Thomaston Public Schools

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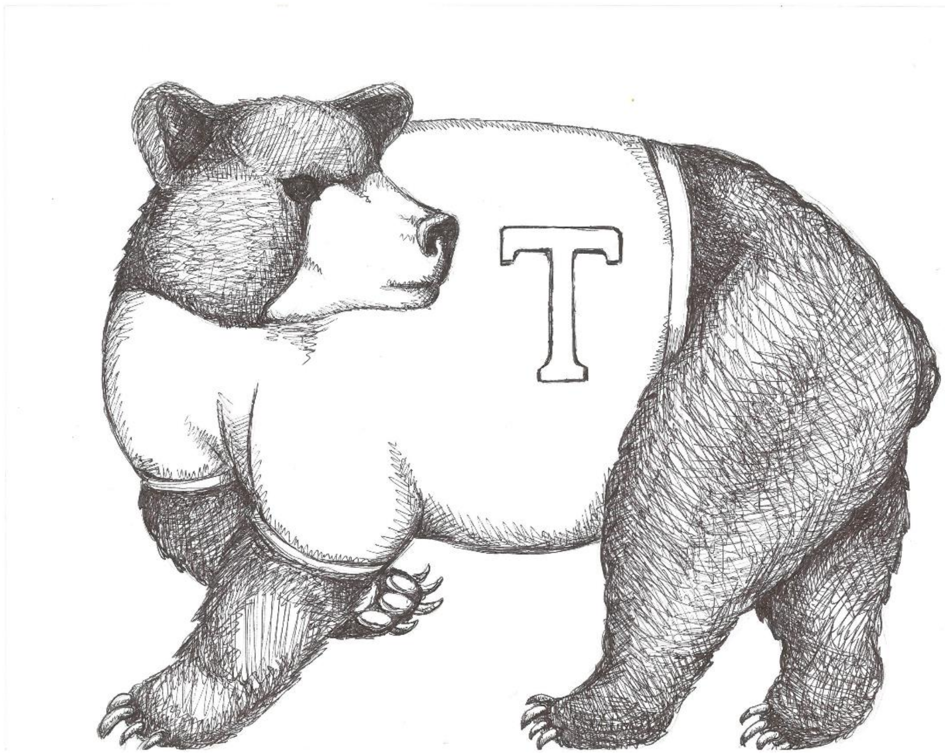

Thomaston Public Schools Curriculum
Thomaston High School
Grade(s): Geometry 2015

Learn to Live….Live to Learn

Acknowledgements

Curriculum Writer(s):

Alisha DiCorpo

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

____*Alisha DiCorpo*____________________________________

Alisha L. DiCorpo
Director of Curriculum and Professional Development

Date of Presentation to the Board of Education: _ August 2015

(Geometry Curriculum)

[Geometry]

Board of Education Mission Statement:

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:
The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main Resource used when writing this curriculum:
NYS COMMON CORE MATHEMATICS CURRICULUM A Story of Units/Ratios/Functions Curriculum. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. A Story of Functions: A Curriculum Overview for Grades 9-12 Date: 7/31/13 5 © 2013 Common Core, Inc. Some rights reserved. commoncore.org

Course Description:
Sequence of Geometry Modules (Units) Aligned with the Standards

Unit 1: Congruence, Proof, and Constructions
Unit 2: Similarity, Proof and Trigonometry
Unit 3: Extending to Three Dimensions
Unit 4: Connecting Algebra and Geometry through Coordinates
Unit 5: Circles With and Without Coordinates

Summary of the Year

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between the Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course, and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Geometry

- Triangle congruence and similarity criteria
- Using coordinates to establish geometric results
- Calculating length and angle measures
- Using geometric representations as a modeling tool
- Using construction tools, physical and computational to draft models of geometry phenomenon

CCSS Major Emphasis Clusters

Congruence

- Understand congruence in terms of rigid motions
- Prove geometric theorems

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems using similarity
- Define trigonometric ratios and solve problems involving right triangles

Expressing Geometric Properties with Equations

- Use coordinates to prove simple geometric theorems algebraically

Modeling with Geometry

- Apply geometric concepts in modeling situations

<table>
<tr><th></th><th>Grade 9 -- Algebra I</th><th>Grade 10 -- Geometry</th><th>Grade 11 -- Algebra II</th><th>Grade 12 -- Precalculus</th><th></th></tr>
<tr><td>20 days</td><td rowspan="2">M1:
Relationships Between Quantities and Reasoning with Equations and Their Graphs
(40 days)</td><td rowspan="3">M1:
Congruence, Proof, and Constructions
(45 days)</td><td rowspan="3">M1:
Polynomial, Rational, and Radical Relationships
(45 days)</td><td rowspan="2">M1:
Complex Numbers and Transformations
(40 days)</td><td>20 days</td></tr>
<tr><td>20 days</td><td>20 days</td></tr>
<tr><td rowspan="2">20 days</td><td rowspan="3">M2: Descriptive Statistics
(25 days)</td><td rowspan="4">M2:
Vectors and Matrices
(40 days)</td><td rowspan="2">20 days</td></tr>
<tr><td rowspan="3">M2:
Similarity, Proof, and Trigonometry
(45 days)</td><td rowspan="2">M2:
Trigonometric Functions
(20 days)</td></tr>
<tr><td rowspan="2">20 days</td><td rowspan="2">20 days</td></tr>
<tr><td>M3:
Linear and Exponential Functions</td><td>M3: Functions
(45 days)</td></tr>
<tr><td rowspan="3">20 days</td><td>State Examinations</td><td>State Examinations</td><td>State Examinations</td><td>State Examinations</td><td rowspan="3">20 days</td></tr>
<tr><td rowspan="3">(35 days)</td><td></td><td rowspan="5"></td><td rowspan="4">M3:
Rational and Exponential Functions
(25 days)</td></tr>
<tr><td rowspan="2">M3: Extending to Three Dimensions (10 days)</td></tr>
<tr><td rowspan="4">20 days</td><td rowspan="4">20 days</td></tr>
<tr><td rowspan="5">M4:
Polynomial and Quadratic Expressions, Equations and Functions
(30 days)</td><td rowspan="4">M4: Connecting Algebra and Geometry through Coordinates (25 days)</td></tr>
<tr><td rowspan="3">M4: Trigonometry
(20 days)</td></tr>
<tr><td rowspan="5">M4:
Inferences and Conclusions from Data
(40 days)</td></tr>
<tr><td rowspan="3">20 days</td><td rowspan="3">20 days</td></tr>
<tr><td rowspan="3">M5:
Circles with and Without Coordinates
(25 days)</td><td rowspan="3">M5:
Probability and Statistics
(25 days)</td></tr>
<tr><td rowspan="2">M5:
A Synthesis of Modeling with Equations and Functions (20 days)</td></tr>
<tr><td rowspan="2">20 days</td><td rowspan="2">20 days</td></tr>
<tr><td rowspan="2">Review and Examinations</td><td rowspan="2">Review and Examinations</td><td rowspan="2">Review and Examinations</td><td rowspan="2">Review and Examinations</td></tr>
<tr><td>20 days</td><td>20 days</td></tr>
</table>

Key:	Number and Quantity and Modeling	Geometry and Modeling	Algebra and Modeling	Statistics and Probability and Modeling	Functions and Modeling

Due to district test decisions that need to be made, the days set aside for testing may be less than what is notated in the chart above. Feel free to modify this pacing guide as needed until district testing is set each year.

Congruence, Proof and Constructions

OVERVIEW

Module 1 embodies critical changes in Geometry as outlined by the Common Core. The heart of the module is the study of transformations and the role transformations play in defining congruence.

Students begin this module with Topic A, Basic Constructions. Major constructions include an equilateral triangle, an angle bisector, and a perpendicular bisector. Students synthesize their knowledge of geometric terms with the use of new tools and simultaneously practice precise use of language and efficient communication when they write the steps that accompany each construction (**G.CO.A.1**).

Constructions segue into Topic B, Unknown Angles, which consists of unknown angle problems and proofs. These exercises consolidate students' prior body of geometric facts and prime students' reasoning abilities as they begin to justify each step for a solution to a problem. Students began the proof writing process in Grade 8 when they developed informal arguments to establish select geometric facts (**8.G.A.5**).

Topics C and D, Transformations/Rigid Motions and Congruence, builds on students' intuitive understanding developed in Grade 8. With the help of manipulatives, students observed how reflections, translations, and rotations behave individually and in sequence (**8.G.A.1**, **8.G.A.2**). In high school Geometry, this experience is formalized by clear definitions (**G.CO.A.4**) and more in-depth exploration (**G.CO.A.3**, **G.CO.A.5**). The concrete establishment of rigid motions also allows proofs of facts formerly accepted to be true (**G.CO.C.9**). Similarly, students' Grade 8 concept of congruence transitions from a hands-on understanding (**8.G.A.2**) to a precise, formally notated understanding of congruence (**G.CO.B.6**). With a solid understanding of how transformations form the basis of congruence, students next examine triangle congruence criteria. Part of this examination includes the use of rigid motions to prove how triangle congruence criteria such as SAS actually work (**G.CO.B.7**, **G.CO.B.8**).

In Topic E, Proving Properties of Geometric Figures, students use what they have learned in Topics A through D to prove properties—those that have been accepted as true and those that are new—of parallelograms and triangles (**G.CO.C.10**, **G.CO.C.11**). The module closes with a return to constructions in Topic F (**G.CO.D.13**), followed by a review of the module that highlights how geometric assumptions underpin the facts established thereafter (Topic G).

Mathematics-Geometry

Rigorous Curriculum Design Template

Unit: 1 Congruence Proof and Constructions

Subject: Mathematics

Grade/Course: Geometry

Pacing: 34 Days

Unit of Study: Congruence, Proof and Constructions

Focus Standards:

Experiment with transformations in the plane.

G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

G-CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

G-CO.C.9 Prove[1] theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

G-CO.C.10 Prove[2] theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

G-CO.C.11 Prove[2] theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*.

Make geometric constructions.

G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G-CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Foundational Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

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[1] Prove *and apply* (in preparation for Regents Exams).

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Math Practice Standards: Focus Standards for Mathematical Practice

MP.3 **Construct viable arguments and critique the reasoning of others**. Students articulate steps needed to construct geometric figures, using relevant vocabulary. Students develop and justify conclusions about unknown angles and defend their arguments with geometric reasons.

MP.4 **Model with mathematics**. Students apply geometric constructions and knowledge of rigid motions to solve problems arising with issues of design or location of facilities.

MP.5 **Use appropriate tools strategically**. Students consider and select from a variety of tools in constructing geometric diagrams, including (but not limited to) technological tools.

MP.6 **Attend to precision**. Students precisely define the various rigid motions. Students demonstrate polygon congruence, parallel status, and perpendicular status via formal and informal proofs. In addition, students will clearly and precisely articulate steps in proofs and constructions throughout the module.

“Unwrapped” Standards

G-CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
G-CO.C.9	Prove[2] theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.*
G-CO.C.10	Prove[2] theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
G-CO.C.11	Prove[2] theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*
G-CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing*

[2] Prove *and apply* (in preparation for Regents Exams).

a line parallel to a given line through a point not on the line. **G-CO.D.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
G-CO.A.1 precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **G-CO.A.2** transformations in the plane transformations that preserve distance and angle to those that do not **G-CO.A.3** rectangle, parallelogram, trapezoid, or regular polygon, the rotations and reflections that carry it onto itself. **G-CO.A.4** definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G-CO.A.5** geometric figure and a rotation, reflection, or translation, draw the transformed figure, sequence of transformations that will carry a given figure onto another. **G-CO.B.6** geometric descriptions of rigid motions to transform figures, effect of a given rigid motion on a given figure; given two figures, definition of congruence in terms of rigid motions to decide if they are congruent. **G-CO.B.7** definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are	Know (L1) Represent (L2) Describe (L2) Compare (L2) Describe (L2) Develop (L2) Specify (L2) Use (L1)

	congruent.	Predict (L3)
G-CO.B.8	criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	Decide (L2) Use (L1)
G-CO.C.9	theorems about lines and angles.	Show (L2)
G-CO.C.10	theorems about triangles.	
G-CO.C.11	theorems about parallelograms.	
G-CO.D.12	formal geometric constructions with a variety of tools and methods	Explain (L1)
G-CO.D.13	equilateral triangle, a square, and a regular hexagon inscribed in a circle.	Prove (L2) Make (L3) Construct (L3)

Essential Questions	Big ideas
Essential Questions: What is rigid motion? How is it used in geometry? What is coordinate geometry? How is it used? How can coordinate geometry describe rigid motion? Explain the significance of undefined terms to the study of geometry?	**Big Ideas:** 1.Rigid motion, or isometries, (rotation, reflection and translation)preserves distance and angle measures 2. Coordinate geometry depicts segment lengths and angle measures. 3.Undefined terms (point, line, and plane) are the building blocks of geometry

Assessments

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	G-CO.A.1, G-CO.A.2, G-CO.A.4, G-CO.A.5, G-CO.B.6, G-CO.C.9, G-CO.D.12
End-of-Module Assessment Task	After Topic G	Constructed response with rubric	G-CO.A.2, G-CO.A.3, G-CO.B.7, G-CO.B.8, G-CO.C.10, G-CO.C.11, G-CO.D.13

Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Since Lesson one is an introductory lesson: Pre-test the following vocabulary: <u>Geometric Construction</u>: A *geometric construction* is a set of instructions for drawing points, lines, circles, and figures in the plane. The two most basic types of instructions are the following: Given any two points □ and □, a ruler can be used to draw the line □□ or segment □□. Given any two points □ and □, use a compass to draw the circle that has its center at □ that passes through □. (Abbreviation: Draw circle □: center □, radius □□.) Constructions also include steps in which the points where lines or circles intersect are selected and labeled. (Abbreviation: Mark the point	Post-test the vocabulary Opening Exercise (give again and reflect on results from first administration prior to the unit) Exploratory Challenge Exit Ticket Student Conferences	Mid-Module and End of Module Assessments (given as prescribed in the pacing guide) see chart above.

of intersection of the lines □□ and □□ by □, etc.) Figure: A (two-dimensional) *figure* is a set of points in a plane. Usually the term figure refers to certain common shapes such as triangle, square, rectangle, etc. However, the definition is broad enough to include any set of points, so a triangle with a line segment sticking out of it is also a figure. Equilateral Triangle: An *equilateral triangle* is a triangle with all sides of equal length. Collinear: Three or more points are collinear if there is a line containing all of		

the points; otherwise, the points are non-collinear. Length of a Segment: The *length of the segment* $\overline{AB}$ is the distance from A to B and is denoted AB. Thus, $AB = dist(A, B)$.		

Performance Task
To be created with teacher input throughout the year.
Engaging Learning Experiences

To be created with teacher input throughout the year.

Instructional Resources
Suggested Tools and Representations ▪ Compass and straightedge ▪ Geometer's Sketchpad or Geogebra Software ▪ Patty paper ▪ IXL Math Representing and Combining Transformations Activity: http://map.mathshell.org/materials/lessons.php?taskid=223#task223 Module for showing how Geometer Sketchpad can be used to prove theorems: http://mtl.math.uiuc.edu/book/export/html/9 Geometry Review Examples: http://www.bsd405.org/portals/0/curriculum/MSPMath/Geometry%20End%20of%20Course%20Packet%203-16-11.pdf Investigating and understanding lines, rays, and segments http://illuminations.nctm.org/ActivityDetail.aspx?ID=

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **Provide Multiple Means of Representation** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use

models and gestures to calculate and explain. For example, a student searching to define “multiplication” may model groups of 6 with drawings or concrete objects and write the number sentence to match.

- Teach students how to ask questions (such as “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…”
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.”
- Adjust wait time for interpreters of deaf and hard-of-hearing students.
- Select numbers and tasks that are “just right” for learners.

	- Model each step of the algorithm before students begin. - Give students a chance to practice the next day's sprint beforehand. (At home, for example.) - Give students a few extra minutes to process the information before giving the signal to respond. - Assess by multiple means, including "show and tell" rather than written. - Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" - Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" - Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** - Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. - Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. - Teach in small chunks so students get a lot of practice with one step at a time. - Know, use, and make the most of Deaf culture and sign language. - Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" - Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. - Incorporate activity. Get students up and

	moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**

New or Recently Introduced Terms **Isometry** (An *isometry* of the plane is a transformation of the plane that is distance-preserving.) **Familiar Terms and Symbols**[3] ▪ Transformation ▪ Translation ▪ Rotation ▪ Reflection ▪ Congruence	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **Provide Multiple Means of Action and Expression** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in

[3] These are terms and symbols students have seen previously.

	thinking and strategy for the solution. Choose numbers and tasks that are “just right” for learners but teach the same concepts. Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom’s Taxonomy with questions such as: “What would happen if...?” “Can you propose an alternative...?” “How would you evaluate...?” “What choice would you have made...?” Ask “Why?” and “What if?” questions.

		Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Similarity, Proof and Trigonometry

OVERVIEW

Just as rigid motions are used to define congruence in Module 1, so dilations are added to define similarity in Module 2.

To be able to define similarity, there must be a definition of similarity transformations and consequently a definition for dilations. Students are introduced to the progression of terms beginning with scale drawings, which they first study in Grade 7 (Module 1, Topic D), but in a more observational capacity than in Grade 10: Students determine the scale factor between a figure and a scale drawing or predict the lengths of a scale drawing, provided a figure and a scale factor. In Topic A, students begin with a review of scale drawings in Lesson 1, followed by two lessons on how to systematically create scale drawings. The study of scale drawings, specifically the way they are constructed under the ratio and parallel methods, gives us the language to examine dilations. The comparison of why both construction methods (MP.7) result in the same image leads to two theorems: the triangle side splitter theorem and the dilation theorem. Note that while dilations are defined in Lesson 2, it is the dilation theorem in Lesson 5 that begins to tell us how dilations behave (**G-SRT.A.1**, **G-SRT.A.4**).

Topic B establishes a firm understanding of how dilations behave. Students prove that a dilation maps a line to itself or to a parallel line and, furthermore, dilations map segments to segments, lines to lines, rays to rays, circles to circles, and an angle to an angle of equal measure. The lessons on proving these properties, Lessons 7–9, require students to build arguments based on the structure of the figure in question and a handful of related facts that can be applied to the situation (e.g., the triangle side splitter theorem is called on frequently to prove that dilations map segments to segments, lines to lines, etc.) (MP.3, MP.7). Students apply their understanding of dilations to divide a line segment into equal pieces and explore and compare dilations from different centers.

In Topic C, students learn what a similarity transformation is and why, provided the right circumstances, both rectilinear and curvilinear figures can be classified as similar (**G-SRT.A.2**). After discussing similarity in general, the scope narrows, and students study criteria for determining when two triangles are similar (**G-SRT.A.3**). Part of studying triangle similarity criteria (Lessons 15 and 17) includes understanding side length ratios for similar triangles, which begins to establish the foundation for trigonometry (**G-SRT.B.5**). The final two lessons demonstrate the usefulness of similarity by examining how two ancient Greek mathematicians managed to measure the circumference of the earth and the distance to the moon, respectively (**G-MG.A.1**).

In Topic D, students are laying the foundation to studying trigonometry by focusing on similarity between right triangles in particular (the importance of the values of corresponding length ratios between similar triangles is particularly apparent in Lessons 16, 21, and 25). Students discover that a right triangle can be divided into two similar sub-triangles (MP.2) to prove the Pythagorean theorem (**G-SRT.B.4**). Two lessons are spent studying the algebra of radicals that is useful for solving for sides of a right triangle and computing trigonometric ratios.

An introduction to trigonometry, specifically right triangle trigonometry and the values of side length ratios within right triangles, is provided in Topic E by defining the sine, cosine, and tangent ratios and using them to find missing side lengths of a right triangle (**G-SRT.B.6**). This is in contrast to studying trigonometry in the context of functions, as is done in Grade 11 of this curriculum. Students explore the relationships between sine, cosine, and tangent using complementary angles and the Pythagorean theorem (**G-SRT.B.7**, **G-SRT.B.8**). Students discover the link between how to calculate the area of a non-right triangle through algebra versus trigonometry. Topic E continues with a study of the laws of sines and cosines to apply them to solve for missing side lengths of an acute triangle (**G-SRT.D.10**, **G-SRT.D.11**). Topic E closes with Lesson 34 which introduces students to the functions □□□□□□□, □□□□□□□, and □□□□□□□, which are formally taught as inverse function in Grade 11. Students use what they know about the trigonometric functions sine, cosine, and tangent to make sense of □□□□□□□, □□□□□□□, and □□□□□□□. Students use these new functions to determine the unknown measures of angles of a right triangle.

Throughout the module students are presented with opportunities to apply geometric concepts in modeling situations. Students will use geometric shapes to describe objects **(G-MG.A.1)**, and apply geometric methods to solve design problems where physical constraints and cost issues arise **(G-MG.A.3)**.

Mathematics-Geometry

Rigorous Curriculum Design Template

Unit: 2 Similarity, Proof and Trigonometry

Subject: Mathematics

Grade/Course: Geometry

Pacing: 34 Days

Unit of Study: Similarity, Proof and Trigonometry

Focus Standards:

Understand similarity in terms of similarity transformations.

G-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity.

G-SRT.B.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles.

G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Apply geometric concepts in modeling situations.

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Focus Standards for Mathematical Practice

MP.3 **Construct viable arguments and critique the reasoning of others**. Critical to this module is the need for dilations in order to define similarity. In order to understand dilations fully, the proofs in Lessons 4 and 5 to establish the triangle side splitter and the dilation theorems require students to build arguments based on definitions and previously established results. This is also apparent in Lessons 7, 8, and 9, when the properties of dilations are being proven. Though there are only a handful of facts students must point to in order to create arguments, how students reason with these facts will determine if their arguments actually establish the properties. It will be essential to communicate effectively and purposefully.

MP.7 **Look for and make use of structure**. Much of the reasoning in Module 2 centers around the interaction between figures and dilations. It is unsurprising then that students must pay careful attention to an existing structure and how it changes under a dilation, for example why it is that dilating the key points of a figure by the ratio method results in the dilation of the segments that join them. The math practice also ties into the underlying idea of trigonometry: how to relate the values of corresponding ratio lengths between similar right triangles and how the value of a trigonometric ratio hinges on a given acute angle within a right triangle.

“Unwrapped” Standards

G-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor:

c. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

d. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

G-SRT.B.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
G-SRT.A.1 properties of dilations given by a center and a scale factor: **G-SRT.A.2** the definition of similarity in terms of similarity transformations similar; using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G-SRT.A.3** the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **G-SRT.B.4** theorems about triangles. **G-SRT.B.5** congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **G-SRT.C.6** similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G-SRT.C.7** the relationship between the sine and cosine of complementary angles. **G-SRT.C.8** trigonometric ratios and the Pythagorean Theorem to right	Verify (L4) Use (L2) Decide (L2) Explain (L1) Use (L2) Prove (L4) Understand (L1) Explain and use (L2)

triangles in applied problems.★ **G-MG.A.1** geometric shapes, their measures, and their properties to objects (e.g., modeling a tree trunk or a human torso as a cylinder).★ **G-MG.A.3** Apply geometric methods to solve design	Use to solve (L2) Use and describe (L2) Apply (L2) Solve (L2)

Essential Questions	**Big ideas**
Essential Questions: What does it mean for two figures to be congruent? How is rigid motion used to prove congruence? How is coordinate geometry used to prove congruence? What are the two types of reasoning that are used to prove statements true? How are they similar and different?	**Big Ideas:** 1. Congruent figures are figures that have corresponding sides are congruent and corresponding angles congruent. 2. Inductive and deductive reasoning are used to prove valid geometric statements true.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module	After Topic C	Constructed response with rubric	G-SRT.A.1, G-SRT.A.2,

Assessment Task			G-SRT.A.3, G-SRT.B.4, G-SRT.B.5, G-MG.A.1, G-MG.A.3
End-of-Module Assessment Task	After Topic E	Constructed response with rubric	G-SRT.B.4, G-SRT.B.5, G-SRT.C.6, G-SRT.C.7, G-SRT.C.8

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest any new vocabulary Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test the vocabulary Opening Exercise (give again and reflect on results from first administration prior to the unit) Exploratory Challenge Exit Ticket Student Conferences	Mid-Module and End of Module Assessments (given as prescribed in the pacing guide)

Performance Task

http://schools.nyc.gov/NR/rdonlyres/C03D80B2-9213-43A9-AAA3-BB0032C62F4F/139657/NYCDOE_G10_ADayattheBeach_FINAL1.pdf

Engaging Learning Experiences

You are at the beach with your friends. You have brought some supplies to make sand castles. These supplies include a pail that has a base with a circumference of 6π inches, is 10 inches tall, and has an opening on top that is twice the diameter of the base. You also have aplastic pyramid mold that has a square base with an edge that measures 4 inches and is 5 inches tall, and an empty soup can with a diameter of 3.25 inches and is 4.5 inches tall.

Instructional Resources

Suggested Tools and Representations

Compass and straightedge

PDF of proofs via isometries
IXL Math

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **<u>Marzano's Strategies</u>** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning.

	Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. ● Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" ● Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

	- Enlarge sprint print for visually impaired learners. - Use student boards to work on one calculation at a time. - Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** - Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. - Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "_______ is ____ hundreds, ____ tens, and ____ ones. - Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting." - Adjust wait time for interpreters of deaf and hard-of-hearing students. - Select numbers and tasks that are "just right" for learners. - Model each step of the algorithm before students begin. - Give students a chance to practice the next day's sprint beforehand. (At home, for example.) - Give students a few extra minutes to process the information before giving the signal to respond. - Assess by multiple means, including "show and tell" rather than written. - Elaborate on the problem-solving process. Read word problems aloud. Post a visual

display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?”
- Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?”
- Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

<u>Provide Multiple Means of Engagement</u>

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!”
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games,

	such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow “everyday” and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**
<u>New or Recently Introduced Terms</u> Cosine (Let θ be the angle measure of an acute angle of the right triangle. The *cosine of* θ *of a right triangle* is the value of the ratio of the length of the adjacent side (denoted *adj*) to the length of the hypotenuse (denoted *hyp*). As a formula, $\cos \theta = adj/hyp$.) Dilation (For $r > 0$, a *dilation with center* O *and scale factor* r is a transformation $D_{O,r}$ of the plane defined as follows: For the center O, $D_{O,r}(O) = O$, and For any other point P, $D_{O,r}(P)$ is the point P' on the ray $\overrightarrow{OP}$ so that $OP' = r \cdot OP$.) Sides of a Right Triangle (The *hypotenuse* of a right triangle is the side opposite the	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...”

right angle; the other two sides of the right triangle are called the *legs*. Let θ be the angle measure of an acute angle of the right triangle. The *opposite side* is the leg opposite that angle. The *adjacent side* is the leg that is contained in one of the two rays of that angle (the hypotenuse is contained in the other ray of the angle).) Similar (Two figures in a plane are *similar* if there exists a similarity transformation taking one figure onto the other figure. A congruence is a similarity with scale factor 1. It can be shown that a similarity with scale factor 1 is a congruence.) Similarity Transformation (A *similarity transformation* (or *similarity*) is a composition of a finite number of dilations or basic rigid motions. The *scale factor* of a similarity transformation is the product of the scale factors of the dilations in the composition; if there are no dilations in the composition, the scale factor is defined to be 1. A similarity is an example of a transformation.) Sine (Let θ be the angle measure of an acute angle of the right triangle. The *sine of θ of a right triangle* is the value of the ratio of the length of the opposite side (denoted *opp*) to the length of the hypotenuse (denoted *hyp*). As a formula, $\sin \theta = opp/hyp$.) Tangent (Let θ be the angle measure of an acute angle of the right triangle. The *tangent of θ of a right triangle* is the value of the ratio of the length of the opposite side (denoted *opp*) to the length of the adjacent side (denoted *adj*). As a formula, $\tan \theta = opp/adj$.)	instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves	Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing

Note that in Algebra II, sine, cosine, and tangent are thought of as functions whose domains are subsets of the real numbers; they are not considered as values of ratios. Thus, in Algebra II, the values of these functions for a given θ are notated as $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ using function notation (i.e., parentheses are included). Familiar Terms and Symbols[4] Composition Dilation Pythagorean theorem Rigid motions Scale drawing Scale factor Slope	questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next **Foundational Standards** **Draw, construct, and describe geometrical figures and describe the relationships between them.** 7.G.A.1 **Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.** **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.A.3 **Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.** 8.G.A.4 **Understand that a two-dimensional**	the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **Provide Multiple Means of Engagement** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support. **Extension**

[4] These are terms and symbols students have seen previously.

	figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **8.G.A.5** **Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.** ***For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.***	**Standards** **Apply trigonometry to general triangles.** **G-SRT.D.9** (+) Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **G-SRT.D.10** (+) Prove the Laws of Sines and Cosines and use them to solve problems. **G-SRT.D.11** (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Extending to Three Dimensions: OVERVIEW

Module 3, Extending to Three Dimensions, builds on students' understanding of congruence in Module 1 and similarity in Module 2 to prove volume formulas for solids.

Topic A studies informal limit arguments to find the area of a rectangle with an irrational side length and of a disk (**G-GMD.A.1**). It also focuses on properties of area that arise from unions, intersections, and scaling. These topics prepare for understanding limit arguments for volumes of solids.

Topic B is introduced by a lesson where students experimentally discover properties of three-dimensional space that are necessary to describe three-dimensional solids such as cylinders and prisms, cones and pyramids, and spheres. Cross-sections of these solids are studied and are classified as similar or congruent (**G-GMD.B.4**). A dissection is used to show the volume formula for a right triangular prism after which limit arguments give the volume formula for a general right cylinder (**G-GMD.A.1**).

In Lesson 10, two-dimensional cross-sections of solids are used to approximate solids by general right cylindrical slices and leads to an understanding of Cavalieri's principle (**G-GMD.A.1**). Congruent cross-sections for a general (skew) cylinder and Cavalieri's principle lead to the volume formula for a general cylinder.

To find the volume formula of a pyramid, a cube is dissected into six congruent pyramids to find the volume of each one. Scaling the given pyramids, according to a scaling principle analogous to the one introduced in Topic A, gives the volume formula for a right rectangular pyramid. The cone cross-section theorem and Cavalieri's principle are then used to find the volume formula of a general cone (**G-GMD.A.1, G-GMD.A.3**).

Cavalieri's principle is used to show that the volume of a right circular cylinder with radius □ and height □ is the sum of the volume of hemisphere of radius □ and the volume of a right circular cone with radius □ and height □. This information leads to the volume formula of a sphere (**G-GMD.A.2, G-GMD.A.3**).

Subject/Grade Level/Unit 3

Rigorous Curriculum Design Template

Unit: Extending to Three Dimensions

Subject: Mathematics

Grade/Course: Geometry

Pacing: 13 Days

Unit of Study: Unit 3: Extending to Three Dimensions

Focus Standards

Explain volume formulas and use them to solve problems.[5]

G-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

G-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Apply geometric concepts in modeling situations.

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g. modeling a tree trunk or a human torso as a cylinder).

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

[5] The (+) standard on the volume of the sphere is an extension of G-GMD.A.1. It is explained by the teacher in this grade and used by students in G-GMD.A.3. Note: Students are not assessed on proving the volume of a sphere formula until Precalculus.

Focus Standards for Mathematical Practice

MP.6 **Attend to precision**. Students will formalize definitions, using explicit language to define terms such as *right rectangular prism* that have been informal and more descriptive in earlier grade levels.

MP.7 **Look for and make use of structure**. The theme of approximation in Module 3 is an interpretation of structure. Students approximate both area and volume (curved two-dimensional shapes and cylinders and cones with curved bases) polyhedral regions. They must understand how and why it is possible to create upper and lower approximations of a figure's area or volume. The derivation of the volume formulas for cylinders, cones, and spheres, and the use of Cavalieri's principle is also based entirely on understanding the structure and sub-structures of these figures.

"Unwrapped" Standards

G-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.★

G-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g. modeling a tree trunk or a human torso as a cylinder).★

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
G-GMD.A.1 informal argument formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. **G-GMD.A.3** for cylinders, pyramids, cones and spheres to solve problems. **G-GMD.B.4** two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **G-MG.A.1** Use geometric shapes, their measures, and their properties (e.g. modeling a tree trunk or a human torso as a cylinder). **G-MG.A.2** density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). **G-MG.A.3** design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	Give an informal argument (L4) Use volume formulas (L2) Identify the shapes of (L1) Use geometric shapes (L2) To describe objects (L1) Apply concepts (L4) Apply geometric methods to solve (L4)

Essential Questions	Big ideas
<u>Essential Questions:</u> What are the three types of measurement? How do you know which measurement to calculate? What are the units of measure associated with each type of measurement? What is similar and different between measuring area and volume? When do you use each? How can the intersection of a solid and a plane be determined?	**<u>Big Ideas:</u>** 1.There are three types of measurement: a. One dimension, length or distance b. Two dimension, area c. Three dimension, volume 2.The intersection of a solid and a plane can be determined by visualizing how the plane slices the solid to form a two-dimensional cross section.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Assessment Task	After Topic B	Constructed response with rubric	G-GMD.A.1, G-GMD.A.3, G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest any new vocabulary Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test the vocabulary Opening Exercise (give again and reflect on results from first administration prior to the unit) Exploratory Challenge Exit Ticket Student Conferences	Mid-Module and End of Module Assessments (given as prescribed in the pacing guide) See chart above for specific information regarding pre and post assessments.

Performance Task
To be created by teacher teams during implementation.
Engaging Learning Experiences To be created by teacher teams during implementation.

Instructional Resources

Suggested Tools and Representations - Three-dimensional models of rectangular prisms, right circular cylinders, right pyramids - Deck of cards - Stack of coins - Images of "sliced" figures, such as a loaf of bread or a stack of deli cheese - IXL Math **Websites:** Building a box http://illuminations.nctm.org/LessonDetail.aspx?id=L570 2D represented as 3D objects http://map.mathshell.org/materials/download.php?fileid=1280

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition	The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students

<table>
<tr>
<td>Nonlinguistic Representations
Homework and Practice
Cooperative Learning
Setting Objectives and Providing Feedback</td>
<td>Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning.
Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations.
Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability.

<u>Provide Multiple Means of Representation</u>

Teach from simple to complex, moving from concrete to representation to abstract at the student’s pace.
Clarify, compare, and make connections to math words in discussion, particularly during and after practice.
Partner key words with visuals (e.g., photo of “ticket”) and gestures (e.g., for “paid”). Connect language (such as ‘tens’) with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with “math-they-can-see,” such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define “multiplication” may model groups of 6 with drawings or concrete objects and write the number sentence to match.
Teach students how to ask questions (such as “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…”
Couple number sentences with models. For example, for equivalent fraction sprint, present
</td>
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	6/8 with: ● Enlarge sprint print for visually impaired learners. ● Use student boards to work on one calculation at a time. ● Invest in or make math picture dictionaries or word walls. **Provide Multiple Means of Action and Expression** ● Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students. ● Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones. ● Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as

	they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” ● Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students

	to focus on content rather than behavior. ● Allow “everyday” and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**
Cavalieri’s Principle (Given two solids that are included between two parallel planes, if every plane parallel to the two planes intersects both solids in cross-sections of equal area, then the volumes of the two solids are equal.) **Cone** (Let □ be a region in a plane □, and □ be a point not in □. The *cone with base* □ *and vertex* □ is the union of all segments <u>□□</u> for all points □ in □. If the base is a polygonal region, then the cone is usually called a *pyramid*.) **General Cylinder** (Let □ and □′ be two parallel planes, let □ be a region in the plane □, and let □ be a line which intersects □ and □′ but not □. At each point □ of □, consider	<u>**Provide Multiple Means of Representation**</u> Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. <u>**Provide Multiple Means of Action**</u>	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. <u>**Provide Multiple Means of Representation**</u> Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. <u>**Provide Multiple Means of Action and Expression**</u>

the segment □□′ parallel to □, joining □ to a point □′ of the plane □′. The union of all these segments is called a *cylinder with base* □.) **Inscribed Polygon** (A polygon is *inscribed in* a circle if all of the vertices of the polygon lie on the circle.) **Intersection** (The *intersection* of □ and □ is the set of all objects that are elements of □ and also elements of □. The intersection is denoted □ ∩ □.) **Rectangular Pyramid** (Given a rectangular region □ in a plane □, and a point □ not in □, the *rectangular pyramid with base* □ *and vertex* □ is the union of all segments □□ for points □ in □.) **Right Rectangular Prism** (Let □ and □′ be two parallel planes. Let □ be a rectangular region in the plane □. At each point □ of □, consider the segment □□′ perpendicular to □, joining □ to a point □′ of the plane □′. The union of all these segments is called a *right rectangular prism*.) **Solid Sphere or Ball** (Given a point □ in the three-dimensional space and	**<u>and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are “just right” for learners but teach the same concepts. Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions.

a number □ > *0*, the *solid sphere (or ball) with center* □ *and radius* □ is the set of all points in space whose distance from point □ is less than or equal to □.) **Sphere** (Given a point □ in the three-dimensional space and a number □ > *0*, the *sphere with center* □ *and radius* □ is the set of all points in space that are distance □ from the point □.) **Subset** (A set □ is a *subset* of a set □ if every element of □ is also an element of □.) **Tangent to a Circle** (A *tangent line to a circle* is a line that intersects a circle in one and only one point.) **Union** (The *union* of □ and □ is the set of all objects that are either elements of □ or of □ or of both. The union is denoted □ ∪ □.) **Familiar Terms and Symbols**[6] Disk Lateral Edge and Face of a Prism	Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next **Foundational Standards** **Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.A.3** Describe the two dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and the area of a circle. **Understand and apply the Pythagorean Theorem.**	Let students write word problems to show mastery and/or extension of the content. **Provide Multiple Means of Engagement** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?" "Can you propose an alternative...?" "How would you evaluate...?" "What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support. **Extension Standards** **Explain volume formulas and use them to solve problems.** **G-GMD.A.2**(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

[6] These are terms and symbols students have seen previously.

	8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **Solve real-life and mathematical problems involving volume of cylinders, cones, and spheres.** **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	

Connecting Algebra and Geometry Through Coordinates-OVERVIEW

In this module, students explore and experience the utility of analyzing algebra and geometry challenges through the framework of coordinates. The module opens with a modeling challenge (**G-MG.A.1**, **G-MG.A.3**), one that reoccurs throughout the lessons, to use coordinate geometry to program the motion of a robot that is bound within a certain polygonal region of the plane—the room in which it sits. To set the stage for complex work in analytic geometry (computing coordinates of points of intersection of lines and line segments or the coordinates of points that divide given segments in specific length ratios, and so on), students will describe the region via systems of algebraic inequalities (**A-REI.D.12**) and work to constrain the robot motion along line segments within the region (**A-REI.C.6**, **G-GPE.B.7**).

The challenge of programming robot motion along segments parallel or perpendicular to a given segment brings in an analysis of slopes of parallel and perpendicular lines and the need to prove results about these quantities (**G-GPE.B.5**). This work highlights the role of the converse of the Pythagorean theorem in the identification of perpendicular directions of motion (**G-GPE.B.4**).

To fully develop the analysis of perimeter and area of a polygon in terms of the coordinates of its vertices
(**G-GPE.B.7**), students will derive the area $\square$ of a triangle with coordinates $(0,0)$, $(\square_1, \square_2)$, and $(\square_2, \square_2)$ as $\square = (\frac{1}{2})|\square_1\square_1 - \square_2\square_2|$ and extend this result to the areas of triangles situated elsewhere in the plane and to simple polygons seen as unions of triangles. Applications to robot motion continue. Students will also find locations on a directed line segment between two given points that partition the segment in given ratios
(**G-GPE.B.6**) and connect this work to proving classical results in geometry (**G-GPE.B.4**). For example, proving that the diagonals of a parallelogram bisect one another, and the medians of a triangle meet at the point $\frac{2}{3}$ of the way from the vertex for each. This study also deepens student understanding of the linear motion of the robot between and beyond two given points.

The module ends with the challenge of locating the point along a line closest to a given point, again given as a robot challenge, and developing the distance formula for a point from a line (**G-GPE.B.4**).

Mathematics/Geometry Unit 4

Rigorous Curriculum Design Template

Unit: 4 Connecting Algebra and Geometry Through Coordinates

Subject: Mathematics

Grade/Course: Mathematics/Geometry

Pacing: 15 Days

Unit of Study: Unit 4 Algebra and Geometry Through Coordinates

Focus Standards

Use coordinates to prove simple geometric theorems algebraically.[7]

G-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.

G-GPE.B.5 Prove[8] the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students start the module with the challenge to understand and develop the mathematics for describing of the motion of a robot bound within a certain polygonal region of the plane—the room in which it sits. This a recurring problem throughout the entire module and with it, and through related problems, students discover the slope criteria for perpendicular and

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parallel lines, the means to find the coordinates of a point dividing a line segment into two lengths in a given ratio, the distance formula of a point from a line, along with a number of geometric results via the tools of coordinate geometry.

MP.2 **Reason abstractly and quantitatively.** Students rotate line segments about their endpoints and discover the general slope criterion for perpendicular lines and articulate this criterion in an abstract setting. Geometric results (such as "the three medians of a triangle are concurrent") are examined in concrete settings and students determine that these results hold in general. They also develop a formula for the area of a triangle based solely on the coordinates of its three vertices and generalize this to an area formula for quadrilaterals and other planar polygons.

MP.4 **Model with mathematics.** Students model the motion of a robot in the plane in two contexts: determining the extent of motion within the bounds of a polygonal region, and determining and moving to the location of the source of beacon signal in the infinite plane.

MP.7 **Look for and make use of structure.** Students determine slope criteria for perpendicular and parallel lines and use these slope conditions to develop the general equation of a line and the formula for the distance of a point from a line. Students determine the area of polygonal regions using multiple methods including Green's theorem and decomposition. Definitive geometric properties of special quadrilaterals are explored and properties of special lines in triangles are examined.

MP.8 **Look for and express regularity in repeated reasoning.** Students use the midpoint to repeatedly separate a segment into proportional parts and derive a formula for calculating the coordinates of a point that will divide a segment into segments of given ratios

"Unwrapped" Standards

G-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.

G-GPE.B.5 Prove[9] the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

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Concepts (What Students Need to Know)	Skills (What Students Need to Be Able to Do) Depth of Knowledge Level
G-GPE.B.4 coordinates, simple geometric theorems algebraically **G-GPE.B.5** parallel and perpendicular lines **G-GPE.B.6** a directed line segment between two given points that partitions the segment in a given ratio. **G-GPE.B.7** perimeters of polygons and areas of triangles and rectangles	Use coordinates (L1) Prove simple theorems (L4) Prove the slope criteria (L4) Use them to solve geometric problems (L2) Find the point on (L1) Use coordinates to compute (L2)

Essential Questions	Big ideas
<u>Essential Questions:</u> What are the triangle congruence postulates/theorems? How do you use them to solve problems? How are rigid motion and dilation used to prove similar figures? How are congruent triangles similar triangles similar and different? How are side lengths or angle measures found in right triangles?	**<u>Big Ideas:</u>** 1.Congruent figures are figures that have corresponding sides are congruent and corresponding angles congruent. 2. Similar figures are figures that have corresponding sides proportional and corresponding angles congruent. 3. Congruent triangles are a specific form of similar triangles whose scale factor is one. 4.The Pythagorean Theorem or trigonometric ratios can be used to find a side length or angle measure of a right triangle

Assessments

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	G-GPE.B.4, G-GPE.B.5, G-GPE.B.7
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7

Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Pretest any new vocabulary Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test the vocabulary Opening Exercise (give again and reflect on results from first administration prior to the unit) Exploratory Challenge Exit Ticket Student Conferences	Mid-Module and End of Module Assessments (given as prescribed in the pacing guide) See chart above for details.

Performance Task
To be created by teachers during implementation next year.
Engaging Learning Experiences
To be created by teachers during implementation next year.

Instructional Resources

Suggested Tools and Representations

- Graph Paper
- Graphing Calculator
- Wolfram Alpha Software
- Geometer's Sketchpad Software
- IXL Math

Dynamic geometry software for creating fractals :

http://illuminations.nctm.org/ActivityDetail.aspx?ID=17

Exploring scale factors:
http://illuminations.nctm.org/ActivityDetail.aspx?ID=176

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **<u>Marzano's Strategies</u>** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of

"ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.

- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:
- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual

	signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” ● Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process.

	- Teach in small chunks so students get a lot of practice with one step at a time. - Know, use, and make the most of Deaf culture and sign language. - Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” - Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. - Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. - Follow predictable routines to allow students to focus on content rather than behavior. - Allow “everyday” and first language to express math understanding. - Re-teach the same concept with a variety of fluency games. - Allow students to lead group and pair-share activities. - Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**

New or Recently Introduced Terms

Normal Segment to a Line (A line segment with one endpoint on a line and perpendicular to the line is called a *normal segment* to the line.)

Familiar Terms and Symbols[10]

- Slope
- Parallel
- Perpendicular
- Distance
- Bisect
- Directed Line Segment

Provide Multiple Means of Representation

Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.

Guide students as they select and practice using their own graphic organizers and models to solve.

Use direct instruction for vocabulary with visual or concrete representations.

Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do."

Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary.

Scaffold complex concepts and provide leveled problems for multiple entry points.

Provide Multiple Means of Action and Expression

First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract.

Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?'

The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

Provide Multiple Means of Representation

Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..."

Incorporate written reflection, evaluation, and synthesis.

Allow creativity in expression and modeling solutions.

Provide Multiple Means of Action and Expression

Encourage students to explain their reasoning both orally and in writing.

Extend exploration of math topics by means of challenging games, puzzles, and brain teasers.

Offer choices of independent or group assignments for early finishers.

Encourage students to notice and

[10] These are terms and symbols students have seen previously.

	Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. <u>Foundational Standards:</u>	explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?" "Can you propose an alternative...?" "How would you

	Solve systems of equations. **A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **Represent and solve equations and inequalities graphically.** **A-REI.D.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. *Apply geometric concepts in modeling situations.* **G-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★ **G-MG.A.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems	evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

	based on ratios).★	

Circles With and Without Coordinates-OVERVIEW

With geometric intuition well established through Modules 1, 2, 3, and 4, students are now ready to explore the rich geometry of circles. This module brings together the ideas of similarity and congruence studied in Modules 1 and 2, the properties of length and area studied in Modules 3 and 4, and the work of geometric construction studied throughout the entire year. It also includes the specific properties of triangles, special quadrilaterals, parallel lines and transversals, and rigid motions established and built upon throughout this mathematical story.

This module's focus is on the possible geometric relationships between a pair of intersecting lines and a circle drawn on the page. If the lines are perpendicular and one passes through the center of the circle, then the relationship encompasses the perpendicular bisectors of chords in a circle and the association between a tangent line and a radius drawn to the point of contact. If the lines meet at a point on the circle, then the relationship involves inscribed angles. If the lines meet at the center of the circle, then the relationship involves central angles. If the lines meet at a different point inside the circle or at a point outside the circle, then the relationship includes the secant angle theorems and tangent angle theorems.

Topic A, through a hands-on activity, leads students first to Thales' theorem (an angle drawn from a diameter of a circle to a point on the circle is sure to be a right angle), then to possible converses of Thales' theorem, and finally to the general inscribed-central angle theorem. Students use this result to solve unknown angle problems. Through this work, students construct triangles and rectangles inscribed in circles and study their properties (**G-C.A.2**, **G-C.A.3**).

Topic B defines the measure of an arc and establishes results relating chord lengths and the measures of the arcs they subtend. Students build on their knowledge of circles from Module 2 and prove that all circles are similar. Students develop a formula for arc length in addition to a formula for the area of a sector and practice their skills solving unknown area problems (**G-C.A.1**, **G-C.A.2**, **G-C.B.5**).

In Topic C, students explore geometric relations in diagrams of two secant lines, or a secant and tangent line (possibly even two tangent lines), meeting a point inside or outside of a circle. They establish the secant angle theorems and tangent-secant angle theorems. By drawing auxiliary lines, students also notice similar triangles and thereby discover relationships between lengths of line segments appearing in these diagrams (**G-C.A.2**, **G-C.A.3**, **G-C.A.4**).

Topic D brings in coordinate geometry to establish the equation of a circle. Students solve problems to find the equations of specific tangent lines or the coordinates of specific points of contact. They also express circles via analytic equations (**G-GPE.A.1**, **G-GPE.B.4**).

The module concludes with Topic E focusing on the properties of quadrilaterals inscribed in circles and establishing Ptolemy's theorem. This result codifies the Pythagorean theorem, curious facts about triangles, properties of the regular pentagon, and trigonometric relationships. It serves as a final unifying flourish for students' year-long study of geometry (**G-C.A.3**).

Mathematics/Geometry/Unit 5

Rigorous Curriculum Design Template

Unit: 5 Circles With and Without Coordinates

Subject: Mathematics

Grade/Course: Mathematics/Geometry

Pacing: 21 Days

Unit of Study: Circles With and Without Coordinates

Focus Standards: Priority Standards

Understand and apply theorems about circles.

G-C.A.1 Prove[11] that all circles are similar.

G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include[12] the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove[2] properties of angles for a quadrilateral inscribed in a circle.

Find arc lengths and areas of sectors of circles.

G-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Translate between the geometric description and the equation for a conic section.

G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically.

G-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students solve a number of complex unknown angles and unknown area geometry problems, work to devise the geometric construction of given objects, and adapt established geometric results to new contexts and to new conclusions.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students must provide justification for the steps in geometric constructions and the reasoning in

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geometric proofs, as well as create their own proofs of results and their extensions.

MP.7 **Look for and make use of structure.** Students must identify features within complex diagrams (e.g., similar triangles, parallel chords, and cyclic quadrilaterals) which provide insight as to how to move forward with their thinking.

“Unwrapped” Standards	
G-C.A.1 Prove[13] that all circles are similar. **G-C.A.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include[14] the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G-C.A.3** Construct the inscribed and circumscribed circles of a triangle, and prove[2] properties of angles for a quadrilateral inscribed in a circle. **G-C.B.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **G-GPE.A.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G-GPE.B.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**

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G-C.A.1	that all circles are similar.	Prove
G-C.A.2	among inscribed angles, radii, and chords. *Include*[15] *the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	Identify and describe relationships
G-C.A.3	the inscribed and circumscribed circles of a triangle, and or a quadrilateral inscribed in a circle.	Construct Prove properties of angles
G-C.B.5	the fact that the length of the arc intercepted by an angle is proportional to the radius, and of the angle as the constant of proportionality; derive the formula for the area of a sector.	Derive using similarity Define the radian measure
G-GPE.A.1	of a circle of given center and radius using the Pythagorean Theorem; to find the center and radius of a circle given by an equation.	Derive the equation Complete the square
G-GPE.B.4	simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.	Use coordinates to prove

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Essential Questions	Big ideas
<u>Essential Questions:</u> 1. How can you prove relationships between angles and arcs of a circle. 2. When lines intersect a circle or within a circle, how do you find the measures of resulting arcs, angle and segments? 3. How do you find the equation of a circle in the coordinate plane? 4. How can relationships between angles and arcs in a circle be proven? 5.How do perimeters and areas of similar figures compare	**<u>Big Ideas:</u>** 1. The measure of an angle that intersect intersects a circle is related to the measure of the arc intersected. 2. Lines and line segments that intersect circles form relationships. 3. The equation of a circle can be written using the circles center and radius and points satisfying that equation line on the circle. 4.The ratio of the perimeters and the ratios of the areas of two similar figures are related to the ratio of the corresponding part

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	G-C.A.1, G-C.A.2, G-C.A.3, G-C.B.5
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	G-C.A.1, G-C.A.2, G-C.A.3, G-GPE.A.1, G-GPE.B.4

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest any new vocabulary Conduct opening exercise Use exit ticket as pre-assessment	Post-test the vocabulary Opening Exercise (give again and reflect on results from first	Mid-Module and End of Module Assessments (given as prescribed in the pacing guide)

and post where applicable	administration prior to the unit) Exploratory Challenge Exit Ticket Student Conferences	
Performance Task To be created by teachers during implementation.		
Engaging Learning Experiences To be created by teachers during implementation.		

Instructional Resources

Suggested Tools and Representations

- Compass and straightedge
- Geometer's Sketchpad or Geogebra Software
- White and colored paper, markers
- IXL Math

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **<u>Marzano's Strategies</u>** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of

"ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.

- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as

	hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” ● Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of

	practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow “everyday” and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms ▪ **Arc Length** (The *length of an arc* is the circular distance around the arc.) ▪ **Central Angle** (A *central angle* of a circle is an angle whose vertex is the center of a circle.)	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions

▪ **Chord** (Given a circle □, let □ and □ be points on □. The $\overline{□□}$ is called a *chord* of □.) ▪ **Cyclic Quadrilateral** (A quadrilateral inscribed in a circle is called a *cyclic quadrilateral*.) ▪ **Inscribed Angle** (An *inscribed angle* is an angle whose vertex is on a circle, and each side of the angle intersects the circle in another point.) ▪ **Inscribed Polygon** (A polygon is *inscribed* in a circle if all vertices of the polygon lie on the circle.) ▪ **Secant Line** (A *secant line* to a circle is a line that intersects a circle in exactly two points.) ▪ **Sector** (Let □□ be an arc of a circle. The *sector of a circle with arc* □□ is the union of all radii of the circle that have an endpoint in arc □□. The arc □□ is called the *arc of the sector*, and the length of any radius of the circle is called the *radius of the sector*.) ▪ **Tangent Line** (A *tangent line to a circle* is a line in the same plane that intersects the circle in one and only one point. This point is called the *point of tangency*.)	vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are “just right” for learners but teach the same concepts. Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>**	(such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint.

Familiar Terms and Symbols[16] - Circle - Diameter - Radius	Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and

[16] These are terms and symbols students have seen previously.

suggestions for ways to extend games.

Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Extension Standards

Apply trigonometry to general triangles.

G-SRT.D.9 (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

Understand and apply theorems about circles.

G-C.A.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Appendix A: Lesson Plan Sample
Module 1 Lesson 1

Lesson 1: Construct an Equilateral Triangle

Student Outcomes

- Students learn to construct an equilateral triangle.
- Students communicate mathematic ideas effectively and efficiently.

Lesson Notes

Most students will have done little more than draw a circle with a compass upon entering 10th grade. The first few lessons on constructions will be a topic where students truly acquire a whole new set of skills.

This lesson begins with a brief Opening Exercise, which requires peer-to-peer conversation and attention to vocabulary. Ensure students understand that, even though the vocabulary terms may be familiar, they should pay careful attention to the **precision of each definition**. For students to develop logical reasoning in geometry, they have to manipulate very exact language, beginning with definitions. Students explore various phrasings of definitions. The teacher guides the discussion until students arrive at a formulation of the standard definition. The purpose of the discussion is to understand why the definition has the form that it does. As part of the discussion, students should be able to test the strength of any definition by looking for possible counterexamples.

Sitting Cats, the main exercise, provides a backdrop to constructing the equilateral triangle. Though students may visually understand where the position of the third cat should be, they will spend time discovering how to use their compass to establish the exact location. (The cat, obviously, will be in a position that approximates the third vertex. The point constructed is the optimal position of the cat—if cats were points and were perfect in their choice of place to sleep.) Students should work without assistance for some portion of the 10 minutes allotted. As students begin to successfully complete the task, elicit discussion about the use of the compass that makes this construction possible.

In the last segment of class, lead students through Euclid's Proposition 1 of Book 1 (Elements 1:1). Have students annotate the text as they read, noting how labeling is used to direct instructions. After reading through the document, direct students to write in their own words the steps they took to construct an equilateral triangle. As part of the broader goal of teaching students to communicate precisely and effectively in geometry, emphasize the need for clear instruction, for labeling in their diagram and reference to labeling in the steps, and for coherent use of relevant vocabulary. Students should begin the process in class together, but should complete the assignment for the Problem Set.

Classwork

Opening Exercise (5 minutes)

Students should test each other's instructions for the construction of an equilateral triangle. The goal is to identify errors in the instructions or opportunities to make the instructions more concise.

Discussion (5 minutes)

- What are common errors? What are concrete suggestions to help improve the instruction-writing process?
  - *Correct use of vocabulary, simple and concise steps (making sure each step involves just one instruction), and clear use of labels.*

It is important for students to describe objects using correct terminology instead of pronouns. Instead of "it" and "they," perhaps "the center" and "the sides" should be used.

Exploratory Challenge one and two (see lesson plan)

For additional sample lessons, please go to: https://www.engageny.org/resource/high-school-geometry

Appendix B: Formative Assessment Sample

Module 1 Lesson 1: Exit Ticket

Lesson 1: Construct an Equilateral Triangle

Exit Ticket

We saw two different scenarios where we used the construction of an equilateral triangle to help determine a needed location (i.e., the friends playing catch in the park and the sitting cats). Can you think of another scenario where the construction of an equilateral triangle might be useful? Articulate how you would find the needed location using an equilateral triangle.

Mid Module Assessment Unit 1: https://www.engageny.org/file/111926/download/geometry-m1-mid-module-assessment.pdf?token=QZOLcBBH4bhkA6LR446n-S-MZg813wf4qchZlVQFNPU

End or Unit Assessment Unit 1: https://www.engageny.org/file/111931/download/geometry-m1-end-of-module-assessment.pdf?token=sG5Y9uLzOT8Af23R-X91L1BsUzUL637cBvEma4vCY4w