we are striving for. Another vitally important component for meeting the needs of all students is the constant flow of data from student work. A Story of Units provides daily tracking through "exit tickets" for each lesson as well as mid- and end-of-module assessment tasks to determine student understanding at benchmark points. These tasks should accompany teacher-made test items in a comprehensive assessment plan. Such data flow keeps teaching practice firmly grounded in student learning and makes incremental forward movement possible. A culture of "precise error correction" in the classroom breeds a comfort with data that that is non-punitive and honest. When feedback is provided with emotional neutrality, students understand that making mistakes is part of the learning process. "Students, for the next five minutes I will be meeting with Brenda, Scott, and Jeremy. They did not remember to rename the remainder in the tens place as 10 ones in their long division on Question 7." Conducting such sessions then provides the teacher the opportunity to quickly assess if students need to start at a simpler level or just need more monitored practice now that their eyes are opened to their mistake. Good mathematics instruction, like any successful coaching, involves demonstration, modeling, and lots of intelligent practice. In math, just as in sports, skill is acquired incrementally; as the student acquires greater skill, more complexity is added and proficiency grows. The careful sequencing of the mathematics and the many scaffolds that have been designed into A Story of Units makes it an excellent curriculum for meeting the needs of all students, including those with special and unique learning modes.

Scaffolds for Students with Disabilities[4]

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding.

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

New or Recently Introduced Terms - Benchmark (e.g., "round" numbers like multiples of 10) - Endpoint (point where something begins or ends) - Estimate (an approximation of a quantity or number) - Hash mark (marks on a ruler or other measurement tool) - Meter (standard unit of length in the metric system) - Meter stick or strip (tool used to measure length) - Number line - Overlap (extend over, or cover partly) - Ruler (tool used to measure length) **Familiar Terms and Symbols** - Centimeter (standard length unit within the metric system) - Combine (join or put together) - Compare (specifically using direct comparison) - Difference (to find the difference between two numbers, subtract the smaller number from the greater number) - Height (vertical distance measurement from bottom to top) - Length (distance measurement from end to end; in a rectangular shape, length can be used to describe any of the four sides) - Length unit (e.g., centimeters, inches)	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** - Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. - Guide students as they select and practice using their own graphic organizers and models to solve. - Use direct instruction for vocabulary with visual or concrete representations. - Use explicit directions with steps and procedures enumerated. - Guide students through initial practice promoting gradual independence. "I do, we do, you do." - Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. - Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** - First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. - Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?'	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** - Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. - Model and post conversation "starters," such as: "I agree because..." "Can you - explain how you solved it?""I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." - Incorporate written reflection, evaluation, and synthesis. - Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** - Encourage students to explain their reasoning both orally and in writing. - Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. - Offer choices of independent or group assignments for early finishers. - Encourage students to notice and explore patterns and to identify rules and relationships in math. - Have students share their observations in discussion and writing (e.g., journaling). - Foster their curiosity about numbers and mathematical ideas. - Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. - Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. - Let students choose their mode of response: written, oral, concrete, pictorial,

	● Encourage students to explain their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

OVERVIEW

In Module 2, students added and subtracted measurement units within 100 (**2.MD.5**, **2.MD.6**), a meaningful application of their work from Module 1 (**2.NBT.5)** and a powerful bridge to the base ten units of Grade 2.
In this 25-day Grade 2 module, students expand their skill with and understanding of units by bundling ones, tens, and hundreds up to a thousand with straws. Unlike the length of 10 centimeters in Module 2, these bundles are discrete sets. One unit can be grabbed and counted just like a banana—1 hundred, 2 hundred, 3 hundred, etc. (**2.NBT.1**).

A number in Grade 1 generally consisted of two different units, tens and ones. Now, in Grade 2, a number generally consists of three units: hundreds, tens, and ones (**2.NBT.1**). The bundled units are organized by separating them largest to smallest, ordered from left to right. Over the course of the module, instruction moves from physical bundles that show the proportionality of the units to non-proportional place value disks and to numerals on the place value chart (**2.NBT.3**).

Furthermore, in this module instruction includes a great deal of counting: by ones, tens, and hundreds (**2.NBT.2**). Counting up using the centimeter tape or a classroom number line shows movement from left to right as the numbers increase. Counting up on the place value chart shows movement from right to left as the numbers increase. For example, as 10 ones are renamed as 1 ten, the larger unit is housed in the place directly to the left. The goal is for students to move back and forth fluidly between these two models, the number line and the place value chart, using either to rename units and compare numbers (**2.NBT.4**).

In this module, the place value story has advanced. Along with changing 10 ones for 1 ten, students now also change 10 tens for 1 hundred. This changing leads to the use of counting strategies to solve word problems (**2.OA.1**). In the next module, this change leads to mental math and the formal algorithms for addition and subtraction. Comparison extends into finding 100 more and 100 less, 10 more and 10 less, etc. Just as in Grade 1, *more* and *less* translate into formal addition and subtraction at the onset of Module 4 (**2.NBT.8**).

The module includes a sequence of engaging problems in which students are asked to change 1 hundred for 10 units of ten and to change 10 units of ten for 1 hundred. The assessment task following Topic G culminates this series with variations on the following problem: "Mrs. Ortiz has 21 students in her second-grade class. All of them have 10 fingers and 10 toes. Write the total number of toes of the students using hundreds, tens, and ones. Explain using words, pictures, or numbers." In order to explain, students must recognize that each child in the problem represents a group of 10 toes. They then count by tens, changing units of ten for 1 hundred as appropriate to find the solution. This transitions into the coming module where students apply their skill of making larger units to work with addition and subtraction.

How is this module's learning foundational to later grades? Understanding 3 tens or 3 units of 10 leads to an understanding of 3 fours or 3 units or groups of four (Grade 3 OA standards), 3 fourths or 3 units of one-fourth (Grade 3 NF standards). Learning that 12 tens = 120 leads to an understanding of 12 tenths = 1.2,
4 thirds = 4/3 = 1 1/3, or even 4 threes = 12. Counting up and down by ones, tens, and hundreds with both the number line and place value chart is essential from Grade 3 forward for rounding and mental math (Grade 3 NBT standards) to meaningful understanding of all operations with base ten whole numbers (Grade 4 NBT standards) and to understanding place value's extension into decimal fractions and operations (Grade 5 NBT standards).

Math Unit 3

Rigorous Curriculum Design Template

Unit 3: Place Value, Counting, and Comparison of Numbers to 1,000

Subject: Mathematics

Grade/Course: Grade 2

Pacing: 25 days

Unit of Study: Unit 3: Place Value, COunting, and Comparison of Numbers to 1,000

Priority Standards:

Understand place value.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens—called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2 Count within 1000; skip-count by 5s, 10s and 100s.

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Foundational Standards:

1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones—called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Math Practice Standards:

MP.2 **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the 6 units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects (exemplified in Topic D).

MP.3 **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can

construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the argument (exemplified in Topics A and E).

MP.6 **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions (exemplified in Topics C and F).

MP.7 **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y (exemplified in Topic B).

MP.8 **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results (exemplified in Topic G).

"Unwrapped" Standards

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens—called a "hundred."
b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Count within 1000; skip-count by 5s, 10s and 100s.

Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	
Concepts - What Students Need to Know (context)	**Skills - What Students Need to Be Able to Do (Depth of Knowledge Level)**
the three digits of a three-digit number represent amounts of hundreds, tens, and ones	Understand (DOK - 2)
Within 1,000	Count (DOK - 1)
By 5's, 10's,and 100's	Skip-Count (DOK - 1)
Numbers to 1,000 (using numerals, number names, and expanded form)	Read and Write (DOK - 1)
Two three-digit numbers based on meanings of the hundreds, tens, and ones digits >, =, and < symbols to record the results of comparisons	Compare (DOK - 2) Use (DOK - 1)

Essential Questions	**Big ideas**
● How do you know what a digit is worth? ● How can you represent a 3-digit number? ● How can you use place value to compare numbers?	● The value of a digit is determined by its place in a number ● numbers may be represented in multiple ways with multiple materials. (a three digit number is represented in amounts of hundreds, tens, and ones) ● numbers can be compared using knowledge of place value.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Lesson Exit tickets for each lesson	Application Problem Student Debriefs Problem Set Data	Exit Ticket Data Mid-Module Assessment End-of Module Assessment

		*See Table Below

*Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic D	Constructed response with rubric	2.NBT.1 2.NBT.2 2.NBT.3
End-of-Module Assessment Task	After Topic G	Constructed response with rubric	2.NBT.1 2.NBT.2 2.NBT.3 2.NBT.4

Performance Task (*To be completed by grade level team)
Overview:
Engaging Learning Experiences
Task 1: Task 2: Task 3: Task 4:

Instructional Resources
Useful Websites: Engage NY K-5 Curriculum overview and guiding documents: https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents Engage NY Grade 2 Resources: https://www.engageny.org/resource/grade-2-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 2nd Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/2nd.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Books:
A Place for Zero: A Math Adventure by Angeline Sparagna LoPresti
Zero by Kathryn Otoshi
1,2,3 Peas by Keith Baker
The King's Commissioners by Aileen Friedman

Suggested Tools and Representations

2 boxes of 1,000 straws per class of 25
Clock number line (details in Lesson 1 Fluency Practice)
Dice, 1 per pair
Dienes blocks
Hide Zero cards (also known as place value cards) showing numbers 1–5, 10–50, and 100—500 (1 small set per student) (Lesson 4 Template 1))

hundreds	tens	ones

Hundreds place value chart (Lesson 4 Template 2)
Meter strip (Lesson 1 Template)
Hundreds Place Value Chart
Number spelling activity sheet (Lesson 7 Activity Sheet)
Personal white boards
Place value box (details in Lesson 4 Concept Development)
Place value cards to 1,000, 1 large teacher set
Place value disks: suggested minimum of one set per pair (18 ones, 18 tens and 18 hundreds, and 1 one thousand)

Play money: $1, $5, $10, and $100 bills (10 ones,

1 five, 12 tens, and 10 hundreds per pair), and a single set of 16 pennies, 13 dimes Rubber bands, 16 per pair Small plastic bags (baggies)

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> ● Critical thinking and problem solving ● Collaboration and leadership ● Agility and adaptability ● Initiative and entrepreneurialism ● Effective oral and written communication ● Accessing and analyzing information ● Curiosity and imagination **<u>Marzano's Nine Instructional Strategies for Effective Teaching and Learning</u>** **1. Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way **2. Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words **3. Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. **4. Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. **5. Nonlinguistic Representations**: has recently been proven to stimulate and increase brain activity. **6. Cooperative Learning**: has been proven to have a positive impact on overall learning. Note: groups should	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. We encourage teachers to pay particular attention to the manner in which knowledge is sequenced in A Story of Units and to capitalize on that sequence when working with special student populations. Most lessons contain a suggested teaching sequence that moves from simple to complex, starting, for example, with an introductory problem for a math topic and building up inductively to the general case encompassing multifaceted ideas. By breaking down problems from simple to complex, teachers can locate specific steps that students are struggling with or stretch out problems for

be small enough to be effective and the strategy should be used in a systematic and consistent manner. **7. Setting Objectives and Providing Feedback**: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied. **8. Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. **9. Cues, Questions, and Advanced Organizers**: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	students who desire a challenge. Throughout A Story of Units, teachers are encouraged to give classwork utilizing a "time frame" rather than a "task frame." Within a given time frame, all students are expected to do their personal best, working at their maximum potential. "Students, you have 10 minutes to work independently." Bonus questions are always ready for accelerated students. The teacher circulates and monitors the work, error-correcting effectively and wisely. Some students will complete more work than others. Neither above nor below grade level students are overly praised or penalized. Personal success is what we are striving for. Another vitally important component for meeting the needs of all students is the constant flow of data from student work. A Story of Units provides daily tracking through "exit tickets" for each lesson as well as mid- and end-of-module assessment tasks to determine student understanding at benchmark points. These tasks should accompany teacher-made test items in a comprehensive assessment plan. Such data flow keeps teaching practice firmly grounded in student learning and makes incremental forward movement possible. A culture of "precise error correction" in the classroom breeds a comfort with data that that is non-punitive and honest. When feedback is provided with emotional neutrality, students understand that making mistakes is part of the learning process. "Students, for the next five minutes I will be meeting with Brenda, Scott, and Jeremy. They did not remember to rename the remainder in the tens place as 10 ones in their long division on Question 7." Conducting such sessions then provides the teacher the opportunity to quickly assess if students need to start at a simpler level or just need more monitored practice now that their eyes are opened to their mistake. Good mathematics instruction, like any successful coaching, involves demonstration, modeling, and lots of intelligent practice. In math, just as in sports, skill is acquired incrementally; as the student acquires greater skill, more complexity is added and proficiency grows. The careful sequencing of the mathematics and the many scaffolds that have been designed into A Story of Units makes it an excellent curriculum for meeting the needs of all students, including those with special and unique learning modes.

Scaffolds for Students with Disabilities[4]

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding.

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

New or Recently Introduced Terms	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.
● Base ten numerals (e.g., a thousand is 10 hundreds, a hundred is 10 tens, starting in Grade 3 a one is 10 tenths, etc.) ● Expanded form (e.g., 500 + 70 + 6) ● Hundreds place (e.g., the 5 in 576 is in the hundreds place) ● One thousand (1,000) ● Place value or number disk (pictured) ● Standard form (e.g., 576) ● Unit form (e.g., 5 hundreds 7 tens 6 ones) ● Word form (e.g., five hundred seventy-six) **Familiar Terms and Symbols** ● =, <, > (equal, less than, greater than) ● Altogether (e.g., 59 centimeters and 17 centimeters; altogether there are 76 centimeters) ● Bundling, grouping (putting smaller units together to make a larger one, e.g., putting 10 ones together to make a ten or 10 tens together to make a hundred) ● How many more/less (the difference between quantities) ● How much more/less (the difference between quantities) ● More than/less than (e.g., 576 is more than 76; 76 is less than 576) ● Number sentence (an equation or inequality that has a true or false value and contains no unknowns, e.g., 3 + 2 = 5) ● Ones place (e.g., the 6 in 576 is in the ones place) ● Place value (the unitary values of the digits in numbers)	**<u>Provide Multiple Means of Representation</u>** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. “I do, we do, you do.” ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ ● Encourage students to explain	**<u>Provide Multiple Means of Representation</u>** ● Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. ● Model and post conversation “starters,” such as: “I agree because...” “Can you ● explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial,

● Renaming, changing (instead of *carrying* or *borrowing,* e.g., a group of 10 ones is renamed a ten when the ones are bundled and moved from the ones to the tens place; if using $1 bills, they may be changed for a $10 bill when there are enough) ● Tens place (e.g., the 7 in 576 is in the tens place) ● Units of ones, tens, hundreds, one thousand (a single one and groups of 10s, 100s, and 1,000)	their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **Provide Multiple Means of Engagement** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **Provide Multiple Means of Engagement** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Addition and Subtraction Within 200 with Word Problems to 100

OVERVIEW

In Module 3, students were immersed in the base ten system as they built a strong foundation of place value understanding through a concrete to pictorial to abstract approach. They bundled groups of 10 and saw that 10 like units could be bundled to produce a new unit that is ten times as large. They progressed from seeing 10 ones as 1 ten (**1.NBT.2a**) to understanding 10 tens as 1 hundred (**2.NBT.2**). Module 4 builds on that place value understanding, which enables students to compose and decompose place value units to add and subtract within 200.

Module 4 is devoted to three major areas of work. The first two are building fluency in two-digit addition and subtraction within 100 (**2.NBT.5**) and applying that fluency to one- and two-step word problems of varying types within 100 (**2.OA.1**). Students' increasing fluency with calculations within 100 allows for word problems to transition from being mere contexts for calculation into opportunities for students to see and analyze the relationships between quantities. Daily Application Problems and specific lessons in Topics A, C, and F provide students with guided and independent practice as they solve a variety of problem types, including more complex comparison problems. Note that most two-step problems involve single-digit addends and do not involve the most difficult comparison problem types. The third major area of work is developing students' conceptual understanding of addition and subtraction of multi-digit numbers within 200 (**2.NBT.7**, **2.NBT.9**) as a foundation for work with addition and subtraction within 1,000 in Module 5.

The final lessons of Module 3 (finding 1 more, 1 less, 10 more, 10 less) transitioned into mental addition and subtraction of 1 and 10 (**2.NBT.8**). In Topic A of Module 4, students work with place value strategies to fluently add and subtract within 100 (**2.NBT.5**). They mentally add and subtract 100 in Topics D and E, as well as during fluency activities throughout the module, as they did in Module 3.

This knowledge is then extended and used to solve problems. For example, students might count on by ones and tens, e.g., 39 + ☐ = 62, so 40, 50, 60, 61, 62. They might use compensation, adding the same amount to the subtrahend as to the minuend to make a multiple of ten, e.g., 62 – 39 = 63 – 40. They might add or subtract a multiple of 10 and adjust the solution as necessary, e.g., 62 – 39 is 4 tens less than 62 but 1 more (**2.NBT.5**). Students explain why these strategies work using place value language, properties of addition and subtraction, and models such as the number line (**2.NBT.9**).

Topic A's strategies lead naturally to work with the algorithms for addition (Topic B) and subtraction (Topic C). Note that the vertical form is used to describe the written numbers, where the algorithm is used to describe the cyclical process of making a larger or smaller unit. In these two topics, students represent place value strategies with place value disks and math drawings (see images with strategy names below). Students work with composing 1 ten from 10 ones or decomposing 1 ten as 10 ones (with minuends within 100).

After the Mid-Module Assessment, students continue working with manipulatives and math drawings to make sense of problems in which they compose or decompose twice. Topic D focuses on addition, with the new complexity of composing 1 hundred from 10 tens within 200 in problems with up to four addends (**2.NBT.6, 2.NBT.7**). Subtraction in Topic E involves subtracting when decomposing 1 hundred for 10 tens and 1 ten for 10 ones (**2.NBT.7**).

Throughout the module, manipulatives and math drawings allow students to see numbers in terms of place value units and serve as a reminder that students must add like units (e.g., knowing that 74 + 38 is 7 tens + 3 tens and 4 ones + 8 ones).

- *Abstract*
- *Concrete*
- *Pictorial*
- Chip model
- Place value disks
- Place value chart with labeled disks
- New groups below

In Module 4, the focus is often on computational strategies with bare numbers (i.e., no context) so that total attention is given to understanding the value of each digit within a number, as well as why the algorithm works. Students use the place value chart as an organizer. Simultaneous use of a vertical form and a place value chart allows students to better recognize both the value of numbers when they are not on the place value chart, and like units. The same is true when students make math drawings and use place value language to relate each step of the drawing to the vertical form (**2.NBT.7**). The different representations serve to solidify the understanding of the composition and decomposition of units, moving from concrete to pictorial to abstract. Throughout the work, students are encouraged to explain their actions and analyses and to use the relationship between addition and subtraction to check their work (**2.NBT.9**).

Throughout the module, students are encouraged to be flexible in their thinking and to use multiple strategies in solving problems, including the use of drawings such as tape diagrams, which they relate to equations. In Topic F, students are introduced to the totals below method and are challenged to explain why both it and the new groups below method work (**2.NBT.9**).

Math Unit 4

Rigorous Curriculum Design Template

Unit 4: Addition and Subtraction Within 200 with Word Problems to 100

Subject: Mathematics

Grade/Course: Grade 2

Pacing: 35 days

Unit of Study: Unit 4: Addition and Subtraction Within 200 with Word Problems to 100

Priority Standards:

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

Foundational Standards:

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

1.OA.3 Apply properties of operations as strategies to add and subtract. (Students need not use formal terms for these properties.) *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones – called a “ten.”

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens – called a “hundred.”

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Math Practice Standards:

MP.1 **Make sense of problems and persevere in solving them.** Students solve two-step word problems and are challenged to make sense of more complex relationships within situations. They flexibly solve problems with a variety of strategies at their disposal, sometimes finding many ways to solve the same problem.

MP.2 **Reason abstractly and quantitatively.** Students reason abstractly when they represent two-step problems and harder problem types with drawings such as tape diagrams and when they relate those drawings to equations. As the module progresses, students move back and forth between concrete, pictorial, and abstract work to make sense of quantities and their relationships in problem situations.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students construct viable arguments when they use place value reasoning and properties of operations to explain why their addition and subtraction strategies work and when they use that reasoning to justify their choice of strategies in solving problems. They critique the reasoning of others when they use those same concepts to disprove or support the work of their peers.

MP.4 **Model with mathematics.** Students model with mathematics when they write equations to solve two-step word problems, make math drawings when solving a vertical algorithm, or when they draw place value charts and disks to represent numbers.

MP.6 **Attend to precision.** Students attend to precision when they label their math drawings and models with specific place value units. They calculate accurately and efficiently when adding numbers within 200 and when using the relationship between addition and subtraction to check their work.

"Unwrapped" Standards *Most Important to this Unit

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)	
Concepts - What Students Need to Know (context)	**Skills - What Students Need to Be Able to Do (Depth of Knowledge Level)**
One- and Two-step word problems (with unknowns in all positions- see table 1 in unit 1) (using addition and subtraction within 100)	Solve (DOK - 2)
Using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (within 100)	Fluently add and subtract (DOK - 2)
10 or 100 to or from a given number 100–900	Mentally add and subtract (DOK - 2)
why addition and subtraction strategies work (using place value and the properties of operations) (Explanations may be supported by drawings or objects.)	Explain (DOK - 3)

Essential Questions	**Big ideas**
● How does place value help us add and subtract multi-digit numbers? ● How can multiples of 10 and 100 be used in mental math? ● How can models and drawings be used to represent addition and subtraction problems?	● Addition and subtraction strategies can be explained using place value. ● multiples of 10 and 100 can be used to understand addition and subtraction problems. ● models and drawings can be used to represent to addition and subtraction problems.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Lesson Exit tickets for each lesson	Application Problem Student Debriefs Problem Set Data	Exit Ticket Data Mid-Module Assessment End-of Module Assessment *See Table Below

*Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	2.OA.1 2.NBT.5 2.NBT.7 2.NBT.8 2.NBT.9
End-of-Module Assessment Task	After Topic F	Constructed response with rubric	2.OA.1 2.NBT.5 2.NBT.6 2.NBT.7 2.NBT.8 2.NBT.9

Performance Task (*To be completed by grade level team)
Overview:
Engaging Learning Experiences
Task 1: Task 2: Task 3: Task 4:

Instructional Resources

Useful Websites:
Engage NY K-5 Curriculum overview and guiding documents:
https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents

Engage NY Grade 2 Resources:
https://www.engageny.org/resource/grade-2-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 2nd Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/2nd.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Books:
The Grapes of Math by Greg Tang
One Hundred Hungry Ants by Elinor J. Pinczes
Amanda Bean's Amazing Dream by Cindy Neuschwander

Suggested Tools and Representations

- *Hide Zero cards*
- Arrow notation (arrow way)
- Chip model (pictured)
- Hide Zero cards (pictured)
- Number bond
- Personal white boards
- Place value chart (Template in Lesson 1)
- Place value disk sets (19 ones, 19 tens, 18 hundreds, 1 one thousand per set)
- Rekenrek
- *Chip model*
- Tape diagram

Note: Students work through a progression of models to represent the addition and subtraction algorithm. Following the use of actual place value disks, students learn to draw the disks to represent numbers. This model provides an added level of support in that students write the value on each disk (see image below left). Because the value is on the disk, there is no need to label the place value chart. Next, students learn the chip model, drawing dots on a labeled place value chart (see image below right). While still pictorial, this model is more abstract because the value of the chip derives from its placement on the chart.

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> ● Critical thinking and problem solving ● Collaboration and leadership ● Agility and adaptability ● Initiative and entrepreneurialism ● Effective oral and written communication ● Accessing and analyzing information ● Curiosity and imagination **<u>Marzano's Nine Instructional Strategies for Effective Teaching and Learning</u>** **1. Identifying Similarities and Differences**: helps students understand more complex problems by analyzing them in a simpler way **2. Summarizing and Note-taking**: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words **3. Reinforcing Effort and Providing Recognition**: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. **4. Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. **5. Nonlinguistic Representations**: has recently been proven to stimulate and increase brain activity. **6. Cooperative Learning**: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner. **7. Setting Objectives and Providing Feedback**: provide	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. We encourage teachers to pay particular attention to the manner in which knowledge is sequenced in A Story of Units and to capitalize on that sequence when working with special student populations. Most lessons contain a suggested teaching sequence that moves from simple to complex, starting, for example, with an introductory problem for a math topic and building up inductively to the general case encompassing multifaceted ideas. By breaking down problems from simple to complex, teachers can locate specific steps that students are struggling with or stretch out problems for students who desire a challenge. Throughout A Story of Units, teachers are encouraged to give classwork utilizing a “time frame” rather than a “task frame.” Within a given time frame, all students are expected to do their

students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied. **8. Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. **9. Cues, Questions, and Advanced Organizers**: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	personal best, working at their maximum potential. "Students, you have 10 minutes to work independently." Bonus questions are always ready for accelerated students. The teacher circulates and monitors the work, error-correcting effectively and wisely. Some students will complete more work than others. Neither above nor below grade level students are overly praised or penalized. Personal success is what we are striving for. Another vitally important component for meeting the needs of all students is the constant flow of data from student work. A Story of Units provides daily tracking through "exit tickets" for each lesson as well as mid- and end-of-module assessment tasks to determine student understanding at benchmark points. These tasks should accompany teacher-made test items in a comprehensive assessment plan. Such data flow keeps teaching practice firmly grounded in student learning and makes incremental forward movement possible. A culture of "precise error correction" in the classroom breeds a comfort with data that that is non-punitive and honest. When feedback is provided with emotional neutrality, students understand that making mistakes is part of the learning process. "Students, for the next five minutes I will be meeting with Brenda, Scott, and Jeremy. They did not remember to rename the remainder in the tens place as 10 ones in their long division on Question 7." Conducting such sessions then provides the teacher the opportunity to quickly assess if students need to start at a simpler level or just need more monitored practice now that their eyes are opened to their mistake. Good mathematics instruction, like any successful coaching, involves demonstration, modeling, and lots of intelligent practice. In math, just as in sports, skill is acquired incrementally; as the student acquires greater skill, more complexity is added and proficiency grows. The careful sequencing of the mathematics and the many scaffolds that have been designed into A Story of Units makes it an excellent curriculum for meeting the needs of all students, including those with special and unique learning modes.

Scaffolds for Students with Disabilities[4]

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding.

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

New or Recently Introduced Terms ● Algorithm (a step-by-step procedure to solve a particular type of problem ● Compose (e.g., to make 1 larger unit from 10 smaller units) ● Decompose (e.g., to break 1 larger unit into 10 smaller units) ● Equation (two expressions with an equal sign between them; that is, an equation is a statement that two expressions are equal; however, there is no guarantee that the statement is true) ● New groups below (show newly composed units on the line below the appropriate place in the addition algorithm, pictured above on page iv) ● Simplifying strategy (e.g., to solve 299 + 6, think 299 + 1 + 5 = 300 + 5 = 305) ● Totals below (pictured above on page iv) **Familiar Terms and Symbols** ● Addend ● Addition ● Bundle, unbundle, regroup, rename, change (compose or decompose a 10 or 100) ● Difference ● Hundreds place (referring to place value) ● Place value (referring to the unit value of each digit in a given number) ● Subtraction ● Units of ones, tens, hundreds, thousands (referring to place value; 10 ones is the same as 1 unit of ten)	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. “I do, we do, you do.” ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ ● Encourage students to explain	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. ● Model and post conversation “starters,” such as: “I agree because...” “Can you ● explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial,

	their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

OVERVIEW

In Module 4, students developed addition and subtraction fluency within 100 and began developing conceptual understanding of the standard algorithm by means of place value strategies. In Module 5, students build upon their mastery of renaming place value units and extend their work with conceptual understanding of the addition and subtraction algorithms to numbers within 1,000, always with the option of modeling with materials or drawings. Throughout the module, students continue to focus on strengthening and deepening conceptual understanding and fluency.

Topic A focuses on place value strategies to add and subtract within 1,000 (2.NBT.7). Students relate *100 more* and *100 less* to addition and subtraction of 100 (2.NBT.8). They add and subtract multiples of 100, including counting on to subtract (e.g., for 650 – 300, they start at 300 and think, "300 more gets me to 600, and 50 more gets me to 650, so… 350"). Students also use simplifying strategies for addition and subtraction: they extend the make a ten strategy to make a hundred, mentally decomposing one addend to make a hundred with the other (e.g., 299 + 6 becomes 299 + 1 + 5, or 300 + 5, which equals 305), and use compensation to subtract from three-digit numbers (e.g., for 376 – 59, add 1 to each, 377 – 60 = 317). The topic ends with students sharing and critiquing solution strategies for addition and subtraction problems. Throughout the topic, students use place value language and properties of operations to explain why their strategies work (2.NBT.9).

In Topics B and C, students continue to build on Module 4's work, now composing and decomposing tens and hundreds within 1,000 (2.NBT.7). As each topic begins, students relate manipulative representations to the algorithm, and then transition to creating math drawings in place of the manipulatives. As always, students use place value reasoning and properties of operations to explain their work.

Throughout Module 5, students maintain addition and subtraction fluency within 100 as they use these skills during their daily application work to solve one- and two-step word problems of all types (2.NBT.5, 2.OA.1). The Application Problem precedes fluency activities in most lessons of Module 5 because this work with smaller numbers does not flow directly into the Concept Development. The focus of the Concept Development is adding and subtracting within 1,000: using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, and relating strategies to a written method (2.NBT.7). Note that a written method can include number bonds, chip models, arrow notation, the algorithm, or tape diagrams. Many students will need to record these strategies to solve correctly. The lessons are designed to provide ample time for discussions that center on student reasoning, explaining why their addition and subtraction strategies work (2.NBT.9). For example, students may use the relationship between addition and subtraction to demonstrate why their subtraction solution is correct.

The module culminates with Topic D, wherein students synthesize their understanding of addition and subtraction strategies and choose which strategy is most efficient for given problems. They defend their choices using place value language and their understanding of the properties of operations (2.NBT.9). Note that, beginning in Topic C, and for the remainder of the year, each day's Fluency Practice includes an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints.

Math Unit 5

Rigorous Curriculum Design Template

Unit 5: Addition and Subtraction Within 1,000 and Word Problems to 100

Subject: Mathematics

Grade/Course: Grade 2

Pacing: 24 days

Unit of Study: Unit 5: Addition and Subtraction Within 1,000 and Word Problems to 100

Priority Standards:

Use place value understanding and properties of operations to add and subtract.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

Foundational Standards:

1.OA.3 Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens—called a "hundred."
b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Math Practice Standards:

MP.3 **Construct viable arguments and critique the reasoning of others.** Students use place value reasoning to explain how each step in their drawing relates to a step in the algorithm. They choose and explain various solution strategies such as number bonds, chip models, vertical form, arrow notation, and tape

diagrams. They critique the reasoning of others when they listen to peers explain their strategies for solving problems, and then discuss the efficacy of those strategies.

MP.6 **Attend to precision.** Students attend to precision when they use place value language to explain their math drawings and calculations. They articulate the arithmetic properties they use to solve a variety of problems. For example, when adding 825 + 80, a student may show understanding of the associative property by saying, "I know that 20 + 80 equals 100, so I added 800 + 100 + 5, which equals 905."

MP.7 **Look for and make use of structure.** Students look for and make use of the base ten structure when composing and decomposing. They extend their understanding from Module 4, viewing 10 tens as forming a new unit called a *hundred,* just as they understand that 10 ones forms 1 ten. They apply this understanding of base ten structure when adding and subtracting three-digit numbers, repeatedly bundling and unbundling groups of ten. Students also make use of structure when they use simplifying strategies, such as compensation, to create a multiple of ten or a hundred.

MP.8 **Look for and express regularity in repeated reasoning.** As students repeatedly manipulate models and record the work abstractly, they recognize the cyclic pattern of the addition or subtraction of like units and the subsequent potential composition or decomposition of units through the place values. They see that the vertical form represents the same cycle they use with the manipulatives.

"Unwrapped" Standards

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)	
Concepts - What Students Need to Know (context)	**Skills - What Students Need to Be Able to Do (Depth of Knowledge Level)**
Using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method (within 1,000)	Add and Subtract (DOK - 2)
In adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	Understand (DOK - 3)
10 or 100 to or from a given number 100–900	Mentally add and subtract (DOK - 2)
why addition and subtraction strategies work (using place value and the properties of operations) (Explanations may be supported by drawings or objects.)	Explain (DOK - 4)

Essential Questions	**Big ideas**
● How does place value help us add and subtract multi-digit numbers? ● How does knowing the properties of operations to help with addition and subtraction? ● How can models and drawings be used to represent addition and subtraction?	● Models and drawings can be used to represent to addition and subtraction problems. ● Place value and properties of operations can be used to explain why addition and subtraction works.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Lesson Exit tickets for each lesson	Application Problem Student Debriefs Problem Set Data	Exit Ticket Data Mid-Module Assessment End-of Module Assessment *See Table Below

*Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	2.NBT.7 2.NBT.8 2.NBT.9
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	2.NBT.7 2.NBT.8 2.NBT.9

Performance Task (*To be completed by grade level team)
Overview:
Engaging Learning Experiences
Task 1: Task 2: Task 3: Task 4:

Instructional Resources
Useful Websites: Engage NY K-5 Curriculum overview and guiding documents: https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents Engage NY Grade 2 Resources: https://www.engageny.org/resource/grade-2-mathematics Eureka Math Module PDFs:

http://greatminds.net/maps/math/module-pdfs

North Carolina 2nd Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/2nd.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Books:
512 Ants on Sullivan Street by Loreen Leedy
Mall Mania by Stuart J. Murphy
Ready, Set, Hop by Stuart J. Murphy

Suggested Tools and Representations

- Arrow notation, arrow way
- Chip model (pictured below)
- Hide Zero cards
- Number bond
- Personal white boards
- Place value charts (pictured above to the right)
- Place value disk sets (19 ones, 19 tens, 10 hundreds, 1 one thousand per set)
- Tape diagram

Note: Students work through a progression of models to represent the addition and subtraction algorithm. Following the use of actual place value disks, students learn to draw the disks to represent numbers. This model provides an added level of support in that students write the value on each disk (pictured below to the left). Because the value is on the disk, there is no need to label the place value chart. Next, students learn the chip model, drawing dots on a labeled place value chart (pictured below to the right). While still pictorial, this model is more abstract because the value of the chip derives from its placement on the chart.

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way

2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words

3. Reinforcing Effort and Providing Recognition: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.

4. Homework and Practice: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments.

5. Nonlinguistic Representations: has recently been proven to stimulate and increase brain activity.

6. Cooperative Learning: has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner.

7. Setting Objectives and Providing Feedback: provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too

The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students.

Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations.

Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.

It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

We encourage teachers to pay particular attention to the manner in which knowledge is sequenced in A Story of Units and to capitalize on that sequence when working with special student populations. Most lessons contain a suggested teaching sequence that moves from simple to complex, starting, for example, with an introductory problem for a math topic and building up inductively to the general case encompassing multifaceted ideas. By breaking down problems from simple to complex, teachers can locate specific steps that students are struggling with or stretch out problems for students who desire a challenge.

Throughout A Story of Units, teachers are encouraged to give classwork utilizing a "time frame" rather than a "task frame." Within a given time frame, all students are expected to do their personal best, working at their maximum potential. "Students, you have 10 minutes to work independently." Bonus questions are always ready for accelerated students. The teacher

much positive feedback, however, the method in which you give that feedback should be varied. **8. Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. **9. Cues, Questions, and Advanced Organizers:** helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	circulates and monitors the work, error-correcting effectively and wisely. Some students will complete more work than others. Neither above nor below grade level students are overly praised or penalized. Personal success is what we are striving for. Another vitally important component for meeting the needs of all students is the constant flow of data from student work. A Story of Units provides daily tracking through "exit tickets" for each lesson as well as mid- and end-of-module assessment tasks to determine student understanding at benchmark points. These tasks should accompany teacher-made test items in a comprehensive assessment plan. Such data flow keeps teaching practice firmly grounded in student learning and makes incremental forward movement possible. A culture of "precise error correction" in the classroom breeds a comfort with data that that is non-punitive and honest. When feedback is provided with emotional neutrality, students understand that making mistakes is part of the learning process. "Students, for the next five minutes I will be meeting with Brenda, Scott, and Jeremy. They did not remember to rename the remainder in the tens place as 10 ones in their long division on Question 7." Conducting such sessions then provides the teacher the opportunity to quickly assess if students need to start at a simpler level or just need more monitored practice now that their eyes are opened to their mistake. Good mathematics instruction, like any successful coaching, involves demonstration, modeling, and lots of intelligent practice. In math, just as in sports, skill is acquired incrementally; as the student acquires greater skill, more complexity is added and proficiency grows. The careful sequencing of the mathematics and the many scaffolds that have been designed into A Story of Units makes it an excellent curriculum for meeting the needs of all students, including those with special and unique learning modes.

Scaffolds for Students with Disabilities[4]

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding.

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

New or Recently Introduced Terms ● Compensation (simplifying strategy where students add or subtract the same amount to or from both numbers to create an equivalent, but simpler, problem) **Familiar Terms and Symbols** ● Addend ● Addition ● Algorithm ● Bundle ● Compose ● Decompose ● Difference ● Equation ● New groups below ● Number bond ● Place value ● Place value chart (pictured to the right) ● Place value or number disk (pictured to the right) ● Rename ● Simplifying strategy ● Subtraction ● Tape diagram ● Total ● Unbundle ● Units of ones, tens, hundreds	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. “I do, we do, you do.” ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ ● Encourage students to explain	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. ● Model and post conversation “starters,” such as: “I agree because...” “Can you ● explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers

	their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Foundations of Multiplication and Division

OVERVIEW

Grade 2 Module 6 lays the conceptual foundation for multiplication and division in Grade 3 and for the idea that numbers other than 1, 10, and 100 can serve as units.

In Topic A, students begin by making equal groups using concrete materials, learning to manipulate a given number of objects to create equal groups (e.g., given 15 objects, they create 3 groups of 5 or 5 groups of 3), and progress to pictorial representations where they may begin by circling a group of 5 stars, adding 5 more, then adding 5 more. They determine the total and relate their drawings to the corresponding repeated addition equation (pictured below). Students calculate the repeated addition sums by adding on to the previous addends, step-by-step, or by grouping the addends into pairs and adding. By the end of Topic A, students draw abstract tape diagrams to represent the total and to show the number in each group as a new unit (pictured below). Hence, they begin their experience toward understanding that any unit may be counted, e.g., 3 dogs, 3 tens, or even 3 fives. This is the bridge between Grades 2 and 3. Grade 2 focuses on the manipulation of place value units, whereas Grade 3 focuses on the manipulation of numbers 1 through 10 as units.

In Topic B, students organize the equal groups created in Topic A into arrays, wherein either a row or column is seen as the new unit being counted. They use manipulatives to compose up to 5 by 5 arrays one row or one column at a time and express the total via repeated addition equations (**2.OA.4**). For example, students might arrange one column of 5 counters, then another, and then another to compose an array of 3 columns of 5, or 15 counters. As they compose and decompose arrays, students create different number sentences yielding the same total (e.g., 5 + 5 + 5 = 15 and 3 + 3 + 3 + 3 + 3 = 15). They find the total number of objects in each array by counting on from left to right. “Three plus 3 is 6. Six plus 3 is 9. Nine plus 3 is 12." As Topic B progresses, students move to the pictorial level to represent arrays and to distinguish rows from columns by separating equal groups horizontally and vertically (e.g., 3 columns of 5 or 5 rows of 3). Then, they use same-size square tiles, moving them closer together in preparation for composing rectangles in Topic C. Topic B concludes with students using tape diagrams to represent array situations and the RDW process to solve word problems.

In Topic C, students build upon their work with arrays to develop the spatial reasoning skills they need in preparation for Grade 3’s area content. They use same-size squares to tile a rectangle with no gaps or overlaps, and then count to find the total number of squares that make up the rectangle (**2.G.2**). After composing rectangles, students partition, or decompose, rectangles. First, they decompose rectangles made of square tiles. Next, they use scissors to cut apart paper rectangles. Finally, they draw and iterate a square unit. In doing so, students begin to see the row or the column as a composite of multiple squares or as a single entity, or unit, which is, in turn, part of the larger rectangle. Students further develop spatial structuring skills by copying and creating drawings on grid paper. Note that the concept of a square unit begins in Grade 3 and is not assessed in Grade 2.

Throughout the topic, students relate repeated addition to the model. They are encouraged to think flexibly and to consider the many ways to construct or partition a given array. Students are not multiplying or dividing in Grade 2; rather, this topic lays the foundation for the relationship between the two operations. As equal parts can be composed to form a whole, likewise, a whole can be decomposed into equal parts.

Topic D focuses on doubles and even numbers (**2.OA.3**), thus setting the stage for the multiplication table of two in Grade 3. As students progress through the lessons, they learn the following interpretations of even numbers.

1. A number that occurs as we skip-count by twos is even: 2, 4, 6, 8…
2. When objects are paired up with none left unpaired, the number is even.
3. A number that is twice a whole number (doubles) is even.
4. A number whose last digit is 0, 2, 4, 6, or 8 is even.

Armed with an understanding of the term *even,* students learn that any whole number that is not even is called *odd,* and that when 1 is added to or subtracted from an even number, the resulting number is odd.

Initially, students arrange pairs into two rows and realize that an even number is the sum of two equal addends, or a repeated sum of twos. They then write number sentences to express the even number (e.g., 2 rows of 7 can be expressed as 7 + 7 or as 2 + 2 + 2 + 2 + 2 + 2 + 2) (**2.OA.3**). Next, students pair objects to make groups of two with none left over, thus discovering one means of determining whether a group of objects (up to 20) has an even or odd number of members. Finally, students learn that any number up to 20 whose last digit is 0, 2, 4, 6, or 8 is even.
After gaining a firm understanding of even numbers, students learn that all other whole numbers are odd. They use the previously learned rules and patterns to identify larger numbers as even or odd and to defend their reasoning. The module concludes with an investigation of what happens when we add two even numbers, two odd numbers, or an odd number with an even number, and the relationship of these pairings to repeated addition (e.g., 3 + 3 is even, but 3 + 3 + 3 is odd).

Rigorous Curriculum Design Template

Unit 6: Foundations of Multiplication and DIvision

Subject: Mathematics

Grade/Course: Grade 2

Pacing: 24 Days

Unit of Study: Unit 6: Foundations of Multiplication and Division

Priority Standards:

Work with equal groups of objects to gain foundations for multiplication.

2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Reason with shapes and their attributes.

2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Foundational Standards:

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

Math Practice Standards:

MP.3 **Construct viable arguments and critique the reasoning of others.** Students explain their thinking using drawings, models, and equations to lay the conceptual foundation for multiplication and division. "If I build an array with 3 columns of 4 objects, then I must have twelve objects because 4 + 4 + 4 = 12. Likewise, if I partition my rectangle into twelve equally sized tiles, I can make 3 equal groups of 4 tiles, or I can make 4 equal groups of 3 tiles." Students defend their reasoning as they prove that a number is even or odd, making connections to the previous concepts of counting by twos, adding on, equal groups, and doubles.

MP.4 **Model with mathematics.** Students learn to organize a set of objects into equal groups and then into rows and columns, or rectangular arrays. They use math drawings to analyze the relationship between rows and columns (e.g., 3 rows of 4 or 4 columns of 3) and to model the array as the sum of equal addends (e.g., 4 + 4 + 4 = 12).

MP.7 **Look for and make use of structure.** As students compose and decompose arrays, they recognize that the array structure is a collection of rows or columns and that either can be seen as a unit. Students

match repeated addition to both the structure of the rows and columns (e.g., 5 + 5 + 5 can be 3 rows or columns of 5, or 3 fives).

MP.8 **Look for and express regularity in repeated reasoning.** As students create equal groups using objects, they recognize that they are repeatedly adding the same number; for example, 3 groups of 4 bears can be expressed as 4 + 4 + 4. Students also discover patterns in odd and even numbers, recognizing the repetition of 0, 2, 4, 6, and 8 in the ones place.

"Unwrapped" Standards	
Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	
Concepts - What Students Need to Know (context)	**Skills - What Students Need to Be Able to Do (Depth of Knowledge Level)**
Whether a group of objects (up to 20) has an odd or even number of members	Determine (DOK - 2)
An equation to express an even number as a sum of two equal addends.	Write (DOK - 2)
Addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns	Use (DOK - 2)
An equation to express the total as a sum of equal addends.	Write (DOK - 2)
A rectangle into rows and columns of same-size squares to find the total number of them.	Partition (DOK - 3) Count (DOK - 1)

Essential Questions	**Big ideas**

• How can you use counting strategies or pairs to determine if a number is odd or even? • How can rectangles be divided into equal sections? • How can addition be used to determine the number of objects in an array?	• Counting strategies and pairs can be used to identify odd and even numbers. • Rectangles can be partitioned into rows and columns of same size squares • Addition can be used to find the total number of objects in an array.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Lesson Exit tickets for each lesson	Application Problem Student Debriefs Problem Set Data	Exit Ticket Data Mid-Module Assessment End-of Module Assessment *See Table Below

*Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	2.OA.4
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	2.OA.3 2.OA.4 2.G.2

Performance Task (*To be completed by grade level team)
Overview:
Engaging Learning Experiences

Task 1:

Task 2:

Task 3:

Task 4:

Instructional Resources

Useful Websites:
Engage NY K-5 Curriculum overview and guiding documents:
https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents

Engage NY Grade 2 Resources:
https://www.engageny.org/resource/grade-2-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 2nd Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/2nd.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Books:
Math Attack by Joan Horton
Math Appeal by Greg Tang

Each Orange Had Eight Slices by Paul Giganti Jr.

Suggested Tools and Representations

- Counters
- Number bond
- Number path
- Personal white board
- Rectangular array
- Square tiles

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills

- Critical thinking and problem solving
- Collaboration and leadership
- Agility and adaptability
- Initiative and entrepreneurialism
- Effective oral and written communication
- Accessing and analyzing information
- Curiosity and imagination

Marzano's Nine Instructional Strategies for Effective Teaching and Learning

1. Identifying Similarities and Differences: helps students understand more complex problems by analyzing them in a simpler way

2. Summarizing and Note-taking: promotes comprehension because students have to analyze what is important and what is not important and put it in their own words

3. Reinforcing Effort and Providing Recognition: showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard.

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Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage.

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Throughout A Story of Units, teachers are encouraged to give classwork utilizing a "time frame" rather than a "task frame." Within a given time frame, all students are expected to do their personal best, working at their maximum potential. "Students, you have 10 minutes to work independently." Bonus questions are always ready for accelerated students. The teacher circulates and monitors the work, error-correcting effectively

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms ● Array (an arrangement of objects in rows and columns) ● Columns (the vertical groups in a rectangular array) ● Even number (a whole number whose last digit is 0, 2, 4, 6, or 8) ● Odd number (any number that is not even) ● Repeated addition (e.g., 2 + 2 + 2) ● Rows (the horizontal groups in a rectangular array) ● Tessellation (tiling of a plane using one or more geometric shapes with no overlaps and no gaps) ● Whole number (e.g., 0, 1, 2, 3…) **Familiar Terms and Symbols** ● Addends ● Doubles ● Equation ● Number path ● Number sentence ● Pair ● Rectangle ● Skip-counting ● Square ● Sum ● Tape diagram ● Total ● Unit	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. “I do, we do, you do.” ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. ● Model and post conversation “starters,” such as: “I agree because...” “Can you ● explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition,

	● Encourage students to explain their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **Provide Multiple Means of Engagement** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **Provide Multiple Means of Engagement** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Problem Solving with Length, Money, and Data

OVERVIEW

Module 7 presents an opportunity for students to practice addition and subtraction strategies within 100 and problem-solving skills as they learn to work with various types of units within the contexts of length, money, and data. Students represent categorical and measurement data using picture graphs, bar graphs, and line plots. They revisit measuring and estimating length from Module 2 but now use both metric and customary units.

Module 7 opens with students representing and interpreting categorical data. In Grade 1, students learned to organize and represent data with up to three categories. Now, in Grade 2, students build upon this understanding by drawing both picture and bar graphs (**2.MD.10**). First, they record category counts in a table, solving problems based on the information in the table. Next, they draw picture graphs in which each picture represents one object. Finally, they represent the same data set in the form of a bar graph, where one axis names the categories, and the other shows a single-unit count scale. Students use the information to solve *put-together, take-apart,* and *compare* problems (**2.MD.10**), making connections to finding sums and differences on a number line diagram. In the final lesson of Topic A, students display money data in the form of a bar graph, thus establishing a connection to word problems with coins in Topic B.

In Topic B, students work with the most popular units of all: bills and coins. Students apply their knowledge of coin values, place value strategies, and the properties of operations to solve addition and subtraction word problems (**2.NBT.5**, **2.MD.8**) to find the total value of a group of coins or bills. Next, they use coins to find multiple ways to represent the same quantity, sometimes using the fewest number of coins. Students then focus on the decomposition of a dollar, where they see that this unit behaves like all others they have seen before (e.g., 100 ones = 1 hundred, 100 cm = 1 m). Students learn how to make change from one dollar using counting on, simplifying strategies (e.g., number bonds), and the relationship between addition and subtraction. As students use coins or bills to solve addition and subtraction word problems within 100, they use drawings and equations to represent the unknown in various situations. The Application Problems throughout this module include solving two-step word problems involving two-digit money amounts (e.g., $28 + $47 or 28¢ + 47¢), as students use this new context to increase fluency with addition and subtraction within 100 (**2.NBT.5**).

After the Mid-Module Assessment, Topic C reviews the measurement concepts and skills presented in Module 2, now with a focus on customary units. Students deepen their understanding of a *length unit* as they lay one-inch square tiles end-to-end to create simple inch rulers, just as they created centimeter rulers in Module 2. They see again that the smaller the unit, the more iterations are necessary to cover a given distance. Students measure the length of various objects with their new unit rulers (**2.MD.1**), applying important concepts such as the understanding that the zero point on a ruler is the beginning of the total length and the number on a ruler means the distance covered by that number of length units.

In Topic D, students apply their measurement skills and knowledge of the ruler to measure a variety of objects using the appropriate measurement tools, such as inch rulers and yardsticks, just as they measured with centimeter rulers, meter sticks, and meter tapes in Module 2 (**2.MD.1**). Students thereby add to their bank of benchmark lengths, such as an inch being the distance across a quarter. By doing so, students develop mental images of an inch, a foot, or a yard, which empowers them to estimate a given length (**2.MD.3**).

In addition, in Topic D, students measure objects using both metric and customary length units, thereby developing an understanding of how the number of units needed depends upon the size of the unit chosen (**2.MD.2**). As in Topic C, students recognize, for example, that the smaller the length unit, the more iterations are necessary to cover a given distance. Topic D concludes with students measuring to determine how much longer one object is than another (**2.MD.4**). Students use addition and subtraction to compare two lengths, subtracting the length of the shorter object from the length of the longer object to determine the difference (e.g., 40 in – 35 in = 5 in, or 35 in + _____ = 40 in).

Whereas in Topic D students used rulers to compare lengths, in Topic E, students use drawings (e.g., tape diagrams and number bonds) and equations with an unknown to represent addition and subtraction word problems (**2.MD.5**). Once they have a solid conceptual understanding of length, students are ready to represent whole numbers as lengths on a number line (**2.MD.6**) and apply their knowledge of the ruler to a number line diagram. In Topic E, they are asked to identify unknown numbers on a number line by using place value, reference points (e.g., 5, 10, 25, and 50), and the distance between points. Students are also asked to represent two-digit sums and differences using the number line as a measurement model for combining and comparing lengths.

Topic F follows naturally, with students generating measurement data and representing it with a line plot (**2.MD.9**). Students position data along a horizontal scale with whole number markings, drawn as a number line diagram (**2.MD.6**). Since students are working with length, the scale on their line plots corresponds to the scale on their rulers. After generating measurement data, students create line plots from different data sets, and then they discuss and interpret the results.

Unit 7: Problem Solving with Length, Money, and Data

Subject: Mathematics

Grade/Course: Grade 2

Pacing: 30 days

Unit of Study: Unit 7: Problem Solving with Length, Money, and Data

Priority Standards:

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Measure and estimate lengths in standard units.

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.MD.3 Estimate lengths using inches, feet, centimeters, and meters.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Foundational Standards:

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number

of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps*.

1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See CCS Glossary, Table 1.)

2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

Math Practice Standards:

MP.1 **Make sense of problems and persevere in solving them.** Students draw to determine the part–whole relationships embedded within various word problem types, and based on their analysis, they persevere to use various addition and subtraction strategies to solve problems. They then persist in making a statement of the solution to answer the question in the original context. In this module, the problem-solving contexts involve length, money, and data.

MP.2 **Reason abstractly and quantitatively.** Students compare measurements using rulers, tape diagrams, and graphs. After they abstract the number of units or length measurements to calculate differences, they reinterpret the difference using the given units within a problem. Students also abstract the value from a set of coins to find the total value and then express that value once again in terms of dollars or cents.

MP.4 **Model with mathematics.** Students create drawings (e.g., tape diagrams) and write equations to model and solve word problems involving units of length, money, and data. Students use appropriate representations (e.g., line plot, bar graph, and picture graph) to visually display data. Students also use the number line to understand numbers and their relationships and to represent sums and differences within 100. Students organize their thinking about money by modeling with dollars and coins to solve addition and subtraction word problems.

MP.5 **Use appropriate tools strategically.** Students apply their measurement skills and knowledge of the ruler to measure a variety of objects using the appropriate measurement tools, such as inch rulers and yardsticks. When conventional measurement tools are not available, students make decisions about which resources might be helpful, such as using iteration with a shoe, a book, or a lima bean, while recognizing the limitations of such tools.

MP.6 **Attend to precision.** Students attend to precision when they iterate a physical unit to create inch rulers. They align the zero point on a ruler as the beginning of the total length, and they use various measurement tools and precise language to describe their experience: "I used an inch as the length unit." Students learn estimation strategies for measurement and make closer and closer approximations to the actual length. They assign specific values to different coins and count up, starting with the largest value. Students generate and represent data in a bar graph, picture graph, or line plot, labeling axes appropriately and specifying the unit of measure.

"Unwrapped" Standards *Most Important to this Unit

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Concepts - What Students Need to Know (context)	**Skills - What Students Need to Be Able to Do (Depth of Knowledge Level)**
Using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (within 100)	Fluently Add and Subtract (DOK - 2)
The length of an object	Measure (DOK - 1)
appropriate tools	Select (DOK - 2)
appropriate tools	Use (DOK - 1)
How two measurements relate to the size of the unit chosen (using units of different lengths for the two measurements)	Describe (DOK - 3)
whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …,	Represent (DOK - 2)
whole-number sums and differences within 100 on a number line diagram.	Represent (DOK - 2)
Word problems (involving money) (using $ and ¢ symbols)	Solve (DOK - 2)
A picture graph and A bar graph (with single-unit scale) to represent a data set (with up to four categories)	Draw (DOK - 2)
Problems using information presented in a bar graph	

(simple put-together, take-apart, and compare)	Solve (DOK - 2)

Essential Questions	**Big ideas**
● How can data be displayed? ● How can graphs help us solve problems? ● How can you show equivalent amounts? ● How can money be represented? ● How do you know you have the right amount of money? ● What strategies can you use to solve a word problem? ● Why do you measure? ● How do you measure an object? ● Why do we use standard units of measurement? ● How does the size of a unit relate to a measurement?	● Line plots, pictographs, and bar graphs can be used to represent measurement data ● Data within graphs can be used to solve problems ● Equivalent amounts of money can be composed in different combinations ● There are many models and strategies for solving problems ● Measurement describes attributes of objects ● Standard units allow people to compare and understand measurement ● The size of a unit should be chosen deliberately

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Lesson Exit tickets for each lesson	Application Problem Student Debriefs Problem Set Data	Exit Ticket Data Mid-Module Assessment End-of Module Assessment *See Table Below

*Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	2.NBT.5 2.MD.8 2.MD.10
End-of-Module Assessment Task	After Topic F	Constructed response with rubric	2.NBT.5 2.MD.1 2.MD.2 2.MD.3 2.MD.4 2.MD.5 2.MD.6 2.MD.8 2.MD.9 2.MD.10

Performance Task (See Appendix A)
Overview: The students are planning and end of the year party and trying to convince the class to pick theirs!
Engaging Learning Experiences

Task 1: The students survey the class about party themes and graph their results

Task 2: The students analyze the information in their graphs to make decisions about their party.

Task 3: The students decide on items to purchase and create a budget for their party

Task 4: The student synthesize their work in the other tasks to create a poster for their party and present it to the class.

Instructional Resources

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Engage NY K-5 Curriculum overview and guiding documents:
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Books:
Follow The Money by Loreen Leedy
The Penny Pot by Stuart J. Murphy
The Great Graph Contest by Loreen Leedy
Lemonade For Sale by Stuart J. Murphy
Tiger Math by Ann Whitehead Nagda
Is a Blue Whale the Biggest Thing There Is? by Robert Wells

Suggested Tools and Representations

- Bar graph
- Centimeter cube
- Centimeter ruler
- Dice
- Grid paper
- Inch and centimeter ruler
- Inch tiles
- Line plot
- Measuring tape
- Meter stick
- Money (i.e., dollars, coins)
- Number bond
- Number line
- Personal white board
- Picture graph
- Table
- Tape diagram
- Yardstick

Instructional Strategies	Meeting the Needs of All Students

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much positive feedback, however, the method in which you give that feedback should be varied. **8. Generating and Testing Hypotheses**: it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. **9. Cues, Questions, and Advanced Organizers**: helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	circulates and monitors the work, error-correcting effectively and wisely. Some students will complete more work than others. Neither above nor below grade level students are overly praised or penalized. Personal success is what we are striving for. Another vitally important component for meeting the needs of all students is the constant flow of data from student work. A Story of Units provides daily tracking through "exit tickets" for each lesson as well as mid- and end-of-module assessment tasks to determine student understanding at benchmark points. These tasks should accompany teacher-made test items in a comprehensive assessment plan. Such data flow keeps teaching practice firmly grounded in student learning and makes incremental forward movement possible. A culture of "precise error correction" in the classroom breeds a comfort with data that that is non-punitive and honest. When feedback is provided with emotional neutrality, students understand that making mistakes is part of the learning process. "Students, for the next five minutes I will be meeting with Brenda, Scott, and Jeremy. They did not remember to rename the remainder in the tens place as 10 ones in their long division on Question 7." Conducting such sessions then provides the teacher the opportunity to quickly assess if students need to start at a simpler level or just need more monitored practice now that their eyes are opened to their mistake. Good mathematics instruction, like any successful coaching, involves demonstration, modeling, and lots of intelligent practice. In math, just as in sports, skill is acquired incrementally; as the student acquires greater skill, more complexity is added and proficiency grows. The careful sequencing of the mathematics and the many scaffolds that have been designed into A Story of Units makes it an excellent curriculum for meeting the needs of all students, including those with special and unique learning modes.

Scaffolds for Students with Disabilities[4]

Individualized education programs (IEP)s or Section 504 Accommodation Plans should be the first source of information for designing instruction for students with disabilities. The following chart provides an additional bank of suggestions within the Universal Design for Learning framework for strategies to use with these students in your class. Variations on these scaffolds are elaborated at particular points within lessons with text boxes at appropriate points, demonstrating how and when they might be used.

Provide Multiple Means of Representation

- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding.

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms ● Bar graph ● Category (group of people or things sharing a common characteristic, e.g., bananas are in the fruit category) ● Data (a set of facts or pieces of information) ● Degree (used to measure temperature, e.g., degrees Fahrenheit) ● Foot (ft, unit of length equal to 12 inches) ● Inch (in, unit of length) ● Legend (notation on a graph explaining what symbols represent) ● Line plot (graphical representation of data—pictured to the right) ● Picture graph (representation of data like a bar graph, using pictures instead of bars) ● Scale (a number line used to indicate the various quantities represented in a bar graph) ● Survey (collecting data by asking a question and recording responses) ● Symbol (picture that represents something else) ● Table (representation of data using rows and columns) ● Thermometer (tool used to measure temperature) ● Yard (yd, unit of length equal to 36 inches or 3 feet) ● Picture Graph **Familiar Terms and Symbols** ● Benchmark (e.g., round numbers like multiples of 10) ● Centimeter (cm, unit of length measure) ● Cents (e.g., 5¢) ● Coins (e.g., penny, nickel, dime, and quarter) ● Compare ● Compose	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. “I do, we do, you do.” ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** ● Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. ● Model and post conversation “starters,” such as: “I agree because...” “Can you ● explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.

● Decompose ● Difference ● Dollars (e.g., $2) ● Endpoint ● Equation ● Estimation (an approximation of the value of a quantity or number) ● Hash mark (the marks on a ruler or other measurement tool) ● Height ● Length ● Length unit ● Meter (m, unit of length measure) ● Meter strip, meter stick ● Number bond ● Number line (a line marked at evenly spaced intervals) ● Overlap (extend over or cover partly) ● Ruler ● Tally mark ● Tape diagram ● Unit ● Value	● Encourage students to explain their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Time, Shapes, and Fractions as Equal Parts of Shapes

OVERVIEW

In Module 8, the final module of the year, students extend their understanding of part–whole relationships through the lens of geometry. As students compose and decompose shapes, they begin to develop an understanding of unit fractions as equal parts of a whole.

In Topic A, students build on their prior knowledge of a shape's defining attributes (**1.G.1**) to recognize and draw categories of polygons with specified attributes: the number of sides, corners, and angles (**2.G.1**). For example, students see that a rectangle has four straight sides, four right angles, and opposite sides with equal length. Students then relate the square, a special rectangle, to the cube by building a cube from six congruent squares. They describe the cube in terms of its attributes, counting the number of edges, faces, and corners (**2.G.1**). Once students are able to describe and analyze polygons and the cube according to their attributes in Topic A, they are ready to combine shapes and build composite shapes in Topic B.

Topic B opens with students using a tangram, a set of seven shapes that compose a square, to create a new shape. Students see that they can arrange two-dimensional shapes to create a new whole, or composite, shape, which can become part of an even larger whole. As students progress through the topic, they build and partition shapes by combining two or more smaller shapes and relating the parts to the whole. For example, they use different pattern blocks to show that a regular hexagon might be composed of two trapezoids or three rhombuses. One might say, "This hexagon is made from two identical trapezoids, or two equal parts." This allows for interpreting equal shares of a whole as a fraction, as students name the equal parts *halves*, *thirds*, or *fourths* (**2.G.3**).

Next, in Topic C, students decompose circles and rectangles into equal parts and describe them as halves (a half of), thirds (a third of), and fourths (a fourth of) or quarters (**2.G.3**). For example, students see that a circle can be partitioned into four quarter-circles, or parts, which can be described as fourths. They learn to describe the whole by the number of equal parts, e.g., one whole circle is composed of 4 fourths. Finally, students decompose a rectangle into four parts that have equal area but different shapes (**2.G.3**).

The module closes with Topic D, where students apply their understanding of partitioning the whole into halves and fourths to tell time to the nearest five minutes (**2.G.3**, **2.MD.7**), using both analog and digital clocks. They construct simple clocks and see the relationship to partitioning a circle into quarters and halves, thereby decomposing 60 minutes. For example, 3 fourths of the circle can be interpreted as 3 intervals of 15 minutes, e.g., 15 + 15 + 15 = 45 (**2.NBT.5, 2.NBT.6**), or 45 minutes. They also use their understanding of skip-counting by fives and tens to tell time on an analog clock (**2.NBT.2**). Finally, they apply their learning by calculating time intervals of hours and half hours and close the year determining the time interval in days before they are third-graders.

Rigorous Curriculum Design Template

Unit 8: Time, Shapes, and Fractions as Equal Parts of Shapes

Subject: Mathematics

Grade/Course: Grade 2

Pacing: 20 Days

Unit of Study: Unit 8: Time, Shapes, and Fractions as Equal Parts of Shapes

Priority Standards:

Work with time and money.

2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

Reason with shapes and their attributes.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)

2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Foundational Standards:

1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks. Recognize and identify coins, their names, and their values.

1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.")

1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

2.NBT.2 Count within 1000; skip-count by 5s[3], 10s, and 100s.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Math Practice Standards:

MP.1 **Make sense of problems and persevere in solving them.** Students are encouraged to persevere when arranging shapes to create specific composite shapes, when recomposing the pieces into different shapes, and when creating even larger shapes from composite shapes. When students partition composite shapes (e.g., circles and rectangles) into equal shares, they ask themselves, "How can I look at this differently?" Students organize their thinking through drawing, and they see, for example, that a circle can be described in terms of halves, thirds, or fourths.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students use drawings and precise language to describe and analyze shapes, and they defend their reasoning as to what makes a quadrilateral, for example, a rhombus. Students also discuss the partitioning of a composite shape (e.g., a hexagon) and relate the different parts, or shares, to halves, thirds, and fourths. They make connections between fraction concepts and telling time, explaining the connection between their work with halves and quarters to the analog clock.

MP.6 **Attend to precision.** Students describe and analyze various two-dimensional shapes by attending to their specific attributes. Students accurately draw shapes using their knowledge of attributes and rulers. Then, while working with a partner, students name and analyze their partner's shape drawings by counting the number of sides or angles. Students also appropriately name parts of a whole using terms such as *halves, thirds,* and *fourths* or *quarters*.

MP.7 **Look for and make use of structure.** Students identify attributes, such as the number of sides and angles, in order to classify shapes such as triangles and quadrilaterals. They make use of the part–whole structure to understand that a whole unit can be partitioned into equal shares, or smaller units (e.g., each of 4 equal shares = a fourth of the whole). Students use their understanding of the partitioning of a circle to tell time to the quarter and half hour. Through previous fluency practice, students use the pattern of skip-counting by fives to tell time on an analog clock.

"Unwrapped" Standards *Most important to this unit	
Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.) Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	
Concepts - What Students Need to Know (context)	**Skills - What Students Need to Be Able to Do (Depth of Knowledge Level)**
Circles and rectangles into two, three, or four equal shares	Partition (DOK - 2)
the shares using the words *halves, thirds, half of, a third of,* etc., and the whole as two halves, three thirds, four fourths.	Describe (DOK - 2)
Equal shares of identical wholes need not have the same shape.	Recognize (DOK - 3)
Shapes having specified attributes, such as a given number of angles or a given number of equal faces.	Recognize and Draw (DOK - 2)
Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	Identify (DOK - 1)

Essential Questions	Big ideas
● How can you describe the specific attributes of shapes? ● How can you compose or decompose a shape using equal shares? ● How can circles and rectangles be used to explain fractional parts? ● How are fractions useful in telling time?	● There is specific language to describe the attributes of shapes. ● Composing and decomposing shapes in equal shares is related to fractional parts ● Fractions and Time are related

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments
Lesson Exit tickets for each lesson	Application Problem Student Debriefs Problem Set Data	Exit Ticket Data Mid-Module Assessment End-of Module Assessment *See Table Below

*Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	2.G.1 2.G.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	2.MD.7 2.G.1 2.G.3

Performance Task (*To be completed by grade level team)
Overview:
Engaging Learning Experiences

Task 1:

Task 2:

Task 3:

Task 4:

Instructional Resources

Useful Websites:
Engage NY K-5 Curriculum overview and guiding documents:
https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents

Engage NY Grade 2 Resources:
https://www.engageny.org/resource/grade-2-mathematics

Eureka Math Module PDFs:
http://greatminds.net/maps/math/module-pdfs

North Carolina 2nd Grade Standards Unpacked:
http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/2nd.pdf

Illustrative Mathematics – problems and tasks by grade and standard
https://www.illustrativemathematics.org/

NCTM Illuminations – problems, tasks and interactives by grade and standard
http://illuminations.nctm.org/Default.aspx

Inside Mathematics – Problems of the Month and Performance Assessment tasks
http://www.insidemathematics.org/

LearnZillion –lesson plans/some with embedded tasks
https://learnzillion.com/resources/17132

SBAC Digital Library

Books:
The Greedy Triangle by Marilyn Burns
Grandfather Tang's Story by Ann Tompert
A Cloak for the Dreamer by Aileen Friedman
Bats Around the Clock by Kathi Appelt

Suggested Tools and Representations

- Cube: three-dimensional shape (real world examples such as a die, alphabet blocks, or a box)
- Geoboards
- Large instructional geared clock
- Pattern blocks

- Rulers
- Spaghetti
- Square tiles
- Student clocks, preferably those with gears which can provide the appropriate hour-hand alignment
- Toothpicks

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills ● Critical thinking and problem solving ● Collaboration and leadership ● Agility and adaptability ● Initiative and entrepreneurialism ● Effective oral and written communication ● Accessing and analyzing information ● Curiosity and imagination **Marzano's Nine Instructional Strategies for Effective Teaching and Learning** **1. Identifying Similarities and Differences:** helps students understand more complex problems by analyzing them in a simpler way **2. Summarizing and Note-taking:** promotes comprehension because students have to analyze what is important and what is not important and put it in their own words **3. Reinforcing Effort and Providing Recognition:** showing the connection between effort and achievement helps students helps them see the importance of effort and allows them to change their beliefs to emphasize it more. Note that recognition is more effective if it is contingent on achieving some specified standard. **4. Homework and Practice**: provides opportunities to extend learning outside the classroom, but should be assigned based on relevant grade level. All homework should have a purpose and that purpose should be readily evident to the students. Additionally, feedback should be given for all homework assignments. **5. Nonlinguistic Representations:** has recently been	The modules that make up A Story of Units propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. We encourage teachers to pay particular attention to the manner in which knowledge is sequenced in A Story of Units and to capitalize on that sequence when working with special student populations. Most lessons contain a suggested teaching sequence that moves from simple to complex,

proven to stimulate and increase brain activity. **6. Cooperative Learning:** has been proven to have a positive impact on overall learning. Note: groups should be small enough to be effective and the strategy should be used in a systematic and consistent manner. **7. Setting Objectives and Providing Feedback:** provide students with a direction. Objectives should not be too specific and should be adaptable to students' individual objectives. There is no such thing as too much positive feedback, however, the method in which you give that feedback should be varied. **8. Generating and Testing Hypotheses:** it's not just for science class! Research shows that a deductive approach works best, but both inductive and deductive reasoning can help students understand and relate to the material. **9. Cues, Questions, and Advanced Organizers:** helps students use what they already know to enhance what they are about to learn. These are usually most effective when used before a specific lesson.	starting, for example, with an introductory problem for a math topic and building up inductively to the general case encompassing multifaceted ideas. By breaking down problems from simple to complex, teachers can locate specific steps that students are struggling with or stretch out problems for students who desire a challenge. Throughout A Story of Units, teachers are encouraged to give classwork utilizing a "time frame" rather than a "task frame." Within a given time frame, all students are expected to do their personal best, working at their maximum potential. "Students, you have 10 minutes to work independently." Bonus questions are always ready for accelerated students. The teacher circulates and monitors the work, error-correcting effectively and wisely. Some students will complete more work than others. Neither above nor below grade level students are overly praised or penalized. Personal success is what we are striving for. Another vitally important component for meeting the needs of all students is the constant flow of data from student work. A Story of Units provides daily tracking through "exit tickets" for each lesson as well as mid- and end-of-module assessment tasks to determine student understanding at benchmark points. These tasks should accompany teacher-made test items in a comprehensive assessment plan. Such data flow keeps teaching practice firmly grounded in student learning and makes incremental forward movement possible. A culture of "precise error correction" in the classroom breeds a comfort with data that that is non-punitive and honest. When feedback is provided with emotional neutrality, students understand that making mistakes is part of the learning process. "Students, for the next five minutes I will be meeting with Brenda, Scott, and Jeremy. They did not remember to rename the remainder in the tens place as 10 ones in their long division on Question 7." Conducting such sessions then provides the teacher the opportunity to quickly assess if students need to start at a simpler level or just need more monitored practice now that their eyes are opened to their mistake. Good mathematics instruction, like any successful coaching, involves demonstration, modeling, and lots of intelligent practice. In math, just as in sports, skill is acquired incrementally; as the student acquires greater skill, more complexity is added and proficiency grows. The careful sequencing of the mathematics and the many scaffolds that have been designed into A Story of Units makes it an excellent curriculum for meeting the needs of all students, including those with special and unique learning modes.

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- Teach from simple to complex, moving from concrete to representation to abstract at the student's pace.
- Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
- Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.
- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

Provide Multiple Means of Action and Expression

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_____ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting."
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are "just right" for learners.
- Model each step of the algorithm before students begin.
- Give students a chance to practice the next day's sprint beforehand. (At home, for example.)
- Give students a few extra minutes to process the information before giving the signal to respond.
- Assess by multiple means, including "show and tell" rather than written.
- Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?"
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?"
- Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!"
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow "everyday" and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding.

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard

New or Recently Introduced Terms ● a.m./p.m. ● Analog clock ● Angle (e.g., figure formed by the corner of a polygon) ● Digital clock ● Parallel (two lines on the same plane are parallel if they do not intersect) ● Parallelogram (quadrilateral with both pairs of opposite sides parallel) ● Polygon (closed figure with three or more straight sides, e.g., triangle, quadrilateral, pentagon, hexagon) ● Quadrilateral (four-sided polygon, e.g., square, rhombus, rectangle, parallelogram, trapezoid) ● Quarter past, quarter to ● Right angle (e.g., a square corner) ● Second (unit for measuring time) ● Third of (shapes), thirds (three equal shares) ● Whole ■ 2 halves ■ 3 thirds ■ 4 fourths **Familiar Terms and Symbols** ● Attributes (characteristics of an object such as number of sides, angles, or faces) ● Cube (three-dimensional shape composed of six squares) ● Face (a two-dimensional side of a three-dimensional shape) ● Fourth of (shapes), fourths (four equal shares) ● Half of (shapes), halves (two equal shares) ● Half past (expression for 30 minutes past a given hour) ● Half hour (interval of time lasting 30 minutes) ● Hour (unit for measuring time,	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are below grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. ● Guide students as they select and practice using their own graphic organizers and models to solve. ● Use direct instruction for vocabulary with visual or concrete representations. ● Use explicit directions with steps and procedures enumerated. ● Guide students through initial practice promoting gradual independence. “I do, we do, you do.” ● Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. ● Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** ● First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. ● Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ ● Encourage students to explain	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** ● Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. ● Model and post conversation “starters,” such as: “I agree because...” “Can you ● explain how you solved it?”“I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” ● Incorporate written reflection, evaluation, and synthesis. ● Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** ● Encourage students to explain their reasoning both orally and in writing. ● Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. ● Offer choices of independent or group assignments for early finishers. ● Encourage students to notice and explore patterns and to identify rules and relationships in math. ● Have students share their observations in discussion and writing (e.g., journaling). ● Foster their curiosity about numbers and mathematical ideas. ● Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. ● Have students compete in a secondary simultaneous competition,

equivalent to 60 minutes or 1/24 of a day) ● Minute (unit for measuring time, equivalent to 60 seconds, 1/60 of an hour) ● O'clock (used to indicate time to a precise hour with no additional minutes) ● Two-dimensional shapes (familiar prior to Grade 2): ■ Circle ■ Half-circle ■ Quarter-circle ■ Hexagon (2 dimensional figure enclosed by six straight sides and six angles) ■ Rectangle (2 dimensional figure enclosed by four straight sides and four right angles) ■ Rhombus (2 dimensional figure enclosed by four straight sides of the same length) ■ Square (rectangle with four sides of the same length) ■ Trapezoid (2 dimensional figure enclosed by four straight sides with only one pair of parallel sides) ■ Triangle (2 dimensional figure enclosed by three straight sides) ● Quarter of (shapes), quarters (4 equal shares)	their thinking and strategy for the solution. ● Choose numbers and tasks that are "just right" for learners but teach the same concepts. ● Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** ● Clearly model steps, procedures, and questions to ask when solving. ● Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). ● Have students work together to solve and then check their solutions. ● Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? ● Practice routine to ensure smooth transitions. ● Set goals with students regarding the type of math work students should complete in 60 seconds. ● Set goals with the students regarding next steps and what to focus on next.	such as skip-counting by 75s, while peers are completing the sprint. ● Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. ● Increase the pace. Offer two word problems to solve, rather than one. ● Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). ● Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. ● Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** ● Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. ● Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). ● Make the most of the fun exercises for practicing skip-counting. ● Accept and elicit student ideas and suggestions for ways to extend games. ● Cultivate student persistence in problem-solving and do not neglect their need for guidance and support

Appendix A: Performance Task

2nd Grade Unit 7 Performance Task- Plan a Party

Priority Standards:

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Unwrapped Standards:

- Solve (DOK - 2) Word problems (involving money) (using $ and ¢ symbols)
- Draw (DOK - 2) A picture graph and A bar graph (with single-unit scale) to represent a data set (with up to four categories)
- Solve (DOK - 2) Problems using information presented in a bar graph (simple put-together, take-apart, and compare)
- Fluently Add and Subtract (DOK - 2) Using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (within 100)

Interdisciplinary Content:

21st Century Skills
Financial Literacy
ELA

Engaging Scenario

Situation: Our class is going to have an end of the year party.
Challenge: You need to decide what type of party to have, how to spend the money for the party without going over your budget, convince your teacher and class to choose your party.
Student's Role: student
Audience: Teacher and Class

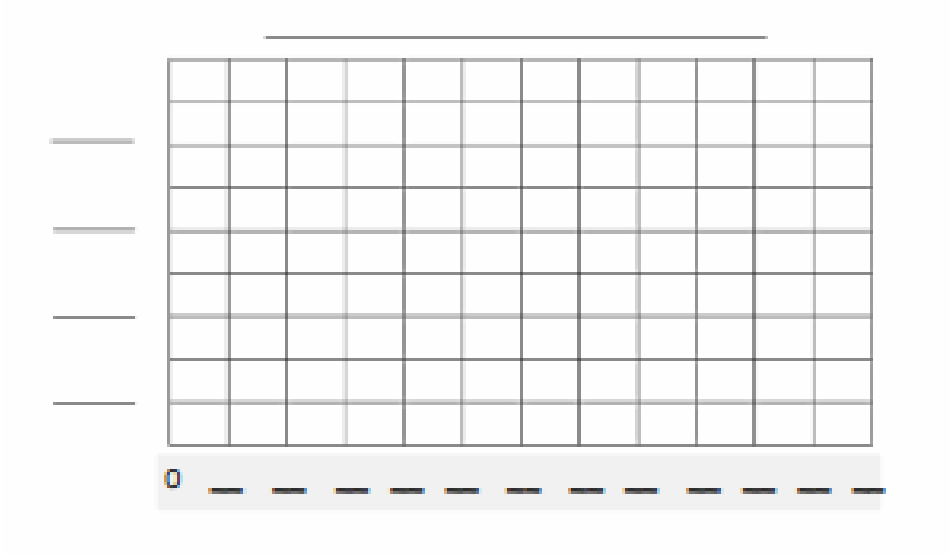
Task 1

Focus Standard: 2.MD.10
Description: Create a survey to decide on a party theme (For example: beach, movie, sports- you may want to brainstorm ideas with the students first about ideas for themes and what activities would go with each theme). Survey the class about the party. Be sure to survey all the students and the teacher. Use a bar graph or picture graph to display the results of your survey
DOK: 2 and 3
Resources:
Student Page (attached below)
Graphing Pages (Multiple Choices from Engage NY attached below- you may want to cut them apart and enlarge them)
Student Rubric (attached below)

Performance Assessment Scoring Rubric

Use this rubric to score students for each task. Be sure to provide feedback in student friendly language for each section

	Understanding	Planning and Execution	Communication	Perseverance
Exemplary 4	Shows complete understanding of the required mathematical knowledge. The solution completely addresses all mathematical components presented in the task.	Uses only the important elements of the task. Uses an appropriate and complete strategy for solving the problem. Uses only relevant information. Uses clear and effective diagrams, tables, charts and graphs.	There is a clear, effective explanation of the solution. All steps are included so the reader does not have to infer how the task was completed. Mathematical representation is actively used as a means of communicating ideas. There is precise and appropriate mathematical terminology and notation.	Works hard on the task and doesn't need much help. Student may extend his thinking beyond the problem and make new connections or create new problems.
Proficient 3	Shows nearly complete understanding of required mathematical knowledge. The solution addresses almost all of the mathematical components presented in the task. There may be minor errors.	Uses most of the important elements of the task. Uses an appropriate but incomplete strategy for solving the problem. Uses most of the relevant data. Appropriate but incomplete use of diagrams, tables, charts and graphs.	There is a clear explanation. There is appropriate use of accurate mathematical representation. There is effective use of mathematical terminology and notation.	Works hard on the task and only gets help after having tried many strategies given throughout. Completes task, working dutifully at the harder parts also.
Progressing 2	Shows some understanding of the required mathematical knowledge The solution addresses some, but not all the mathematical components presented in the task.	Uses some important elements of the task. Uses an inappropriate strategy or application of strategy is unclear. Uses some relevant data. Limited use or misuse of diagrams, tables, charts, and graphs.	There is an incomplete explanation; it may not be clearly represented. There is some use of appropriate mathematical representation. There is some use of mathematical notation appropriate to the task.	Can do simple parts of the problem with little help. Starts working on the harder parts, but unless there is help, gives up
Not yet Meeting 1	Shows limited or no understanding of the problem, perhaps only re-copying the given data. The solution addresses none of the mathematical components required to solve the task	Uses none of the important elements of the task. Works haphazardly with no particular strategy for solving the problem. Uses irrelevant data. Does not show use of diagrams, tables, charts or graphs	There is no explanation of the solution.The explanation cannot be understood, or is unrelated to the task. There is no use or inappropriate use of mathematical representations. There is no use, or mostly inappropriate use, of mathematical terminology and notation.	Needs help, even for the very simple tasks. Gives up quickly, often just wanting someone to give the answer.



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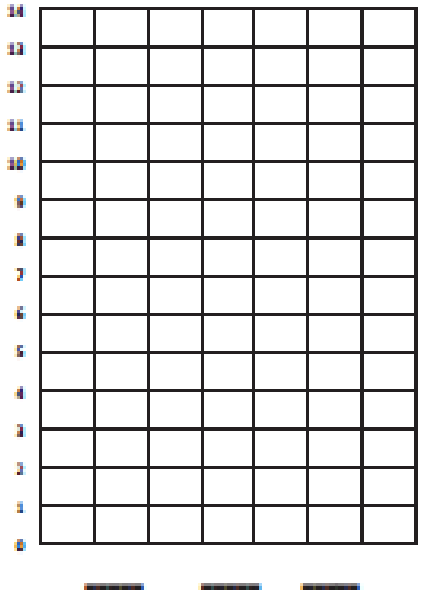
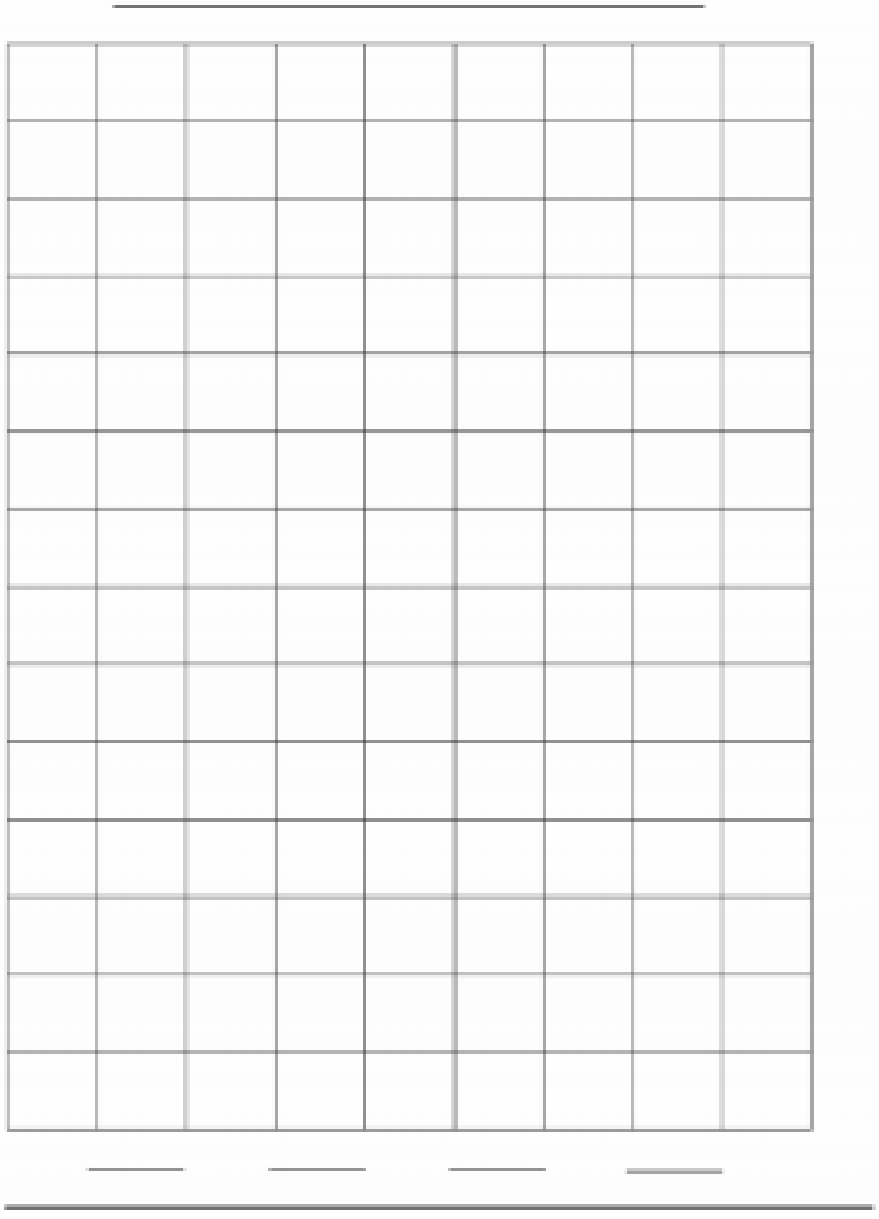


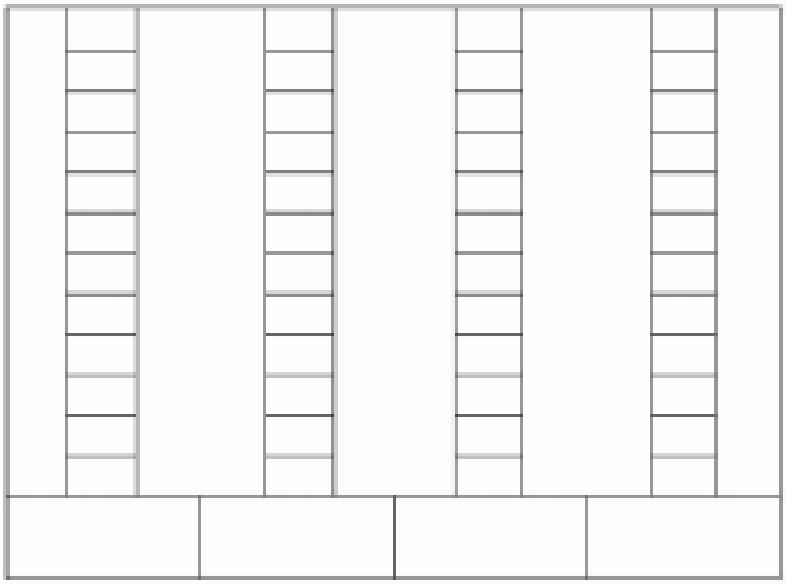
horizontal and vertical bar graphs



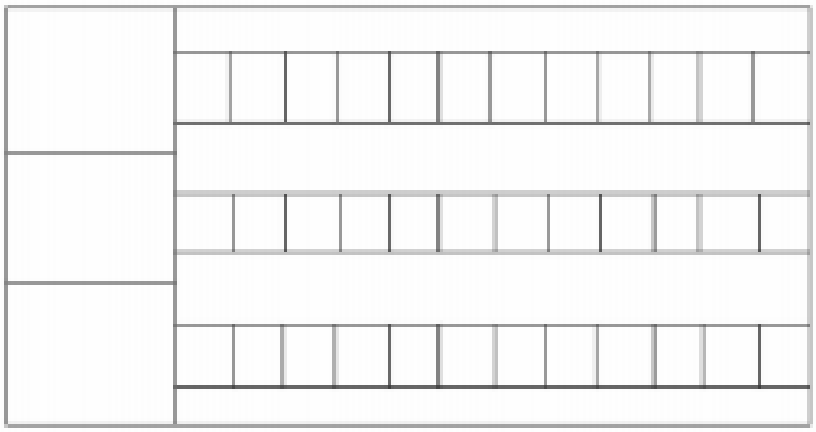
Legend: _____

_____

vertical picture graph



Legend: _____



Legend: _____

vertical and horizontal picture graphs

Name _____ Date _____

Performance Assessment Task - Planning a Party
Task 1: Party Theme

Directions:
Choose 3 or 4 themes that you would like for our party. Survey the whole class and the teacher about their favorite theme. Record your survey results in the table below.

Survey Results:

Think about your data and what type of graph would be best for displaying your results. Choose one of the graphing pages and create a graph that shows your results. Remember this graph will be used to help convince us to have your party. You want it to look good!

Explain what decisions you made about your graph and why.

_____

_____

_____

_____

_____

_____

_____

Name _____

Performance Assessment Task - Planning a Party
Scoring for Task 1: Party Theme

Self- Assessment Checklist

When doing this task I:

- ❏ Surveyed the whole class and my teacher about the themes
- ❏ Used a tally chart to keep track of the survey results
- ❏ Determined which graph would be best to show my results
- ❏ Created a graph to display the results of my survey
    - ❏ My graph is accurate
    - ❏ My graph has a title, labels, and a legend
- ❏ Explained my ideas clearly using math vocabulary
- ❏ Tried my best and revised my ideas when I needed to

Teacher Feedback:

Use the Performance Assessment Scoring Rubric to score students for each task. Be sure to provide feedback in student friendly language for each section.

Understanding:	1	2	3	4
Planning and Execution:	1	2	3	4
Communication:	1	2	3	4
Perseverance:	1	2	3	4

Comments:

_____

_____

_____

_____

_____

Task 2

Focus Standard: 2.MD.10

DOK: 1,2, and 3

Description: The students analyze their graph. They answer questions about it and use the data to make decisions about their party plan.

Resources:
Graph Created in Task 1
Student pages
Scoring Rubric (See task 1)

Name _____ Date _____

Performance Assessment Task - Planning a Party

Task 2: Party Theme Decision

Directions: Use the graph you made about the party theme to answer the questions and make decisions.

1. Did you survey the entire class and your teacher?

_____

Justify your answer using your graph and words or equations.

2. Which theme had the most votes? _____

3. Which theme had the least votes? _____

4. How many more students voted for your most popular theme than for your least popular? Show or explain your thinking.

Answer:_____

5. Looking your graph, which theme will you choose for your party?_____

Why? _____

6. Keeping in mind that you are trying to convince the class to pick your party. Look at the theme on your graph that had the most votes **and** the choices on your graph that were not the most popular. What activities could you do with the theme you picked to make even more people vote for your party? Remember the activities should not cost money!

List 3 activities you will have at your party and explain why you chose them.

Activity 1 _____

Explanation: _____

_____

_____

Activity 2 _____

Explanation: _____

_____

_____

Activity 3 _____

Explanation: _____

_____

_____

Name _____

Performance Assessment Task - Planning a Party
Scoring for Task 2: Party Theme Decision

Self- Assessment Checklist

When doing this task I:

- ❑ Used my graph to answer **all** the questions

- ❏ Showed my thinking clearly using drawings, models, equations, or words
- ❏ Explained my ideas clearly using math vocabulary
- ❏ Tried my best and revised my ideas when I needed to

Teacher Feedback:

Use the Performance Assessment Scoring Rubric to score students for each task. Be sure to provide feedback in student friendly language for each section.

Understanding:	1	2	3	4
Planning and Execution:	1	2	3	4
Communication:	1	2	3	4
Perseverance:	1	2	3	4

Comments:

_____

_____

_____

_____

_____

_____

Task 3

Focus Standards: 2.MD.8, 2.NBT.5

DOK: 2 and 3

Description: Now that students have a theme and 3 activities they will use shopping lists and a budget to determine what to purchase for their party.

Resources:

Student page

Price List

Budget Worksheet

Oriental Trading Catalogues or website

Scoring Rubric (See task 1)

Name _____ Date _____

Performance Assessment Task - Planning a Party

Task 3: Party Supplies

Directions: You have $45.00 to spend on the party. Use the price list and the Oriental Trading Catalogue or Website to decide what you will buy for your party.

You need to buy:

- ❏ at least one drink for each person in the class
- ❏ at least one food item for each person in the class
- ❏ at least one decoration
- ❏ Items that will match your party theme and make people vote for yours!

Use the bottom of this page and the back of this page to do your work.

When you have decided what you will buy, complete the budget worksheet.

Name _____

Performance Assessment Task - Planning a Party

Task 3: Price List

Drinks		Food	
24 Bottles of Water	$3.00	Party Size Pizza	$18.00
10 Juice Boxes	$3.00	24 Pack of Cookies	$4.75
10 Capri Suns	$2.50	24 Cupcakes	$10.00
Gallon of Fruit Punch	$2.00	12 Ice Cream Bars	$3.50
		30 Popsicles	$4.25
		Bag of Chips	$3.50
		Fruit Tray	$12.00
		Veggie Tray	$11.00
		Cheese and Crackers	$7.00
		Pop Corn	$3.00
Decorations		Other Items	

Balloons (bag of 12)	$2.50	16 Plates	$2.00
6 Helium Balloons	$10.00	16 Napkins	$1.50
Pinata with Candy	$18.00	30 pack of Cups	$3.50
Streamers	$2.50	10 pack of bubbles	$3.30
Theme Poster Set	$8.00	12 sticker pages	$5.00
Party Music CD	$7.25	18 Tootsie Pops	$3.00
		24 Assorted Prizes	$7.50

Name _____

Performance Assessment Task - Planning a Party

Scoring for Task 3: Budget Worksheet

Name of Item	Price	How many it comes with	How many you will buy	Total Price for item

The total cost for my party items is: _____

Show your work:

Name _____

Performance Assessment Task - Planning a Party
Scoring for Task 3: Party Supplies

Self- Assessment Checklist

When doing this task I:

- ❏ Bought each of the items I needed
- ❏ Did not spend more money than I had
- ❏ Chose items that would make my party the best
- ❏ Showed my thinking clearly using drawings, models, equations, or words
- ❏ Tried my best and revised my ideas when I needed to

Teacher Feedback:

Use the Performance Assessment Scoring Rubric to score students for each task. Be sure to provide feedback in student friendly language for each section.

Understanding:	1	2	3	4
Planning and Execution:	1	2	3	4
Communication:	1	2	3	4
Perseverance:	1	2	3	4

Comments:

_____

_____

_____

_____

_____

_____

Task 4

Focus Standards: 2.MD.10, 2.MD.8, 2.NBT.5

DOK: 4

Description: Create a poster that will convince your class and teacher to choose your party and present it to the class.

Resources:

Student Pages

Poster Paper

Art Supplies

Work from previous tasks

Scoring Rubric (See task 1)

Name _____ Date _____

Performance Assessment Task - Planning a Party
Task 4: Pick My Party

Directions: Now that you have used your data to decide on a theme and activities and budgeted for your party supplies, you need to convince everyone to pick your party! You are going to make a poster for your party and present it to the class. Then we will vote for our favorite party and you're not allowed to vote for your own!

When you make your poster be sure to include:

- ❏ The theme of the party
- ❏ The activities we will do
- ❏ The food that will be served
- ❏ Anything else that will convince people to pick your party
    - ❏ Remember to make it colorful and exciting!

When you present your poster to the class don't forget to:

- ❏ Explain how you chose your theme and activities
- ❏ Explain which items you chose to purchase
    - ❏ Why did you choose those items?
    - ❏ What is the total cost of your party?
- ❏ Be convincing!

Name _____

Performance Assessment Task - Planning a Party
Scoring for Task 4: Pick My Party

Self- Assessment Checklist

When doing this task I:

- ❏ Included all the parts in my poster
- ❏ Explained all the parts to the class
- ❏ Explained my ideas clearly using math vocabulary
- ❏ Used convincing words and pictures
- ❏ Tried my best and revised my ideas when I needed to

Teacher Feedback:

Use the Performance Assessment Scoring Rubric to score students for each task. Be sure to provide feedback in student friendly language for each section.

Understanding:	1	2	3	4
Planning and Execution:	1	2	3	4
Communication:	1	2	3	4
Perseverance:	1	2	3	4

Comments:

_____

_____

_____

_____

_____

_____

Appendix B: Three representative sample CFAs

Unit 4, Lesson 30 Exit Ticket used for pre-test and post-test:

Name _____ Date _____

1. Kevin solved 166 + 25 using totals below. Solve the same problem another way.

166 + 25 11 80 100 191	

2. Explain how Kevin's work and your work are similar.

_____

_____

_____

Unit 4 MId-Module Assessment:

Name _____ Date _____

1. Solve. Show your mental math strategy.

a. 35 + 25 = _____	b. _____ = 27 + 46	c. _____ = 19 + 73
d. 89 − 52 = _____	e. 61 − _____ = 32	f. 75 − _____ = 29
g. 32 $\xrightarrow{+1}$ _____ $\xrightarrow{+\_\_\_\_}$ 43	h. 60 $\xrightarrow{-\_\_\_\_}$ _____ $\xrightarrow{-\_\_\_\_}$ 49	i. _____ $\xrightarrow{+10}$ _____ $\xrightarrow{+1}$ 73

2. Solve and show your work with a model.

a. 116 + 74 = _____ Model:	b. 147 + 28 = _____ Model:

c.	d.
84 − 59 = _____ Model:	62 − 45 = _____ Model:

3. Label each as true or false. Use a place value strategy to show how you know.

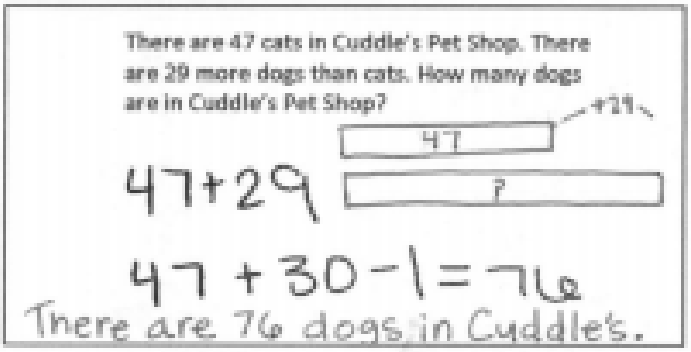
   a. 23 − 14 = 14 + 23 _____

   b. 45 − 19 = 22 + 4 _____

   c. 93 − 56 = 84 − 37 _____

   d. 8 ones + 5 tens = 85 _____

4. Sarah solved the word problem below.



a. Explain why Sarah's addition strategy worked.

b. There are 18 fewer cats than birds. How many birds are in Cuddle's Pet Shop? Use another place value strategy to find the answer. Show your work.

Name _____ Date _____

1. Solve mentally:

a. 72 + 10 = _____	b. _____ − 73 = 10	c. _____ + 10 = 174
d. 83 + 100 = _____	e. _____ − 182 = 100	f. _____ − 100 = 81
g. 65 + 40 = _____	h. _____ − 166 = 40	i. 127 + _____ = 167
j. 85 + 42 = _____	k. _____ − 186 = 41	l. 189 − 47 = _____

2. Solve:
   a. Find the solution and model how you found your answer.

87 + 56 =	Model:
38 + 68 + 71 + 12 =	Model:

b. Solve and explain your answer using place value.

91 − 24 =	154 + 27 =
105 − 42 =	86 + 45 =

c. Susan and James solved 125 + 32 in different ways. Explain why both ways are correct.

Susan's Way:	*James' Way:*
125 + 32 $125 \xrightarrow{+10} 135 \xrightarrow{+10} 145 \xrightarrow{+10} 155 \xrightarrow{+2} \boxed{157}$	125 + 32 $125 + 30 + 2 = \boxed{157}$
Explanation:	Explanation:

3. Find the missing numbers to make each statement true. Show your mental math strategy.

a. $98 \xrightarrow{-10} \_\_\_\_ \xrightarrow{+\_\_} 109$

b. 6 tens + 4 ones = 70 = _____

c. 25 + 75 = _____ + 30

d. 39 + _____ = 82

e. 100 = _____ = 45 + 15 + 32

4. Sally went shopping. She spent $86 on groceries and $39 on clothing.
   a. How much more did Sally spend on groceries than on clothing? Show your work.

   b. After Sally's shopping trip she had $12 left. How much money did she have to begin with? Show your work.

c. If Sally hadn't purchased the clothing would she have been able to afford a $55 necklace? Explain your answer.

d. How much money would Sally need to buy the groceries, clothing, and the necklace? Show your work with a model.

Appendix C: Three representative model lessons

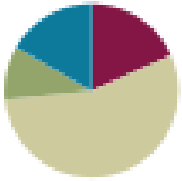
Unit 2, Lesson 7:

Lesson 7

Objective: Measure and compare lengths using standard metric length units and non-standard length units; relate measurement to unit size.

Suggested Lesson Structure

Fluency Practice	(11 minutes)
Application Problem	(6 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (11 minutes)

- Which Is Shorter? 2.MD.4 (2 minutes)
- Sprint: Subtraction 2.NBT.5 (9 minutes)

Which Is Shorter? (2 minutes)

Note: Students prepare for comparing lengths by identifying the shorter length and providing the number sentence to find the difference.

T: I am going to say two lengths. Tell me which length is shorter. Ready? 6 centimeters and 10 centimeters.

S: 6 centimeters.

T: Give the number sentence to find how much shorter.

S: 10 cm − 6 cm = 4 cm.

Continue with the following possible sequence: 12 cm and 22 cm, 16 cm and 20 cm, 20 cm and 13 cm, 20 cm and 9 cm, 9 cm and 19 cm, 24 cm and 14 cm, 12 cm and 24 cm, 23 cm and 15 cm, and 18 cm and 29 cm.

Sprint: Subtraction (9 minutes)

Materials: (S) Subtraction Sprint

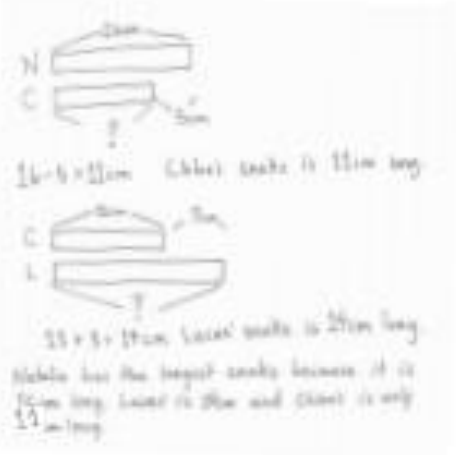
Note: Students practice their simple subtraction skills in preparation for the lesson content.

Application Problem (6 minutes)

Natalia, Chloe, and Lucas are making clay snakes. Natalia's snake is 16 centimeters long. Chloe's snake is 5 centimeters shorter than Natalia's. How long is Chloe's snake? Draw a picture and use numbers to explain your thinking.

Lucas's snake is 3 centimeters longer than Chloe's snake. Who has the longest snake: Natalia, Lucas, or Chloe? Add to your picture and use numbers to explain your thinking.

Note: This multiple-step problem presents a challenge for students to extend their understanding of measuring and comparing. Students are asked to connect addition and subtraction concepts to comparison language and to draw a conclusion.



Concept Development (33 minutes)

Materials: (S) Personal white board, 1 30 centimeter ruler (various types, e.g., wood, plastic, tape, etc.), 1 baggie per pair (containing 1 straw, 1 new crayon, 1 wedge eraser, 1 square sticky note, 30 paper clips)

Note: Prepare half of the baggies with small paper clips and half the baggies with large paper clips. Use only one size paper clip per table so partners don't see that they are different sizes.

T: Measure your straw with your paper clips.

S: (Measure.)

T: How long is the straw?

S: 6 paper clips long. → About 5 paper clips long.

T: (Record measurements on the board.)

MP.3 T: Why do you think the measurements are different? Turn and talk.

S: Maybe they didn't start at the beginning of the straw. → They measured wrong.

T: Take out your crayon and measure with your paper clips. Share your measurement with your partner.

Students continue to measure the other items in their baggies. After each item, discuss and record the unit measure (in paper clips) of each item. Notice that measurements are different, but do not explain why.

T: Let's switch baggies with our neighbors and measure again.

Tables now switch bags and measure all items in the baggie using the other size paper clip. Record

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Extend thinking by connecting to real world experiences. Ask students, "What are some other items you might use to measure your straw?" Students will identify objects that are easy to use as a measure: erasers, fingers, crayons, etc., either by using mark and move forward or by laying multiple copies.

measurements on the board. Have students discuss the difference between the measurements made using the large paper clips and those using the small paper clips.

T: Do you know why your measurements were different?

S: We had different size paper clips!

T: Why does the size of my paper clip matter?

S: You can fit more small paper clips than big paper clips along the side of the item.

T: What does that tell you about measurement and unit size?

MP.3

S: If it's a small unit size, you get a bigger measurement number.

T: Let's measure again using small and big paper clips mixed together.

S: (Use varying amounts of small and big paper clips to measure their straws.)

T: What were your results? (Record results.)

T: Why are all these measurements different?

S: We all had different sizes. → Some people had lots of big paper clips.

T: So, if I wanted to order a table and I told you I want it to be 80 paper clips long, what might happen?

S: They wouldn't know which one you want. → You could get a big table or a tiny table.

T: (Pass out different types of centimeter rulers, e.g., tape measures, wooden rulers, plastic rulers. Have students re-measure each object in their baggies. Record the measurements on the board in centimeters.)

T: What do you notice about the measurement of the object when you use a centimeter ruler?

S: The measurements for each object are the same even if the ruler looks different.

T: What is the same about all the rulers?

S: They are the same, except one is wood and one is plastic. → The rulers all have centimeters. → The centimeters are all the same size.

T: Why is it more efficient to measure with a centimeter instead of paper clips?

S: Because everyone knows how big a centimeter is. → All centimeters are the same.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Inverse relationships require thoughtful consideration because they seem to challenge logic and reasoning.

Post sentence frames for English language learners for reference during the Debrief:

"The _____ the unit, the _____ number of units in a given measurement."

Invite students to brainstorm real-life examples of inverse relationships. (e.g., The longer you sleep in the morning, the less time you have to get ready for school.)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

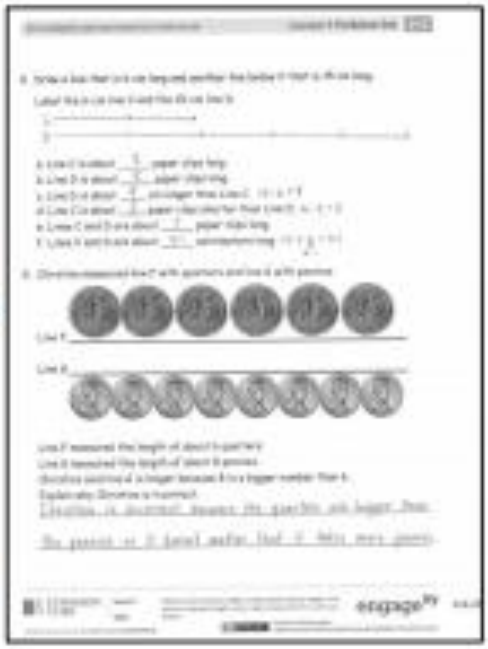
Lesson Objective: Measure and compare lengths using standard metric length units and non-standard length units; relate measurement to unit size.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Turn to your partner and compare your answers to Problems 1 and 2. Which math strategies did you use to determine which line was longer or shorter?
- Look at Problem 5. Turn and talk to your partner about why Christina's answer is incorrect.
- Do you think that paper clips are a reliable measurement tool? Is a ruler a better measurement tool? Why?
- What did you notice about the relationship between the unit of length (e.g., paper clips, centimeters) and the number of units needed to measure the lines? Use comparative words (*bigger, smaller, greater, fewer*) in your response.
- Let's think back to our Application Problem. Would it have been possible to accurately compare the lengths of the clay snakes with a non-standard length unit? What challenges can you predict?



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

A # Correct _____

Subtract.

1	3 - 1 =		23	8 - 7 =	
2	13 - 1 =		24	18 - 7 =	
3	23 - 1 =		25	58 - 7 =	
4	53 - 1 =		26	62 - 2 =	
5	4 - 2 =		27	9 - 8 =	
6	14 - 2 =		28	19 - 8 =	
7	24 - 2 =		29	29 - 8 =	
8	64 - 2 =		30	69 - 8 =	
9	4 - 3 =		31	7 - 3 =	
10	14 - 3 =		32	17 - 3 =	
11	24 - 3 =		33	77 - 3 =	
12	74 - 3 =		34	59 - 9 =	
13	6 - 4 =		35	9 - 7 =	
14	16 - 4 =		36	19 - 7 =	
15	26 - 4 =		37	89 - 7 =	
16	96 - 4 =		38	99 - 5 =	
17	7 - 5 =		39	78 - 6 =	
18	17 - 5 =		40	58 - 5 =	
19	27 - 5 =		41	39 - 7 =	
20	47 - 5 =		42	28 - 6 =	
21	43 - 3 =		43	49 - 4 =	
22	87 - 7 =		44	67 - 4 =	

B Improvement _____ # Correct _____

Subtract.

1	2 - 1 =		23	8 - 7 =	
2	12 - 1 =		24	18 - 7 =	
3	22 - 1 =		25	68 - 7 =	
4	52 - 1 =		26	32 - 2 =	
5	5 - 2 =		27	9 - 8 =	
6	15 - 2 =		28	19 - 8 =	
7	25 - 2 =		29	29 - 8 =	
8	65 - 2 =		30	79 - 8 =	
9	4 - 3 =		31	8 - 4 =	
10	14 - 3 =		32	18 - 4 =	
11	24 - 3 =		33	78 - 4 =	
12	84 - 3 =		34	89 - 9 =	
13	7 - 4 =		35	9 - 7 =	
14	17 - 4 =		36	19 - 7 =	
15	27 - 4 =		37	79 - 7 =	
16	97 - 4 =		38	89 - 5 =	
17	6 - 5 =		39	68 - 6 =	
18	16 - 5 =		40	48 - 5 =	
19	26 - 5 =		41	29 - 7 =	
20	46 - 5 =		42	38 - 6 =	
21	23 - 3 =		43	59 - 4 =	
22	67 - 7 =		44	77 - 4 =	

Name _____ Date _____

Measure each set of lines with one small paper clip, using mark and move forward.
Measure each set of lines in centimeters using a ruler.

1. Line A _____

   Line B _____

   a. Line A is about _____ paper clips long.

   b. Line A is about _____ cm long.

   c. Line B is about _____ paper clips long.

   d. Line B is about _____ cm long.

   e. Line B is about _____ paper clips shorter than Line A.

   f. Line A is about _____ cm longer than Line B.

2. _____ Line L

   _____ Line M

   a. Line L is about _____ paper clips long.

   b. Line L is about _____ cm long.

   c. Line M is about _____ paper clips long.

   d. Line M is about _____ cm long.

   e. Line L is about _____ paper clips longer than Line M.

   f. Line M doubled is about _____ cm shorter than Line L.

3. Draw a line that is 18 cm long and another line below it that is 12 cm long.
   Label the 18 cm line R and the 12 cm line S.

   a. Line R measured about _____ paper clips.
   b. Line S measured about _____ paper clips.

4. Draw a line that is 6 cm long and another line below it that is 15 cm long.

   Label the 6 cm line *C* and the 15 cm line D.

   a. Line *C* is about _____ paper clips long.
   b. Line D is about _____ paper clips long.
   c. Line D is about _____ cm longer than Line *C*.
   d. Line *C* is about _____ paper clips shorter than Line D.
   e. Lines *C* and D together are about _____ paper clips long.
   f. Lines *C* and D together are about _____ centimeters long.

5. Christina measured Line F with quarters and Line *G* with pennies.

   Line F _____

   Line *G* _____

   Line F measured the length of about 6 quarters.
   Line *G* measured the length of about 8 pennies.
   Christina said Line *G* is longer because 8 is a bigger number than 6.
   Explain why Christina is incorrect.

   _____

   _____

**Note that the exit ticket will be used as a pre-assessment and post assessment

Name _____ Date _____

Measure the lines with small paper clips and then with a centimeter ruler. Then, answer the questions below.

Line 1 _____

Line 2 _____

Line 3 _____

a. Line 1 is about _____ paper clips long.

b. Line 1 is about _____ cm long.

c. Line 2 is about _____ paper clips long.

d. Line 2 is about _____ cm long.

e. Line 3 is about _____ paper clips long.

f. Line 3 is about _____ cm long.

Explain why each measurement has more centimeters than paper clips.

_____

_____

_____

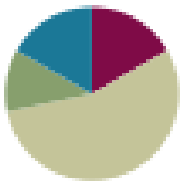
Unit 6, Lesson 13:

Lesson 13

Objective: Use square tiles to decompose a rectangle.

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Concept Development	(33 minutes)
Application Problem	(7 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Making the Next Ten to Add 2.OA.2, 2.NBT.5 (5 minutes)
- Grade 2 Core Fluency Practice Sets 2.OA.2 (5 minutes)

Making the Next Ten to Add (5 minutes)

Note: This fluency activity reviews making a ten to add.

T: When I say 9 + 4, you say 10 + 3. Ready? 9 + 4.

S: 10 + 3.

T: Answer.

S: 13.

Continue with the following possible sequences:

19 + 4, 29 + 4, 29 +14, 59 + 14	9 + 6, 19 + 6, 19 + 16, 49 + 16	8 + 3, 18 + 3, 18 + 13
8 + 5, 18 + 5, 18 + 15, 38 + 15	7 + 6, 17 + 6, 17 + 16, 37 + 16	7 + 4, 17 + 4, 67 + 4

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (G2–M6–Lesson 12 Core Fluency Practice Sets)

Note: During Topic C and for the remainder of the year, each day's fluency activities include an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 12.

Concept Development (33 minutes)

Materials: (T/S) 25 square tiles, personal white board, ruler, square tiles (Template)

For the following Concept Development, model the work for students using an overhead projector or document camera.

T: With your partner, use the tiles to construct a rectangle with 4 rows of 5 on your personal white board. Tell your partner the total number of tiles in your rectangle and how you know.

T: (Model a rectangle with 4 rows of 5 using the overhead projector.)

S: There are 20 tiles because 5 + 5 + 5 + 5 = 20. → 20 because 4 + 4 + 4 + 4 + 4 = 20.

T: Write the number of rows and the number in each row as the whole in your number bond as I do. (Model writing 4 rows of 5 under the rectangle.)

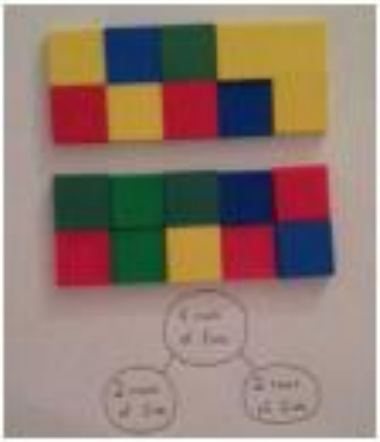
S: (Write 4 rows of 5, as pictured.)

T: Turn and talk: How can we decompose this rectangle into two equal parts?

S: I know that 10 + 10 makes 20, so we could put 10 on one side and 10 on the other. → If we split it across the middle like how we spread out the rows of lima beans with a ruler, we can make it half and half. → Two equal parts would be 2 rows of 5 on one side and 2 rows of 5 on the other.

T: Use your ruler to break your rectangle into two equal parts as I do. (Model using the ruler to split the rectangle.)

T: (Circulate as students decompose the rectangle as pictured.)



T: How many rows do you have in each part?

S: Two rows!

T: How many tiles in each row?

S: 5 tiles!

T: Write 2 rows of 5 for each part of your number bond.

T: (Model writing 2 rows of 5 in each part of the number bond.)

T: If 5 + 5 + 5 + 5 represented the rectangle before we decomposed it, what number sentence can you use to describe each part?

S: 5 + 5 = 10.

T: Write 5 + 5 = 10 below the parts of the number bond. (Model writing the number sentences under each part.)

S: (Write number sentences.)

T: Tell your partner the two parts and the whole using a number sentence.

S: Two rows of 5 and 2 rows of 5 make 4 rows of 5.

Repeat the process with 6 columns of 2, decomposing by columns rather than by rows.

T: With your partner, count out 16 tiles. Then, construct a rectangle on your desk with 4 rows.

T: (Circulate as students work.)

T: How many rows did you make?

S: 4 rows!

T: How many tiles are in each row?

S: 4 tiles!

T: Say the repeated addition sentence.

S: 4 + 4 + 4 + 4 =16.

T: What do 4 rows of 4 equal?

MP.7 S: 16.

T: Take away a row.

T: Turn and talk: What is the new total for the rectangle and how do you know?

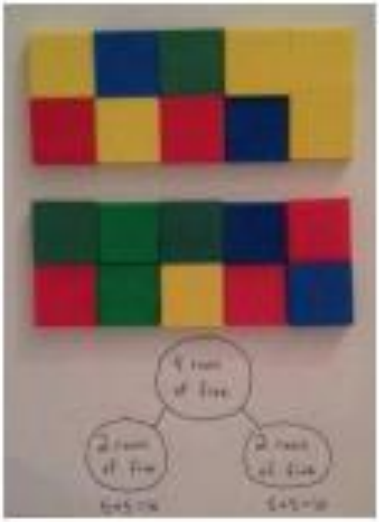
S: 12 because 4 + 4 + 4 = 12. → 12 because 16 − 4 = 12. → 3 fours is 12.

T: Remove one column.

T: Turn and talk: How many tiles do you have now, and how do you know?

S: 9 because there are 3 rows of 3 and 3 + 3 + 3 = 9. → I see 3 threes and that's 9. → 9, because 12 − 3 = 9.

Repeat the above process with a rectangle of 25 tiles.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some students will be able to express their arrays in multiplication number sentences and will be eager to show off their expertise. Encourage them to write both types of number sentences and share how they know with another student.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Application Problem (7 minutes)

Note: This Application Problem provides an opportunity for students to apply understandings from today's lesson, so it follows the Concept Development. If necessary, provide manipulatives for students to use when solving the problem.

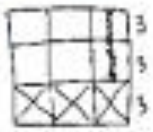
Ellie bakes a square pan of lemon bars, which she cut into nine equal pieces. Her brothers eat 1 row of her treats. Then, her mom eats 1 column.

a. Draw a picture of Ellie's lemon bars before any are eaten. Write a number sentence to show how to find the total.

b. Write an X on the bars that her brothers eat. Write a new number sentence to show how many are left.

c. Draw a line through the bars that her mom eats. Write a new number sentence to show how many are left.

d. How many bars are left? Write a statement.



a) 3+3+3=9

b) 9-3=6 or 3+3=6

c) 6-2=4 or 2+2=4

d) There are 4 lemon bars left.

Student Debrief (10 minutes)

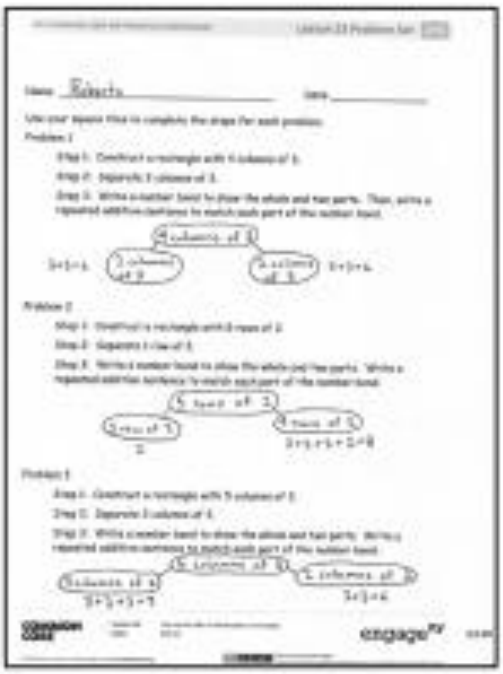
Lesson Objective: Use square tiles to decompose a rectangle.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How does your number bond show how you decomposed, or broke apart, your rectangle in Problem 1?
- In Problem 2, what do you notice is the same in the whole and parts of your number bond? (A unit of two.) How does your repeated addition sentence change without one row?



- In Problem 3, defend how you know that a rectangle can be decomposed into smaller rectangles. Describe the two smaller rectangles that you found in 5 columns of 3. Use the terms rows, columns, units, and repeated addition.
- What was your strategy for composing a rectangle with 12 squares for Problem 4? How many different possibilities are there?
- For Problem 5, how is removing a row from a rectangle with 2 rows of 10 different from removing a row from 5 rows of 4? Which one will leave you with more squares?
- For Problem 6, share with a partner all of the different ways that you could break apart a rectangle made up of 16 square tiles.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

Use your square tiles to complete the steps for each problem.

Problem 1

Step 1: Construct a rectangle with 4 columns of 3.

Step 2: Separate 2 columns of 3.

Step 3: Write a number bond to show the whole and two parts. Then, write a repeated addition sentence to match each part of the number bond.

Problem 2

Step 1: Construct a rectangle with 5 rows of 2.

Step 2: Separate 1 row of 2.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of the number bond.

Problem 3

Step 1: Construct a rectangle with 5 columns of 3.

Step 2: Separate 3 columns of 3.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of the number bond.

4. Use 12 square tiles to construct a rectangle with 3 rows.

   a. _____ rows of _____ = 12

   b. Remove 1 row. How many squares are there now? _____

   c. Remove 1 column from the new rectangle you made in 4(b). How many squares are there now? _____

5. Use 20 square tiles to construct a rectangle.

   a. _____ rows of _____ = _____

   b. Remove 1 row. How many squares are there now? _____

   c. Remove 1 column from the new rectangle you made in 5(b). How many squares are there now? _____

6. Use 16 square tiles to construct a rectangle.

   a. _____ rows of _____ = _____

   b. Remove 1 row. How many squares are there now? _____

   c. Remove 1 column from the new rectangle you made in 6(b). How many squares are there now? _____

*Exit ticket to be used as Pre-Assessment and Post-Assessment

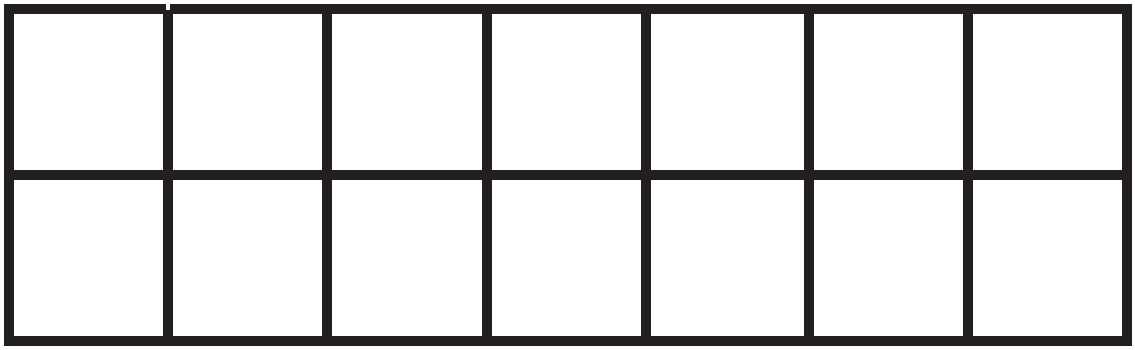
Name _____ Date _____

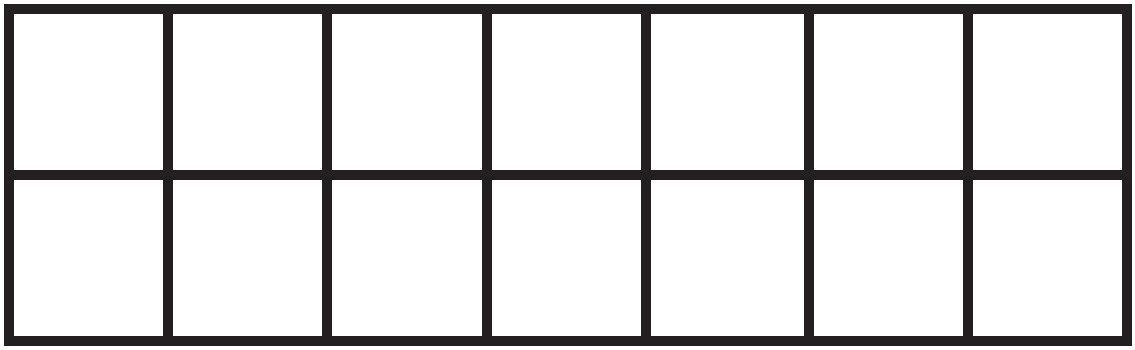
Use your square tiles to complete the steps for each problem.

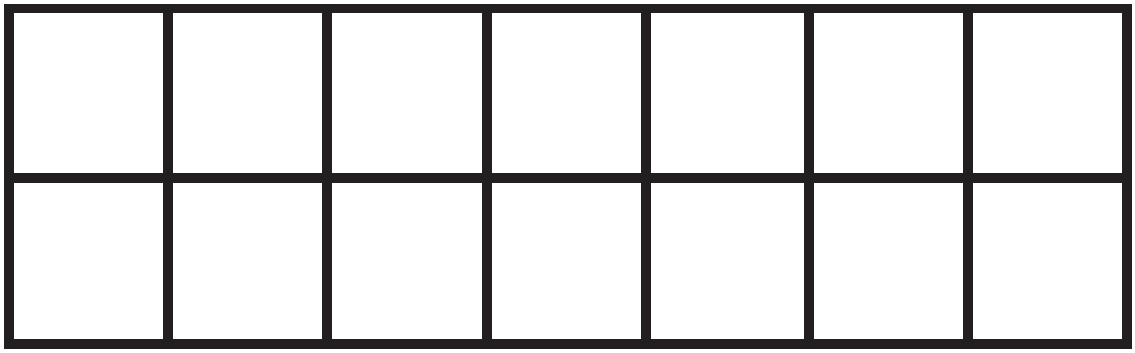
Step 1: Construct a rectangle with 3 columns of 4.

Step 2: Separate 2 columns of 4.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of the number bond.







_____

square tiles

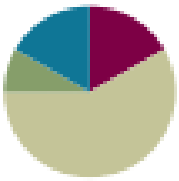
Unit 7, Lesson 13:

Lesson 13

Objective: Solve two-step word problems involving dollars or cents with totals within $100 or $1.

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Application Problem	(5 minutes)
Concept Development	(35 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Grade 2 Core Fluency Differentiated Practice Sets 2.OA.2 (5 minutes)
- Decomposition Tree 2.NBT.5 (5 minutes)

Grade 2 Core Fluency Differentiated Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (Lesson 1 Core Fluency Practice Sets)

Note: During Topic B and for the remainder of the year, each day's Fluency Practice includes an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. The process is detailed, and Practice Sets are provided in Lesson 1.

Decomposition Tree (5 minutes)

Materials: (S) Decomposition tree (Lesson 6 Fluency Template)

Note: Students are given 90 seconds to decompose a dollar.

T: (Distribute tree Template.)

T: You are going to break apart $1 on your Deco Tree for 90 seconds. Do as many problems as you can. Go!

S: (Work for 90 seconds.)

T: Now, exchange your tree with your partner, and check each other's work. (Allow students 30–45 seconds to check.)

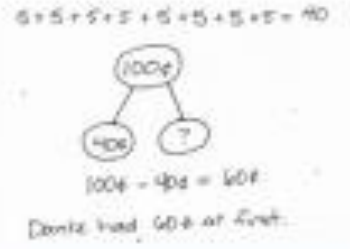
T: Return each other's papers. Did you see another way to make $1 on your partner's paper? (Allow students to share for another 30 seconds.)

T: Turn your paper over. Let's break apart $1 for another minute.

Application Problem (5 minutes)

Dante had some money in a jar. He puts 8 nickels into the jar. Now, he has 100 cents. How much money was in the jar at first?

Note: In this *add to with start unknown* problem, students must pay close attention to the question, as they may incorrectly jump to the conclusion that they should subtract 100 − 8. Ask questions that guide students toward seeing that 100 cents equals 20 nickels, or guide them toward calculating the value of 8 nickels and subtracting that from 100.



Concept Development (35 minutes)

Materials: (T) Document camera (if available)
(S) Personal white board

Part 1: Solve an *add to with change unknown* type problem.

Gary has 2 dimes, 5 nickels, and 13 pennies. His brother gives him one more coin. Now, he has 68 cents. What coin did his brother give him?

T: What do we do first when we see a word problem?

S: Read it.

T: Yes. Let's read the problem together.

T/S: (Read aloud.)

T: What can you draw?

S: Gary's coins. → We can draw 2 dimes, 5 nickels, 13 pennies, and a question mark coin. → A tape diagram.

T: Great! Do it. (Pause while students draw.)

T: Turn and talk: Look at your drawing. What are you trying to find?

S: The value of the coin Gary's brother gave him. → We need to find the value of the question mark coin.

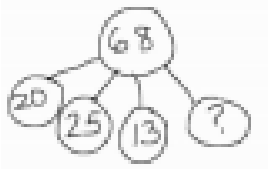
T: Go ahead and do that. Write a number sentence and statement to match your work. (Allow students time to work.) Explain to your partner how you solved the problem.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Support students who are performing below grade level by talking them through the Application Problem one step at a time: "How much money did Dante put in the jar? How much does he have now? Are nickels and cents the same unit? Can we add or subtract different units? What can we do to make them the same unit so that we can solve the problem?" And, if necessary, "What is the value of 8 nickels?"

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

At times, students can discuss what they will draw before drawing. At other times, they might go ahead and draw. Use professional judgment to adapt to varying circumstances.



S: I skip-counted by tens, then fives, then ones: 10, 20, 25, 30, 35, 40, 45, 46, ...58. Then, I counted 10 more to get to 68. → First, I found the value of the dimes, nickels, and pennies. 20 + 25 + 13 = 58. I know 68 is 10 more than 58, so the coin is a dime. → First, I counted up the coins I know and got 58¢. 68¢ − 58¢ = 10¢.

T: What was the value of Gary's money before his brother gave him a coin?

S: 58¢.

T: What's your number sentence?

S: 58¢ + ___ = 68¢. → 68¢ − 58¢ = 10¢.

T: And what is the statement of your solution?

S: Gary's brother gave him a dime.

T: Yes! Look how we can also represent this problem with a number bond (pictured above to the right).

T: Turn and talk. Use part–whole language to describe how your drawing matches mine.

S: My tape diagram shows two parts and a whole. → Your diagram shows each coin as a different part. That's how I added to find the value of Gary's coins.

T: Great work! Let's do another one.

Part 2: Solve a two-step problem.

Hailey bought a pretzel stick for a dime and a nickel. She also bought a juice box for 18 cents more than the pretzel stick. How much did she spend on the pretzel and juice box?

T: What do we do first when we see a word problem?

S: Read it.

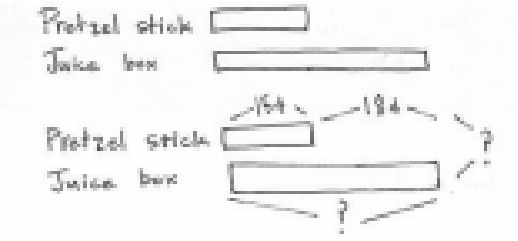
T: Yes. Let's read the problem together.

T/S: (Read aloud.)

T: What can you draw?

S: The juice box and pretzel stick. → I'm going to write how much they cost, too. → A tape diagram for both.

T: Go ahead and draw. (Pause while students draw.)



T: Turn and talk: Look at your drawing. What are you trying to find?

S: How much Hailey spent on the pretzel and juice box. → First, you need to know how much the juice box cost.

T: Go ahead and do that. Write a number sentence and statement to match your work. (Allow students time to work.) Explain to your partner how you solved.

S: I made two tape diagrams that were the same size. Then, I made the juice box tape diagram longer to show the extra 18¢. → I added 15¢ + 18¢ = 33¢ to find out the cost of the juice box. → To find the total, I added 30 + 10 + 3 + 5 = 48.

T: How much did the juice box cost?

S: 33 cents!

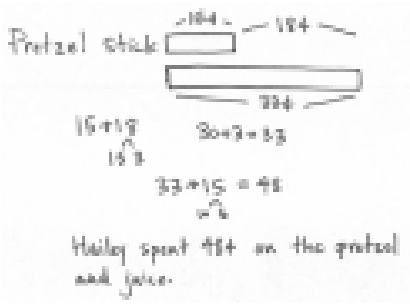
T: What's your number sentence to find the total?

S: 15¢ + 33¢ = 48¢.

T: And what is the statement of your solution?

S: Hailey spent 48¢ on the pretzel and juice.

T: Terrific! Let's work on one more problem together.



Part 3: Solve a *take from with start unknown* type problem.

Wendell bought a game at the store for $16. He had 2 five-dollar bills and 4 one-dollar bills left over. How much money did he have before buying the game?

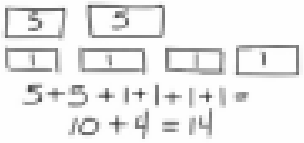
T: Read the problem to me, everyone.

S: (Read chorally.)

T: Can you draw something?

S: Yes!

T: Do that. (Provide work time.)



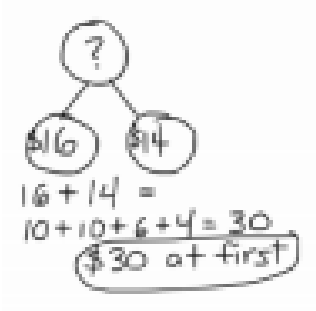
T: Turn and talk: Look at your drawing. What are you trying to find?

S: The amount of money he had before he bought the game. → We need to find the value of his change to know.

T: Go ahead and do that. Write a number sentence and statement to match your work. (Allow students time to work.) Explain to your partner how you solved the problem.

MP.1

S: First, I drew Wendell's bills and counted by fives and ones. He got $14 in change. → I drew a number bond. The cost of the game is one part and the change is the other part. I made 16 + 14 into 10 + 10 + 6 + 4. That's 3 tens, or 30. → I added $16 + $10 + $4 = $30.



T: What was the value of Wendell's change?

S: $14.

T: What's your number sentence?

S: $16 + $14 = $30.

Wendell had $30 at first.

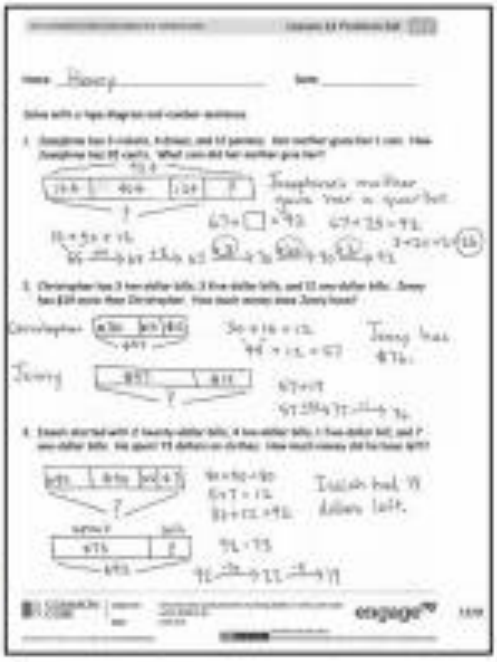
T: And, what is the statement of your solution?

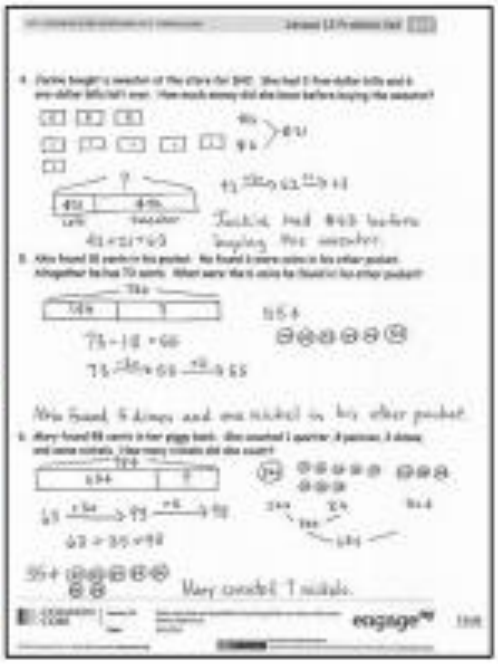
S: Wendell had $30 before buying the game.

T: Great. You're now ready to work on the Problem Set. Remember the strategies we have been practicing.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.





Student Debrief (10 minutes)

Lesson Objective: Solve two-step word problems involving dollars or cents with totals within $100 or $1.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Before you begin solving a word problem, what are some things you should think about? (What type of models to use, whether there is more than one part to the problem, what operations to use, and what strategies I can use to help me.)
- Look at Problem 1 of your Problem Set. Could skip-counting help you solve one part of the problem quickly?
- Look at Problem 2. Tell your partner what you did first. Take your partner through your entire solution path.
- Talk to your partner about the models you used to solve word problems today. Share with your partner how you used a model on your Problem Set.
- Share your strategy for figuring out the coins Akio found in his pocket.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

Solve with a tape diagram and number sentence.

1. Josephine has 3 nickels, 4 dimes, and 12 pennies. Her mother gives her 1 coin. Now, Josephine has 92 cents. What coin did her mother give her?

2. Christopher has 3 ten-dollar bills, 3 five-dollar bills, and 12 one-dollar bills. Jenny has $19 more than Christopher. How much money does Jenny have?

3. Isaiah started with 2 twenty-dollar bills, 4 ten-dollar bills, 1 five-dollar bill, and 7 one-dollar bills. He spent 73 dollars on clothes. How much money does he have left?

4. Jackie bought a sweater at the store for $42. She had 3 five-dollar bills and 6 one-dollar bills left over. How much money did she have before buying the sweater?

5. Akio found 18 cents in his pocket. He found 6 more coins in his other pocket. Altogether he has 73 cents. What were the 6 coins he found in his other pocket?

6. Mary found 98 cents in her piggy bank. She counted 1 quarter, 8 pennies, 3 dimes, and some nickels. How many nickels did she count?

***Exit ticket to be used as a pre-assessment and post-assessment**

Name _____ Date _____

Solve with a tape diagram and number sentence.

Gary went to the store with 4 ten-dollar bills, 3 five-dollar bills, and 7 one-dollar bills. He bought a sweater for $26. What bills did he leave the store with?