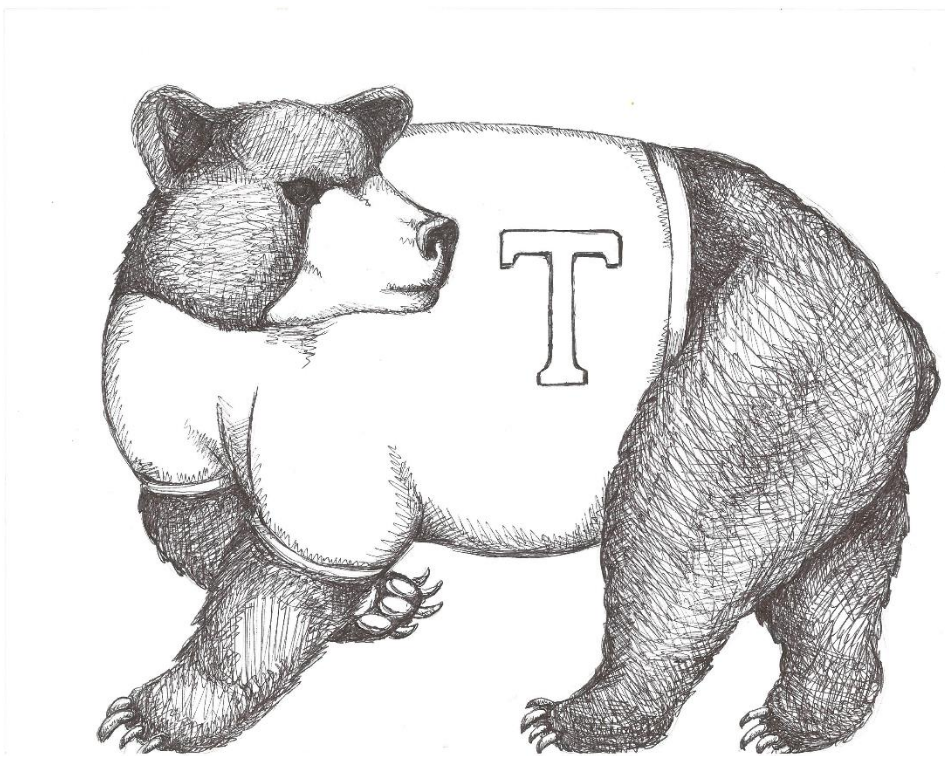
Thomaston Public Schools

158 Main Street
Thomaston, Connecticut 06787
www.thomastonschools.org – 860-283-4796



Thomaston Public Schools Curriculum
Thomaston High School
Algebra II
2015

Learn to Live….Live to Learn

Acknowledgements

<u>Curriculum Writer(s):</u>

Alisha DiCorpo

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

____*Alisha DiCorpo*____________________________________

Alisha L. DiCorpo
Director of Curriculum and Professional Development

Date of Presentation to the Board of Education: _August 2015__________

(Math Curriculum Algebra II)

Algebra II Curriculum Thomaston Public Schools

Board of Education Mission Statement:

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:

The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main Resource used when writing this curriculum:
NYS COMMON CORE MATHEMATICS CURRICULUM A Story of Units/Ratios/Functions Curriculum. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. A Story of Functions: A Curriculum Overview for Grades 9-12 Date: 7/31/13 5 © 2013 Common Core, Inc. Some rights reserved. commoncore.org

Course Description:

Unit 1: Polynomial, Rational, and Radical Relationships
Unit 2: Trigonometric Functions
Unit 3: Functions
Unit 4: Inferences and Conclusions from Data

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and together, with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

<u>Recommended Fluencies for Algebra II</u>

- Divide polynomials with remainder by inspection in simple cases.
- See structure in expressions and use this structure to rewrite expressions (e.g. factoring, grouping).
- Translate between recursive definitions and closed forms for problems involving sequences and series.

	Grade 9 -- Algebra I	Grade 10 -- Geometry	Grade 11 -- Algebra II	Grade 12 -- Precalculus	
20 days	M1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)	M1: Congruence, Proof, and Constructions (45 days)	M1: Polynomial, Rational, and Radical Relationships (45 days)	M1: Complex Numbers and Transformations (40 days)	20 days
20 days					20 days
20 days	M2: Descriptive Statistics (25 days)	M2: Similarity, Proof, and Trigonometry (45 days)	M2: Trigonometric Functions (20 days)	M2: Vectors and Matrices (40 days)	20 days
20 days	M3: Linear and Exponential Functions		M3: Functions (45 days)		20 days
	State Examinations	State Examinations	State Examinations	State Examinations	
20 days	(35 days)			M3: Rational and Exponential Functions (25 days)	20 days
20 days	M4: Polynomial and Quadratic Expressions, Equations and Functions (30 days)	M3: Extending to Three Dimensions (10 days) M4: Connecting Algebra and Geometry through Coordinates (25 days)		M4: Trigonometry (20 days)	20 days
20 days			M4: Inferences and Conclusions from Data (40 days)	M5: Probability and Statistics (25 days)	20 days
20 days	M5: A Synthesis of Modeling with Equations and Functions (20 days)	M5: Circles with and Without Coordinates (25 days)			20 days
20 days	Review and Examinations	Review and Examinations	Review and Examinations	Review and Examinations	20 days

Key:	Number and Quantity and Modeling	Geometry and Modeling	Algebra and Modeling	Statistics and Probability and Modeling	Functions and Modeling

Due to district test decisions that need to be made, the days set aside for testing may be less than the chart above. Feel free to modify this pacing guide as needed until district testing is set each year.

Major Emphasis Clusters
The Real Number System ● Extend the properties of exponents to rational exponents Seeing Structure in Expressions ● Interpret the structure of expressions ● Write expressions in equivalent forms to solve problems Arithmetic with Polynomials and Rational Expressions ● Understand the relationship between zeros and factors of polynomials Reasoning with Equations and Inequalities ● Understand solving equations as a process of reasoning and explain the reasoning ● Represent and solve equations and inequalities graphically Interpreting Functions ● Interpret functions that arise in applications in terms of context Building Functions ● Build a function that models a relationship between two quantities Making Inferences and Justifying Conclusions ● Make inferences and justify conclusions from sample surveys, experiments and observational studies

Unit 1 Overview: Polynomial, Rational, and Radical Relationships

OVERVIEW

In this unit, students draw on their foundation of the analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property (**A-SSE.B.2, A-APR.A.1**). Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers (**A-APR.A.1**, **A-APR.D.6**). Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations (**A-APR.B.3**). Students explore the role of factoring, as both an aid to the algebra and to the graphing of polynomials (**A-SSE.2**, **A-APR.B.2**, **A-APR.B.3**, **F-IF.C.7c**). Students continue to build upon the reasoning process of solving equations as they solve polynomial, rational, and radical equations, as well as linear and non-linear systems of equations (**A-REI.A.1**, **A-REI.A.2**, **A-REI.C.6**, **A-REI.C.7**). The module culminates with the fundamental theorem of algebra as the ultimate result in factoring. Students pursue connections to applications in prime numbers in encryption theory, Pythagorean triples, and modeling problems.

An additional theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students use appropriate tools to analyze the key features of a graph or table of a polynomial function and relate those features back to the two quantities that the function is modeling in the problem (**F-IF.C.7c**).

Math Unit –Algebra II

Rigorous Curriculum Design Template

Unit 1: Unit 1: Polynomial, Rational and Radical Relationships

Subject: Mathematics

Grade/Course: Algebra II

Pacing: 40 Days

Unit of Study: Unit 1: Polynomial, Rational and Radical Relationships

Priority Standards: Focus Standards

Reason quantitatively and use units to solve problems.

N-Q.A.2[1] Define appropriate quantities for the purpose of descriptive modeling.★

Perform arithmetic operations with complex numbers.

N-CN.A.1 Know there is a complex number i such that $i^2 = –1$, and every complex number has the form
$a + bi$ with a and b real.

N-CN.A.2 Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

N-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

Interpret the structure of expressions.

A-SSE.A.2[2] Use the structure of an expression to identify ways to rewrite it. *For example, see* x^4 –

---

1 This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation and then choose to work with peak amplitude.

2 In Algebra II, tasks are limited to polynomial, rational, or exponential expressions. Examples: see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center $(– 1, 0)$. See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.

y^4 as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Understand the relationship between zeros and factors of polynomials.

A-APR.B.2[3] Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-APR.B.3[4] Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

A-APR.C.4 Prove[5] polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

Rewrite rational expressions.

A-APR.D.6[6] Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1[7] Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

A-REI.B.4[8] Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

---

[3] Include problems that involve interpreting the Remainder Theorem from graphs and in problems that require long division.

[4] In Algebra II, tasks include quadratic, cubic, and quadratic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$.

[5] Prove *and apply*

[6] Include rewriting rational expressions that are in the form of a complex fraction.

[7] In Algebra II, tasks are limited to simple rational or radical equations.

[8] In Algebra II, in the case of equations having roots with nonzero imaginary parts, students write the solutions as a ± bi, where *a* and *b* are real numbers.

Solve systems of equations.

A-REI.C.6[9] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.

Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph (by hand in simple cases and using technology for more complicated cases).★

c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.

Translate between the geometric description and the equation for a conic section.

G-GPE.A.2 Derive the equation of a parabola given a focus and directrix.

Foundational Standards

Use properties of rational and irrational numbers.

N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Reason quantitatively and use units to solve problems.

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

Interpret the structure of expressions.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of P and a factor not depending on P.*

---

9 In Algebra II, tasks are limited to 3 x 3 systems.

Write expressions in equivalent forms to solve problems.

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

Perform arithmetic operations on polynomials.

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Create equations that describe numbers or relationships.

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.★

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A-CED.A.3 Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.★

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning used in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R*.★

Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Solve systems of equations.

A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and
$y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Translate between the geometric description and the equation for a conic section.

G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students discover the value of equating factored terms of a polynomial to zero as a means of solving equations involving polynomials. Students solve rational equations and simple radical equations, while considering the possibility of extraneous solutions and verifying each solution before drawing conclusions about the problem. Students solve systems of linear equations and linear and quadratic pairs in two variables. Further, students come to understand that the complex number system provides solutions to the equation $x^2 + 1 = 0$ and higher-degree equations.

MP.2 **Reason abstractly and quantitatively.** Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students also learn to make sense of remainders in polynomial long division problems.

MP.4 **Model with mathematics.** Students use primes to model encryption. Students transition between verbal, numerical, algebraic, and graphical thinking in analyzing applied polynomial problems. Students model a cross-section of a riverbed with a polynomial, estimate fluid flow with their algebraic model, and fit polynomials to data. Students model the locus of points at equal distance between a point (focus) and a line (directrix) discovering the parabola.

MP.7 **Look for and make use of structure.** Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. Students recognize structure in the graphs of polynomials in factored form and develop refined techniques for graphing. Students discern the structure of rational expressions by comparing to analogous arithmetic problems. Students perform geometric operations on parabolas to discover congruence and similarity.

MP.8 **Look for and express regularity in repeated reasoning.** Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques. Further, students understand that all quadratics can be written as a product of linear factors in the complex realm.

"Unwrapped" Standards

N-Q.A.2[10] Define appropriate quantities for the purpose of descriptive modeling.★

N-CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N-CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

A-SSE.A.2[11] Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

A-APR.B.2[12] Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-APR.B.3[13] Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-APR.C.4 Prove[14] polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate*

---

[10] This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation and then choose to work with peak amplitude.

[11] In Algebra II, tasks are limited to polynomial, rational, or exponential expressions. Examples: see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center $(-1, 0)$. See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.

[12] Include problems that involve interpreting the Remainder Theorem from graphs and in problems that require long division.

[13] In Algebra II, tasks include quadratic, cubic, and quadratic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$.

[14]

Pythagorean triples.

A-APR.D.6[15] Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x),* where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-REI.A.1[16] Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.B.4[17] Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A-REI.C.6[18] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph (by hand in simple cases and using technology for more complicated cases).★

c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.

G-GPE.A.2 Derive the equation of a parabola given a focus and directrix.

Concepts (What Students Need to Know	Skills (What Students Need to Be Able to Do)

15 Include rewriting rational expressions that are in the form of a complex fraction.

16 In Algebra II, tasks are limited to simple rational or radical equations.

17 In Algebra II, in the case of equations having roots with nonzero imaginary parts, students write the solutions as a ± bi, where *a* and *b* are real numbers.

18 In Algebra II, tasks are limited to 3 x 3 systems.

	Depth of Knowledge Level
N-Q.A.2[19] appropriate quantities for the purpose of descriptive modeling.	Define quantities (L2)
N-CN.A.1 complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.	Know (L1)
N-CN.A.2 relation $i^2 = -1$ and the commutative, associative, and distributive properties complex numbers.	Use the relation (L2)
N-CN.C.7 quadratic equations with real coefficients that have complex solutions.	Add, Subtract and Multiply (L2) Solve (L2)
A-SSE.A.2[20] ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	Use the structure of an expression to identify (L2)
A-APR.B.2[21] For a polynomial *p(x)* and a number *a*, the remainder on division by $x - a$ is *p(a)*, so *p(a)* = 0 if and only if $(x - a)$ is a factor of *p(x)*.	Know and apply the Remainder Theorem (L2)
A-APR.B.3[22] when suitable factorizations are available, rough graph of the function defined by the polynomial.	Identify zeros of polynomials (L1) Use the zeros to construct a (L3)
A-APR.C.4 numerical relationships	Prove[27] polynomial identities and use them to describe (L3)
A-APR.D.6[23] Simple rational expressions in different forms; *a(x)/b(x)* in the form *q(x) + r(x)/b(x),* where *a(x), b(x), q(x),* and *r(x)* are polynomials with the	Rewrite (L2)

19 This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation and then choose to work with peak amplitude.

20 In Algebra II, tasks are limited to polynomial, rational, or exponential expressions. Examples: see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center (– 1, 0). See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.

21 Include problems that involve interpreting the Remainder Theorem from graphs and in problems that require long division.

22 In Algebra II, tasks include quadratic, cubic, and quadratic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$.

23 Include rewriting rational expressions that are in the form of a complex fraction.

	degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.	Explain each step (L2)
A-REI.A.1[24]	In solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution, viable argument solution method.	Construct (L3) Justify (L3) Solve (L2) Give examples (L2)
A-REI.A.2	Simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Solve (L2) Solve systems (L2)
A-REI.B.4[25]	Quadratic equations in one variable.	Solve a simple system (L2)
A-REI.C.6[26]	Linear equations	
A-REI.C.7	linear equation and a quadratic equation in two variables algebraically and graphically.	Graph functions (L2)
F-IF.C.7	key features of the graph (by hand in simple cases and using technology for more complicated cases).★	Derive the equation (L2)
G-GPE.A.2	parabola given a focus and directrix.	

27 Prove *and apply*

24 In Algebra II, tasks are limited to simple rational or radical equations.

25 In Algebra II, in the case of equations having roots with nonzero imaginary parts, students write the solutions as a ± bi, where *a* and *b* are real numbers.

26 In Algebra II, tasks are limited to 3 x 3 systems.

Essential Questions	Big ideas

Essential Questions:
 	
● When and how is mathematics used in solving real world problems?
 	
● What characteristics of problems would determine how to model the situation and develop a problem solving strategy?
  o *What is the role of complex numbers in the equation solving process?*
 	
● When and why is it necessary to follow set rules/procedures/properties when manipulating numeric or algebraic expressions?
  o *How do the ordered pairs on the graph of an equation relate to the equation itself and then to a system which contains the given equation?*
  o *What are some similarities and differences between the algorithms used for performing operations on rational numbers and the algorithms used for*
	Big Ideas:
	
	● Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities.
	  o *For a given set of numbers there are relationships that are always true and these are the rules that govern arithmetic and algebra*
	  o *Similarities exist between base-ten computation and the arithmetic of polynomials*
	  o *Similarities exist between the arithmetic of rational numbers and the arithmetic of rational expressions.*
	
	● Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations
	  o *The coordinates of the point(s) where the graphs of equations intersect represent the solution(s) to the system of equations formed*

performing operations on rational expressions? o *Why does the equation solving process sometimes produce extraneous solutions?*	*by the equations.* o *There is a connection between the zeros of the polynomial and solutions of polynomial equations.* ● Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. ● Multiple representations may be used to model given real world relationships. ● Mathematics can be used to solve real world problems and can be used to communicate solutions to stakeholders.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pre-test vocabulary Exit ticket as pre-assessment Student interview	Post-test the vocabulary Opening Exercise (give again and reflect on results from first administration prior to the unit) Exploratory Challenge Exit Ticket Student Conferences	(see table below)

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	N-Q.A.2, A-SSE.A.2, A-APR.B.2, A-APR.B.3, A-APR.C.4, A-REI.A.1, A-REI.B.4b, F-IF.C.7c
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	N-Q.A.2, A.SSE.A.2, A.APR.B.2, A-APR.B.3,

					A-APR.C.4, A-APR.D.6, A-REI.A.1, A-REI.A.2, A-REI.B.4b, A-REI.C.6, A-REI.C.7, F-IF.C.7c, G-GPE.A.2

Mid-Module and End of Module Assessments (given as prescribed in the pacing guide) see chart above.

Performance Task
To be created with teacher in 2015-2016 during implementation.
Engaging Learning Experiences
To be created with teacher in 2015-2016 during implementation.

Instructional Resources
Suggested Tools and Representations Graphing Calculator Wolfram Alpha Software Geometer's Sketchpad Software IXL Math

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **<u>Marzano's Strategies</u>** Identifying Similarities and Differences	## Extension Standards The (+) standards below are provided as an extension to Module 1 of the Algebra II course to provide coherence to the curriculum. They are used to introduce themes and concepts that will be fully covered in the Precalculus course. *Note: None of the (+) standards below will be assessed on the Regents Exam or PARCC Assessments until Precalculus.* **Use complex numbers in polynomial**

Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	**identities and equations.** **N-CN.C.8** (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.* **N-CN.C.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **Rewrite rational expressions.** **A-APR.C.7** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the

	best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. ● Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" ● Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with: ● Enlarge sprint print for visually impaired learners. ● Use student boards to work on one calculation at a time. ● Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** ● Provide a variety of ways to respond: oral;

	choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students. ● Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones. ● Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” ● Focus on students’ mathematical reasoning (i.e.,

their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language.

Provide Multiple Means of Engagement

- Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems.
- Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process.
- Teach in small chunks so students get a lot of practice with one step at a time.
- Know, use, and make the most of Deaf culture and sign language.
- Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!”
- Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.
- Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently.
- Follow predictable routines to allow students to focus on content rather than behavior.
- Allow “everyday” and first language to express math understanding.
- Re-teach the same concept with a variety of fluency games.
- Allow students to lead group and pair-share activities.
- Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
<u>**New or Recently Introduced Terms**</u> ▪ **A Square Root of a Number** (*A square root of a number* □ is a number whose square is □. In symbols, a square root of □ is a number □ such that $□^2 = □$. Negative numbers	<u>**Provide Multiple Means of Representation**</u> Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used.

do not have any square roots, zero has exactly one square root, and positive numbers have two square roots.) ▪ **The Square Root of a Number** (Every positive real number x has a unique positive square root called *the square root* or *principle square root* of x; it is denoted $\sqrt{x}$. The square root of zero is zero.) ▪ **Pythagorean Triple** (A *Pythagorean triple* is a triplet of positive integers (a, b, c) such that $a^2 + b^2 = c^2$. The triplet $(3, 4, 5)$ is a Pythagorean triple but $(1, 1, \sqrt{2})$ is not, even though the numbers are side lengths of an isosceles right triangle.) ▪ **End Behavior** (Let f be a function whose domain and range are subsets of the real numbers. The end behavior of a function f is a description of what happens to the values of the function o as x approaches positive infinity, and o as x approaches negative infinity.) ▪ **Even Function** (Let f be a function whose domain and range is a subset of the real numbers. The function f is called *even* if the equation, $f(x) = f(-x)$, is true for every number x in the domain. Even-degree polynomial	Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2.	**<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-

functions are sometimes even functions, such as $f(x) = x^{10}$, and sometimes not, such as $f(x) = x^2 - x$.) ▪ **Odd Function** (Let f be a function whose domain and range is a subset of the real numbers. The function f is called *odd* if the equation, $f(-x) = -f(x)$, is true for every number x in the domain. Odd-degree polynomial functions are sometimes odd functions, such as $f(x) = x^{11}$, and sometimes not, such as $f(x) = x^3 - x^2$.) ▪ **Rational Expression** (A *rational expression* is either a numerical expression or a variable symbol, or the result of placing two previously generated rational expressions into the blanks of the addition operator (__+__), the subtraction operator (__−__), the multiplication operator (__×__), or the division operator (__÷__).) ▪ **Parabola** (A *parabola* with *directrix line* L and *focus point* F is the set of all points in the plane that are equidistant from the point F and line L.) ▪ **Axis of Symmetry** (The *axis of symmetry of a parabola* given by a focus point and a directrix is the perpendicular line to the directrix that passes through the focus.)	**<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend

▪ **Vertex of a Parabola** (The *vertex of a parabola* is the point where the axis of symmetry intersects the parabola.) ▪ **Dilation at the Origin** (A dilation at the origin D_k is a horizontal scaling by $k > 0$ followed by a vertical scaling by the same factor k. In other words, this dilation of the graph of $y = f(x)$ is the graph of the equation $y = kf(\frac{1}{k}x)$. A dilation at the origin is a special type of a dilation.) **Familiar Terms and Symbols**[28] ▪ Sequence ▪ Arithmetic Sequence ▪ Numerical Symbol ▪ Variable Symbol ▪ Algebraic Expression ▪ Numerical Expression ▪ Polynomial Expression ▪ Monomial ▪ Degree of a Monomial ▪ Binomial ▪ Trinomial ▪ Coefficient of a Monomial ▪ Terms of a Polynomial ▪ Like Terms of a Polynomial ▪ Standard Form of a Polynomial in One Variable ▪ Degree of a Polynomial in One Variable ▪ Equivalent Polynomial Expressions		games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

[28] These are terms and symbols students have seen previously.

▪ Polynomial Identity ▪ Function ▪ Polynomial Function ▪ Degree of a Polynomial Function ▪ Constant Function ▪ Linear Function ▪ Quadratic Function ▪ Discriminant of a Quadratic Function ▪ Cubic Function ▪ Zeros or Roots of a Function ▪ Increasing/Decreasing ▪ Relative Maximum ▪ Relative Minimum ▪ Graph of □ ▪ Graph of □ = □(□)		

Unit 2: Trigonometric Functions

OVERVIEW

Unit 2 builds on students' previous work with units (**N-Q.A.1**) and with functions (**F-IF.A.1**, **F-IF.A.2**, **F-IF.B.4**, **F-IF.C.7e**, **F-BF.A.1**, **F-BF.B.3**) from Algebra I, and with trigonometric ratios and circles (**G-SRT.C.6**, **G-SRT.C.7**, **G-SRT.C.8**) from high school Geometry. Included in Topic A is preparation for extension standard **F-TF.A.3**. Extension standard **F-TF.C.9** is also discussed in Topic B as preparation for the Pre-Calculus and Advanced Topics course.

Topic A starts by asking students to graph the height of a Ferris wheel as a function of time and uses that study to help define the sine, cosine, and tangent functions as functions from all (or most) real numbers to the real numbers. A precise definition of sine and cosine (as well as tangent and the co-functions) is developed using transformational geometry from high school Geometry. This precision leads to a discussion of a mathematically *natural* unit of measurement for angle measures, a radian, and students begin to build fluency with values of sine, cosine, and tangent at $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$, etc. The topic concludes with students graphing the sine and cosine functions and noticing various aspects of the graph, which they write down as simple trigonometric identities.

In Topic B, students make sense of periodic phenomena as they model them with trigonometric functions. They identify the periodicity, midline, and amplitude from graphs of data and use them to construct sinusoidal functions that model situations from both the biological and physical sciences. They extend the concept of polynomial identities to trigonometric identities and prove simple trigonometric identities such as the Pythagorean identity; these identities are then used to solve problems.

The Mid-Unit Assessment follows Topic A. The End-of-Unit Assessment follows Topic B.

Math Unit –Algebra II Unit 2

Rigorous Curriculum Design Template

Unit 2: Trigonometric Functions

Subject: Mathematics

Grade/Course: Algebra II

Pacing: 17 Days

Unit of Study: Unit 2: Trigonometric Functions

Priority Standards: Focus Standards

Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Extend the domain of trigonometric functions using the unit circle.

F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.

F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

Prove and apply trigonometric identities.

F-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Summarize, represent, and interpret data on two categorical and quantitative variables.

S-ID.B.6a Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

Foundational Standards

Reason quantitatively and use units to solve problems.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

Understand the concept of a function and use function notation.

F-IF.A.1 **Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.**

F-IF.A.2 **Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.**

Build a function that models a relationship between two quantities.

F-BF.A.1 Write a function that describes a relationship between two quantities.★

b. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

(+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.*

Build new functions from existing functions.

F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Define trigonometric ratios and solve problems involving right triangles.

G-SRT.C.6 **Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.**

G-SRT.C.7 **Explain and use the relationship between the sine and cosine of complementary angles.**

G-SRT.C.8 **Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★**

Math Practice Standards: Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students look for entry points into studying the "height" of the sun above the ground, first by realizing that no such quantity exists, then by surmising that the notion can still be profitably analyzed in terms of trigonometric ratios. They use this and other concrete situations to extend concepts of trigonometry studied in previous years, which were initially limited to angles between zero and ninety-degrees, to the full range of inputs; they also solve challenges about circular motion.

MP.2 **Reason abstractly and quantitatively.** Students extend the study of trigonometry to the domain of all (or almost all) real inputs. By focusing only on the linear components of circular motion (the vertical or the horizontal displacement of a point in orbit), students develop the means to analyze periodic phenomena. Students also extend a classic proof of the Pythagorean theorem to discover trigonometric addition formulas.

MP.3 **Construct viable arguments and critique the reasoning of others.** The vertical and horizontal displacements of a Ferris wheel passenger car are both periodic. Students conjecture how these functions are related to the trigonometric ratios they studied in geometry, making plausible arguments by modeling the Ferris wheel with a circle in the coordinate plane. Also, students construct valid arguments to extend trigonometric identities to the full range of inputs.

MP.4 **Model with mathematics.** The main modeling activity of this module is to analyze the vertical and horizontal displacement of a passenger car of a Ferris wheel. As they make assumptions and simplify the situation, they discover the need for sine and cosine functions to model the periodic motion using sinusoidal functions. Students then model a large number of other periodic phenomena by fitting sinusoidal functions to data given about tides, sound waves, and daylight hours; they then solve problems using those functions in the context of that data.

MP.7 **Look for and make use of structure.** Students recognize the periodic nature of a phenomenon and look for suitable values of midline, amplitude, and frequency for it. The periodicity and properties of cyclical motion shown in graphs helps students to recognize different trigonometric identities, and structure in standard proofs (of the Pythagorean theorem, for example) provides the means to extend familiar trigonometric results to a wider range of input values.

MP.8 **Look for and express regularity in repeated reasoning.** In repeatedly graphing different sinusoidal functions, students identify how parameters within the function give information about the amplitude, midline, and frequency of the function. They express this regularity in terms of a general formula for sinusoidal functions and use the formula to quickly write functions that model periodic data.

“Unwrapped” Standards

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

F-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

S-ID.B.6a Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

c. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
F-IF.C.7 Expressed symbolically, by hand, in simple cases for more complicated cases.★ Functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-TF.A.1** Radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.A.2** coordinate plane enables the extension of trigonometric functions	Graph functions (L2) Show key features of the graph (L1) Using technology (L1) Graph exponential and logarithmic (L2) Understand (L1)

	to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Explain how the unit circle (L1)
F-TF.B.5	specified amplitude, frequency, and midline.★	Choose trigonometric functions (L2) Model periodic phenomena (L2)
F-TF.C.8	Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	Prove the Pythagorean identity (L3)
S-ID.B.6a	Quantitative variables on a scatter plot, variables are related.	Represent data (L2) Describe the variables (L2)

Essential Questions	**Big ideas**
Essential Questions: How can you determine if a relation is a function using a variety of methods? How can real-world situations influence the domain and range of a function? When will a function have an inverse? What can the inverse of a function be used for? How are the critical features of function graphs affected after a transformation?	**Big Ideas:** **Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities.** o *Proving identities requires the use of the rules of arithmetic and algebra to produce equivalent expressions.* o *Evaluating trigonometric functions requires the use of arithmetic and algebraic rules and geometric analysis to understand degree and radian measure.* ● **Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations** o *Trigonometric functions and features such as amplitude, frequency, and midline can be used to model periodic phenomena.* ● **Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.** ● **Mathematics can be used to solve real world problems and can be used to communicate solutions to stakeholders.**

Assessments

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic A	Constructed response with rubric	F-IF.C.7e, F-TF.A.1, F-TF.A.2
End-of-Module Assessment Task	After Topic B	Constructed response with rubric	F-IF.C.7e, F-TF.B.5, F-TF.C.8, S-ID.B.6a

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pre-test vocabulary Exit ticket as pre-assessment Student interview	Post-test the vocabulary Opening Exercise (give again and reflect on results from first administration prior to the unit) Exploratory Challenge Exit Ticket Student Conferences	see table below

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	N-Q.A.2, A-SSE.A.2, A-APR.B.2, A-APR.B.3, A-APR.C.4, A-REI.A.1, A-REI.B.4b, F-IF.C.7c
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	N-Q.A.2, A.SSE.A.2, A.APR.B.2, A-APR.B.3, A-APR.C.4,

					A-APR.D.6, A-REI.A.1, A-REI.A.2, A-REI.B.4b, A-REI.C.6, A-REI.C.7, F-IF.C.7c, G-GPE.A.2

Performance Task
http://schools.nyc.gov/NR/rdonlyres/1EC8789F-28D5-4D77-8AFB-030108B929AD/165318/G11_MinimizingtheMetalinaCan_final.pdf

ALGEBRA II: MINIMIZING THE METAL IN A CAN

UNIT OVERVIEW

This packet contains a curriculum-embedded Common Core–aligned task and instructional supports. The final task assesses student mastery of the Algebra II standards relating to building a function that models a relationship between two quantities, interpreting functions that arise in applications in terms of the context and defining appropriate quantities for the purpose of descriptive modeling.

TASK DETAILS

Task Name: Minimizing the Metal in a Can

Grade: 11

Subject: Algebra II

Depth of Knowledge: 4

Task Description: This task requires students to use an equation and a graph of a function to represent real-world objects and situations. It also requires students to interpret functions, to reason quantitatively, and to use units to solve problems.

Standards Assessed:

F.BF.1 Write a function that describes a relationship between two quantities.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

Standards for Mathematical Practice:

MP.2 Reason abstractly and quantitatively.

MP.4 Model with mathematics.

Engaging Learning Experiences

This task requires students to use an equation and a graph of a function to represent real-world objects and situations. It also requires students to interpret functions, to reason quantitatively, and to use units to solve problems.

Instructional Resources
Suggested Tools and Representations Graphing Calculator Wolfram Alpha Software Geometer's Sketchpad Software IXL Math

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **<u>Marzano's Strategies</u>** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	**Extension Standards** **Extend the domain of trigonometric functions using the unit circle.** **F-TF.A.3** (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. **Prove and apply trigonometric identities.** **F-TF.C.9** (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning.

	Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of "ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. ● Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" ● Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with: ● Enlarge sprint print for visually impaired learners.

	● Use student boards to work on one calculation at a time. ● Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** ● Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. ● Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones. ● Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in "Happy Counting." ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are "just right" for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day's sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including "show and tell" rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we

	counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" ● Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games.

	• Allow students to lead group and pair-share activities. • Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms ▪ **Radian** (A *radian* angle is the angle subtended by an arc of a circle that is equal in length to the radius of the circle. A *radian* (*1* rad) is a unit of rotational measure given by a rotation by a radian angle.) ▪ **Periodic Function** (A function □ whose domain is a subset of the real numbers is said to be *periodic* with period □ > 0 if the domain of □ contains □ $+$ □ whenever it contains □, and if □(□ $+$ □) $=$ □(□) for all real numbers □ in its domain.) ▪ **Sine** (Let □ be any real number. In the Cartesian plane, rotate the initial ray by □ radians about the origin. Intersect the resulting terminal ray with the unit circle to get a point (□$_□$, □$_□$). The value of	<u>**Provide Multiple Means of Representation**</u> Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. <u>**Provide Multiple Means of Action**</u>	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. <u>**Provide Multiple Means of Representation**</u> Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. <u>**Provide Multiple Means of Action and Expression**</u> Encourage students to explain

▯▯▯(▯) is ▯▯.) **Cosine** (Let ▯ be any real number. In the Cartesian plane, rotate the initial ray by ▯ radians about the origin. Intersect the resulting terminal ray with the unit circle to get a point (▯▯, ▯▯). The value of ▯▯▯(▯) is ▯▯.) **Sinusoidal Function** (A periodic function is *sinusoidal* if it can be written in the form ▯(▯) = ▯▯▯▯ (▯(▯ − ▯)) + ▯ for real numbers ▯, ▯, ▯, and ▯. In this form, ● |▯| is called the *amplitude* of the function, ● 2▯/|▯| is the *period* of the function, ● |▯|/2▯ is the *frequency* of the function, ● ▯ is called the *phase shift*, and ● the graph of ▯ = ▯ is called the *midline*. Furthermore, we can see that the graph of the *sinusoidal function* ▯ is obtained by first vertically scaling the graph of the sine function by ▯, then horizontally scaling the resulting graph by 1/▯, and, finally, by horizontally and vertically translating the resulting graph by ▯ and ▯ units, respectively.) **Period** (The *period* ▯ is the	**<u>and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make?	their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of

distance between two consecutive maximal points, or two consecutive minimal points, on the graph of the sinusoidal function.) **Amplitude** (The *amplitude* is the distance between a maximal point of the graph of the sinusoidal function and the midline.) ▪ **Frequency** (The *frequency* of a periodic function is the unit rate of the constant rate defined by the number of cycles per unit length.) ▪ **Midline** (The *midline* is the horizontal line that is halfway between the maximal line and the minimal line.) ▪ **Trigonometric Identity** (A *trigonometric identity* is a statement that two trigonometric functions are equivalent.) ▪ **Tangent** (Let □ be any real number such that □ ≠ □/2 + □□ for all integers □. In the Cartesian plane, rotate the initial ray by □ radians about the origin. Intersect the resulting terminal ray with the unit circle to get a point (□□, □□). The value of □□□(□) is □□/□□.) ▪ **Secant** (Let □ be any real number such that □ ≠ □/2 + □□ for all integers □. In the Cartesian plane,	Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	the content. **Provide Multiple Means of Engagement** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

rotate the initial ray by ▯ radians about the origin. Intersect the resulting terminal ray with the unit circle to get a point (▯$_▯$, ▯$_▯$). The value of ▯▯▯(▯) is $\frac{1}{▯_▯}$.) ▪ **Cosecant** (Let ▯ be any real number such that ▯ ≠ ▯▯ for all integers ▯. In the Cartesian plane, rotate the initial ray by ▯ radians about the origin. Intersect the resulting terminal ray with the unit circle to get a point (▯$_▯$, ▯$_▯$). The value of ▯▯▯(▯) is $\frac{1}{▯_▯}$.) ▪ **Cotangent** (Let ▯ be any real number such that ▯ ≠ ▯▯ for all integers ▯. In the Cartesian plane, rotate the initial ray by ▯ radians about the origin. Intersect the resulting terminal ray with the unit circle to get a point (▯$_▯$, ▯$_▯$). The value of ▯▯▯(▯) is $\frac{▯_▯}{▯_▯}$.) **Familiar Terms and Symbols**[29]		

[29] These are terms and symbols students have seen previously.

Degree Sine, Cosine, Tangent Circle Rotation Identity Asymptote Even and Odd Functions		

Unit 3 Overview: Exponential and Logarithmic Functions

OVERVIEW

In this unit, students synthesize and generalize what they have learned about a variety of function families. They extend the domain of exponential functions to the entire real line (**N-RN.A.1**) and then extend their work with these functions to include solving exponential equations with logarithms (**F-LE.A.4**). They explore (with appropriate tools) the effects of transformations on graphs of exponential and logarithmic functions. They notice that the transformations on a graph of a logarithmic function relate to the logarithmic properties (**F-BF.B.3**). Students identify appropriate types of functions to model a situation. They adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as, "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions," is at the heart of this module. In particular, through repeated opportunities in working through the modeling cycle (see page

61 of the CCLS), students acquire the insight that the same mathematical or statistical structure can sometimes model seemingly different situations.

This unit builds on the work in Algebra I, Units 3 and 5, where students first modeled situations using exponential functions and considered which type of function would best model a given real world situation. The unit also introduces students to the extension standards relating to inverse functions and composition of functions to further enhance student understanding of logarithms.

Topic E is a culminating project spread out over several lessons where students consider applying their knowledge to financial literacy. They plan a budget, consider borrowing money to buy a car and a home, study paying off a credit card balance, and finally, decide how they could accumulate one million dollars.

Math Unit –Algebra II Unit 3

Rigorous Curriculum Design Template

Unit: 3-Exponential and Logarithmic Functions

Subject: Algebra II

Grade/Course: Algebra II

Pacing: 35 Days

Unit of Study: Unit 3: Exponential and Logarithmic Functions

Priority Standards: Focus Standards

Extend the properties of exponents to rational exponents.

N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{\frac{1}{3}}$ *to be the cube root of 5 because we want* $\left(5^{\frac{1}{3}}\right)^3 = 5^{\left(\frac{1}{3}\right)^3}$ *to hold, so* $\left(5^{\frac{1}{3}}\right)^3$ *must equal 5.*

N-RN.A.2[30] Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Reason quantitatively and use units to solve problems.

N-Q.A.2[31] Define appropriate quantities for the purpose of descriptive modeling.★

Write expressions in equivalent forms to solve problems.

A-SSE.B.3[32] Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^{t} *can be rewritten as* $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

A-SSE.B.4[33] Derive the formula for the sum of a finite geometric series (when the common ratio is not *1*), and use the formula to solve problems. *For example, calculate mortgage payments*.★

Create equations that describe numbers or relationships.

A-CED.A.1[34] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.★

Represent and solve equations and inequalities graphically.

30 Including expressions where either base or exponent may contain variables.

31 This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.

32 Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation, such that choosing and producing an equivalent form of the expression reveals something about the situation. In Algebra II, tasks include exponential expressions with rational or real exponents.

33 This standard includes using the summation notation symbol.

34 Tasks have a real-world context. In Algebra II, tasks include exponential equations with rational or real exponents, rational functions, and absolute value functions.

A-REI.D.11[35] Explain why the $\square$-coordinates of the points where the graphs of the equations $\square = \square(\square)$ and $\square = \square(\square)$ intersect are the solutions of the equation $\square(\square) = \square(\square)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $\square(\square)$ and/or $\square(\square)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Understand the concept of a function and use function notation.

F-IF.A.3[36] Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $\square(0) = \square(1) = 1,\ \square(\square + 1) = \square(\square) + \square(\square - 1)$ *for* $\square \geq 1$.

Interpret functions that arise in applications in terms of the context.

F-IF.B.4[37] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* $\square(\square)$ *gives the number of person-hours it takes to assemble* $\square$ *engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6[9] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.C.8[38] Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

---

35 In Algebra II, tasks may involve any of the function types mentioned in the standard.

36 This standard is Supporting Content in Algebra II. This standard should support the Major Content in F-BF.2 for coherence.

37 Tasks have a real-world context. In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.

38 Tasks include knowing and applying A=Pert and A=P(1+rn)ntA=P(1+rn)nt.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{\frac{t}{10}}$, *and classify them as representing exponential growth or decay.*

F-IF.C.9[39] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Build a function that models a relationship between two quantities

F-BF.A.1 Write a function that describes a relationship between two quantities.★

Determine an explicit expression, a recursive process, or steps for calculation from a context.[40]

Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*[41]

F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Build new functions from existing functions.

F-BF.B.3[42] Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\, f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F-BF.B.4 Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2\, x^3$ *or* $f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$.

Construct and compare linear, quadratic, and exponential models and solve problems.

---

39 In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions

40 Tasks have a real-world context. In Algebra II, tasks may involve linear functions, quadratic functions, and exponential functions.

41 Combining functions also includes composition of functions.

42 In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. Tasks may involve recognizing even and odd functions.

F-LE.A.2[43] Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

F-LE.A.4[44] For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★

Interpret expressions for functions in terms of the situation they model.

F-LE.B.5[45] Interpret the parameters in a linear or exponential function in terms of a context.★

Foundational Standards

Use properties of rational and irrational numbers.

N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Interpret the structure of expressions.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Create equations that describe numbers or relationships.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*★

Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

---

43 In Algebra II, tasks will include solving multi-step problems by constructing linear and exponential functions.

44 Students learn terminology that logarithm without a base specified is base 10 and that natural logarithm always refers to base *e*.

45 Tasks have a real-world context. In Algebra II, tasks include exponential functions with domains not in the integers.

Understand the concept of a function and use function notation.

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

Math Practice Standards: Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of rational and real number exponents and in doing so are able to apply exponential functions to solve problems involving exponential growth and decay for continuous domains such as time. They explore logarithms numerically and graphically to understand their meaning and how they can be used to solve exponential equations. Students have multiple opportunities to make connections between information presented graphically, numerically, and algebraically and search for similarities between these representations to further understand the underlying mathematical properties of exponents and logarithms. When presented with a wide variety of information related to financial planning, students make sense of the given information and use appropriate formulas to effectively plan for a long-term budget and savings plan.

MP.2 **Reason abstractly and quantitatively.** Students consider appropriate units when exploring the properties of exponents for very large and very small numbers. They reason about quantities when solving a wide variety of problems that can be modeled using logarithms or exponential functions. Students relate the parameters in exponential expressions to the situations they model. They write and solve equations and then interpret their solutions within the context of a problem.

MP.4 **Model with mathematics.** Students use exponential functions to model situations involving exponential growth and decay. They model the number of digits needed to assign identifiers using logarithms. They model exponential growth using a simulation with collected data. The application of exponential functions and logarithms as a means to solve an exponential equation is a focus of several lessons that deal with financial literacy and planning a budget. Here, students must make sense of several different quantities and their relationships as they plan and prioritize for their future financial solvency.

MP.7 **Look for and make use of structure.** Students extend the laws of exponents for integer exponents to rational and real number exponents. They connect how these laws are related to the properties of logarithms and understand how to rearrange an exponential equation into logarithmic form. Students analyze the structure of exponential and logarithmic functions to understand how to sketch graphs and see how the properties relate to transformations of these types of functions. They analyze the structure of expressions to reveal properties such as recognizing when a function models exponential growth versus decay. Students use the structure of equations to understand how to identify an appropriate solution method.

MP.8 **Look for and express regularity in repeated reasoning.** Students discover the properties of logarithms and the meaning of a logarithm by investigating numeric examples. They develop formulas that involve exponentials and logarithms by extending patterns and examining tables and graphs. Students generalize transformations of graphs of logarithmic functions by examining several different cases.

"Unwrapped" Standards

N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{\frac{1}{3}}$ *to be the cube root of 5 because we want* $\left(5^{\frac{1}{3}}\right)^3 = 5^{\left(\frac{1}{3}\right)^3}$ *to hold, so* $\left(5^{\frac{1}{3}}\right)^3$ *must equal* 5.

N-RN.A.2[46] Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-Q.A.2[47] Define appropriate quantities for the purpose of descriptive modeling.★

A-SSE.B.3[48] Choose and produce an equivalent form of an expression to reveal and explain

[46] Including expressions where either base or exponent may contain variables.

[47] This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.

[48] Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation, such that choosing and producing an equivalent form of the expression reveals something about the situation. In Algebra II, tasks include exponential expressions with rational or real exponents.

properties of the quantity represented by the expression.★

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^{t} can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

A-SSE.B.4[49] Derive the formula for the sum of a finite geometric series (when the common ratio is not *1*), and use the formula to solve problems. *For example, calculate mortgage payments.*★

A-CED.A.1[50] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

A-REI.D.11[51] Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

F-IF.A.3[52] Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.*

F-IF.B.4[53] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6[9] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in

---

49 This standard includes using the summation notation symbol.

50 Tasks have a real-world context. In Algebra II, tasks include exponential equations with rational or real exponents, rational functions, and absolute value functions.

51 In Algebra II, tasks may involve any of the function types mentioned in the standard.

52 This standard is Supporting Content in Algebra II. This standard should support the Major Content in F-BF.2 for coherence.

53 Tasks have a real-world context. In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.

simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.C.8[54] Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{\frac{t}{10}}$, *and classify them as representing exponential growth or decay.*

F-IF.C.9[55] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F-BF.A.1 Write a function that describes a relationship between two quantities.★

Determine an explicit expression, a recursive process, or steps for calculation from a context.[56]

Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*[57]

F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

F-BF.B.3[58] Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F-BF.B.4 Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2\ x^3$ *or*

---

54 Tasks include knowing and applying A=Pert and A=P(1+rn)ntA=P(1+rn)nt.

55 In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions

56 Tasks have a real-world context. In Algebra II, tasks may involve linear functions, quadratic functions, and exponential functions.

57 Combining functions also includes composition of functions.

58 In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. Tasks may involve recognizing even and odd functions.

$f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$.

F-LE.A.2[59] Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

F-LE.A.4[60] For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★

F-LE.B.5[61] Interpret the parameters in a linear or exponential function in terms of a context.★

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)** **Depth of Knowledge Level**
N-RN.A.1 definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	Explain (L1) Rewrite Expressions (L2)
N-RN.A.2[62] expressions involving radicals and rational exponents the properties of exponents.	
N-Q.A.2[63] appropriate quantities for the purpose of descriptive modeling.	Define (L1)
A-SSE.B.3[64] equivalent form of an expression to reveal, properties of the quantity represented by the	Choose and produce (L1 and L2)

59 In Algebra II, tasks will include solving multi-step problems by constructing linear and exponential functions.

60 Students learn terminology that logarithm without a base specified is base 10 and that natural logarithm always refers to base *e*.

61 Tasks have a real-world context. In Algebra II, tasks include exponential functions with domains not in the integers.

62 Including expressions where either base or exponent may contain variables.

63 This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.

64 Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation, such that choosing and producing an equivalent form of the expression reveals something about the situation. In Algebra II, tasks include exponential expressions with rational or real exponents.

	expression.★	Explain (L1)
A-SSE.B.4[65]	formula for the sum of a finite geometric series (when the common ratio is not *1*), and use the formula to solve problems. *For example, calculate mortgage payments*.★	Derive(L3) Create (L2)
A-CED.A.1[66]	equations and inequalities in one variable.	Solve Problems (L2) Explain why (L1)
A-REI.D.11[67]	the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately values	Recognize that (L1)
F-IF.A.3[68]	sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	Interpret (L3)
F-IF.B.4[69]	For a function that models a relationship between two quantities, key features of graphs and tables in terms of the quantities, graphs showing key features given a verbal description of the relationship.	Sketch graphs (L2) Relate (L3)
F-IF.B.5	domain of a function to its graph and, where applicable, to the quantitative relationship it describes	Calculate and Interpret (L3) Estimate rate of change (L2)
F-IF.B.6[9]	the average rate of change of a function (presented symbolically or as a table) over a specified interval. the rate of change from	Graph functions (L2) Show key features (L2)

65 This standard includes using the summation notation symbol.

66 Tasks have a real-world context. In Algebra II, tasks include exponential equations with rational or real exponents, rational functions, and absolute value functions.

67 In Algebra II, tasks may involve any of the function types mentioned in the standard.

68 This standard is Supporting Content in Algebra II. This standard should support the Major Content in F-BF.2 for coherence.

69 Tasks have a real-world context. In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.

	a graph.★	Write a function (L2)
F-IF.C.7	functions expressed symbolically and key features of the graph, by hand in simple cases and using technology for more complicated cases.★	Explain (L1) Compare (L3)
F-IF.C.8[70]	defined by an expression in different but equivalent forms to reveal and different properties of the function.	Write a function (L2) Write (L2)
F-IF.C.9[71]	properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	Identify the effect (L2)
F-BF.A.1	function that describes a relationship between two quantities.	
F-BF.A.2	arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	
F-BF.B.3[72]	the effect on the graph of replacing □(□) by □(□) + □, □ □(□), □(□□), and □(□ + □) for specific values of □ (both positive and negative); find the value of □ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	Find inverse (L2) Construct (L3) Express as a logarithm the solution to (L2)

70 Tasks include knowing and applying A=Pert and A=P(1+rn)ntA=P(1+rn)nt.

71 In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions

72 In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. Tasks may involve recognizing even and odd functions.

F-BF.B.4 inverse functions. **F-LE.A.2**[73] linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **F-LE.A.4**[74] For exponential models, □□$^{□□}$ = □ where □, □, and □ are numbers and the base □ is 2, 10, or □; evaluate the logarithm using technology.★ **F-LE.B.5**[75] in a linear or exponential function in terms of a context.★	Interpret the parameters (L3)

Essential Questions	**Big ideas**

[73] In Algebra II, tasks will include solving multi-step problems by constructing linear and exponential functions.
[74] Students learn terminology that logarithm without a base specified is base 10 and that natural logarithm always refers to base *e*.
[75] Tasks have a real-world context. In Algebra II, tasks include exponential functions with domains not in the integers.

Essential Questions:	Big Ideas:
1. How do you determine when more than one equation is needed to solve a problem? *2. What are the different methods of solving systems of equations? How do you determine the best method to apply?* 3. *What are the different possible solution set of a system of linear equations?*	*1. Solutions are not always unique.* *2. Multiple methods can be used to solve problems.* *3. Sometimes, more than one function is needed to solve some problems.*

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	N-RN.A.1, N-RN.A.2, N-Q.A.2, A.CED.A.1, F-IF.B.6, F-BF.A.1a, F-LE.A.4
End-of-Module Assessment Task	After Topic E	Constructed response with rubric	A-SSE.B.3c, A-SSE.B.4, A-CED.A.1, A-REI.D.11, F-IF.A.3, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7e, F-IF.C.8b, F-IF.C.9, F-BF-A.1a, F-BF.A.1b, F-BF.A.2, F-BF.B.3, F-BF.B.4a, F-LE.A.2, F-LE.A.4, F-LE.B.5

Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Pretest any new vocabulary Conduct opening exercise Use exit ticket as pre-assessment	Post-test the vocabulary Opening Exercise (give again and reflect on results from first administration prior to the unit)	**Please see chart above**

and post where applicable	Exploratory Challenge Exit Ticket Student Conferences	

Performance Task
To be created with teachers throughout the 2015-2016 school year.
Engaging Learning Experiences
To be created with teachers throughout the 2015-2016 school year.

Instructional Resources
Suggested Tools and Representations Graphing calculator or Desmos online calculator simulation Wolfram Alpha Software GeoGebra or Geometer's Sketchpad Software Excel or other spreadsheet software, such as Calc (part of the OpenOffice suite) IXL Math

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **<u>Marzano's Strategies</u>** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of

"ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.

- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:

- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as hand pointed downward means count backwards in

	"Happy Counting." ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are "just right" for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day's sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including "show and tell" rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" ● Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones

	up first/Make a bundle if you can!” ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as “Say ‘right angle’ and show me a right angle with your legs,” and “Make groups of 5 right now!” Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as “Happy Counting.” Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow “everyday” and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**
New or Recently Introduced Terms **Arithmetic Series** (An *arithmetic series* is a series whose terms form an arithmetic sequence.) **Geometric Series** (A *geometric series* is a series whose terms form a geometric sequence.) **Invertible Function** (Let ⍰ be a function whose domain is the set ⍰, and whose image is the set ⍰. Then ⍰ is *invertible* if there exists a function ⍰ with domain	<u>**Provide Multiple Means of Representation**</u> Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. <u>**Provide Multiple Means of Representation**</u> Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?”

⍰ and image ⍰ such that ⍰ and ⍰ satisfy the property: For all ⍰ ∈ ⍰ and ⍰ ∈ ⍰, ⍰(⍰) = ⍰ if and only if ⍰(⍰) = ⍰. The function ⍰ is called the *inverse* of ⍰, and is denoted ⍰$^{-1}$. The way to interpret the property is to look at all pairs (⍰, ⍰) ∈ ⍰ × ⍰: If the pair (⍰, ⍰) makes ⍰(⍰) = ⍰ a true equation, then ⍰(⍰) = ⍰ is a true equation. If it makes ⍰(⍰) = ⍰ a false equation, then ⍰(⍰) = ⍰ is false. If that happens for each pair in ⍰ × ⍰, then ⍰ and ⍰ are invertible and are inverses of each other.) **Logarithm** (If three numbers, ⍰, ⍰, and ⍰ are related by ⍰ = ⍰$^{⍰}$, then ⍰ is the *logarithm base ⍰ of ⍰*, and we write ⍰ = ⍰⍰⍰$_{⍰}$ (⍰). That is, the value of the expression ⍰⍰⍰$_{⍰}$(⍰) is the power of ⍰ needed to be equivalent to ⍰. Valid values of ⍰ as a base for a logarithm are $0 <$ ⍰ < 1 and ⍰ > 1.) **Series** (Let ⍰$_1$, ⍰$_2$, ⍰$_3$, ⍰$_4$, … be a sequence of numbers. A sum of the form ⍰$_1$ + ⍰$_2$ + ⍰$_3$ + ⋯ + ⍰$_{⍰}$ for some positive integer ⍰ is called a *series,* or *finite series,* and is denoted ⍰$_{⍰}$. The ⍰$_{⍰}$'s are	students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly	"I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip- counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one.

called the *terms* of the series. The number that the series adds to is called the *sum* of the series. Sometimes S_n is called the n^{th} *partial sum*.) • e (Euler's number, e, is an irrational number that is approximately equal to $e \approx 2.7182818284590$.) • Σ (The Greek letter sigma, Σ , is used to represent the sum. There is no rigid way to use Σ to represent a summation, but all notations generally follow the same rules. We will discuss the most common way it is used. Given the sequence $a_1, a_2, a_3, a_4, \ldots$, we can write the sum of the first n terms of the sequence using the expression: $\sum_{i=1}^{n} a_i$.) **Familiar Terms and Symbols**[76]	for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for

76 These are terms and symbols students have seen previously.

▪ Compound Interest ▪ Exponential Decay ▪ Exponential Expression ▪ Exponential Growth ▪ Scientific Notation		practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Unit 4 Overview: Inferences and Conclusions from Data

OVERVIEW

The concepts of probability and statistics covered in Grade 11 build on students' previous work in Grades 7 and 9. Topics A and B address standards **S-CP.A.1–5** and **S-CP.B.6–7**, which deal primarily with probability. In Topic A, fundamental ideas from Grade 7 are revisited and extended to allow students to build a more formal understanding of probability. More complex events are considered (unions, intersections, complements)
(**S-CP.A.1**). Students calculate probabilities based on two-way data tables and interpret them in context (**S-CP.A.4**). They also see how to create "hypothetical 1000" two-way tables as a way of calculating probabilities. Students are introduced to conditional probability (**S-CP.A.3**, **S-CP.A.5**), and the important concept of independence is developed (**S-CP.A.2**, **S-CP.A.5**). The final lessons in this topic introduce probability rules (**S-CP.B.6**, **S-CP.B.7**).

Topic B is a short topic consisting of four lessons. This topic introduces the idea of using a smooth curve to model a data distribution, describes properties of the normal distribution, and asks students to distinguish between data distributions for which it would be reasonable to use a normal distribution as a model and those for which a normal distribution would not be a reasonable model. In the final two lessons of this topic, students use tables and technology to find areas under a normal curve and interpret these areas in the context of modeling a data distribution (**S-ID.A.4**).

Topics C and D develop students' understanding of statistical inference and introduce different types of statistical studies (observational studies, surveys, and experiments) (**S-IC.B.3**). In Topic C, students explore using data from a random sample to estimate a population mean or a population proportion. Building on what they learned about sampling variability in Grade 7, students use simulation to create an understanding of margin of error. Students calculate the margin of error and interpret it in context (**S-IC.B.4**). Students also evaluate reports from the media using sample data to estimate a population mean or proportion (**S-IC.B.6**).

Topic D focuses on drawing conclusions based on data from a statistical experiment. Given data from a statistical experiment, students use simulation to create a randomization distribution and use it to determine if there is a significant difference between two treatments (**S-IC.B.5**). Students also critique and evaluate published reports based on statistical experiments that compare two treatments (**S-IC.B.6**).

Math Unit –Algebra II Unit 4

Rigorous Curriculum Design Template

Unit: 4 Inferences and Conclusions from Data

Subject: Algebra II

Grade/Course: Algebra II

Pacing: 30 Days

Unit of Study: Unit:4 Inferences and Conclusions from Data

Priority Standards: Focus Standards

Summarize, represent, and interpret data on a single count or measurement variable.

S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Understand and evaluate random processes underlying statistical experiments.

S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S-IC.B.6 Evaluate reports based on data.

Understand independence and conditional probability and use them to interpret data.

S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

S-CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP.A.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare results.*

S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S-CP.B.7 Apply the Addition Rule, $P(A \text{ or } B) – P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model.

Foundational Standards

Using random sampling to draw inferences about a population.

7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 [illegible] greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type blood, what is the probability that it will take at least 4 donors to find one with type blood?*

Summarize, represent, and interpret data on a single count or measurement variable.

S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables.

S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Math Practice Standards: Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively**. Students use data from a sample to estimate a population mean or proportion and generalize from a sample to the population. They associate a margin of error with estimates based on a sample and interpret them in the context of generalizing from a sample to the population. Students also make conjectures or claims about independence and use arguments based on probabilities to support them.

MP.3 **Construct viable arguments and critique the reasoning of others**. Students test conjectures about treatment differences in the context of a statistical experiment. Students critique and evaluate reports based on data from random samples and reports based on data from experiments. Students frequently develop conjectures and use statistical reasoning to evaluate them.

MP.4 **Model with mathematics**. Students use smooth curves to model data distributions. Students use the normal distribution as a model in order to answer questions about a data distribution. Students use probability models to describe real-world contexts.

MP.5 **Use appropriate tools strategically**. Students use technology to carry out simulations in order to study sampling variability. Students also use technology to compute estimates of population characteristics (such as the mean and standard deviation) and to calculate margin of error. Students also use simulation to investigate statistical significance in the context of comparing treatments in a statistical experiment.

“Unwrapped” Standards

S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S-IC.A.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
S-IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S-IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
S-IC.B.6	Evaluate reports based on data.
S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
S-CP.A.3	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a*

randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare results.

S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

S-CP.B.6 Find the conditional probability of □ given □ as the fraction of □'s outcomes that also belong to □, and interpret the answer in terms of the model.

S-CP.B.7 Apply the Addition Rule, □(□ □□ □)– □(□) + □(□) – □(□ □□□ □), and interpret the answer in terms of the model.

Concepts (What Students Need to Know)		Skills (What Students Need to Be Able to Do) Depth of Knowledge Level
S-ID.A.4	the mean and standard deviation of a data set to a normal distribution and population percentages there are data sets for which such a procedure is not appropriate, calculators, spreadsheets, and tables, areas under the normal curve.	Use the mean (L1) Fit mean and standard deviation (L2) Estimate population percentages (L2) Recognize (L1)
S-IC.A.1	statistics as a process for making inferences about population parameters based on a random sample from that population.	Use calculators, spreadsheets and tables (L2) Estimate areas (L2)
S-IC.A.2	specified model is consistent with results from a given data-generating process,	Understand statistics (L1) Decide if a model (L2)
S-IC.B.3	purposes of and differences among sample surveys, experiments, and observational studies; randomization	Recognize the purposes and differences (L1) Explain how randomization….(L1)
S-IC.B.4	data from a sample survey population mean or proportion; margin of error through the use of simulation models for random sampling.	Use data (L2) Estimate a population (L2)
S-IC.B.5	data from a randomized experiment two treatments; use	Develop a margin of error (L2)

	simulations differences between parameters are significant.	Use data to compare (L2)
S-IC.B.6	reports based on data.	Use simulations to decide (L2) Evaluate reports (L3)
S-CP.A.1	events as subsets of a sample space (the set of outcomes) characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	Describe events (L2)
S-CP.A.2	two events □ and □ are independent if the probability of □ and □ occurring together is the product of their probabilities, characterization to determine if they are independent.	Understand two events (L1)
S-CP.A.3	conditional probability of □ given □ as □(□ □□□ □)/□(□), and interpret independence of □ and □ as saying that the conditional probability of □ given □ is the same as the probability of □, and the conditional probability of □ given □ is the same as the probability of □.	Use the characterization (L2) Determine (L2) Understand the probability (L1) Interpret independence (L3) Construct and interpret (L3)
S-CP.A.4	two-way frequency tables of data when two categories are associated with each object being classified. two-way table as a sample space, decide if events are independent and to approximate conditional probabilities.	Decide if events (L2) Recognize and explain concepts (L1) Find (L1) Apply (L2)
S-CP.A.5	concepts of conditional probability and independence in everyday language and everyday situations.	Interpret the answer (L3)
S-CP.B.6	conditional probability of □ given □ as the fraction of □’s outcomes that also belong to □, and the answer in terms of the model.	
S-CP.B.7	Addition Rule,	

□(□ □□ □)– □(□) + □(□) – □(□ □□□ □), and the answer in terms of the model.	

Essential Questions	**Big ideas**
What are some different ways a data set can be organized and displayed for others to analyze? In what ways do sampling techniques help us? What sampling technique would be best for gathering this data and why are they best?	Data displays allow us to make conclusions about a set of data. We can make predictions and inferences about an entire population using sampling techniques. There is a specific sample technique that we use when gathering specific sets of data.

Assessments: Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric.	S-ID.A.4, S-IC.A.2, S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7
End-of-Module Assessment Task	After Topic D	Constructed response with rubric.	S-ID.A.4, S-IC.A.1, S-IC.A.2, S-IC.B.3, S-IC.B.4, S-IC.B.5, S-IC.B.6

Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest any new vocabulary	Post-test the vocabulary	Mid-Module and End of Module

Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Opening Exercise (give again and reflect on results from first administration prior to the unit) Exploratory Challenge Exit Ticket Student Conferences	Assessments (given as prescribed in the pacing guide) See chart above for details.

Performance Task
To be created by teachers during the 2015-2016 school year during implementation.
Engaging Learning Experiences To be created by teachers during the 2015-2016 school year during implementation.

Instructional Resources

Suggested Tools and Representations

Graphing calculator or graphing software

Random-number tables

Random-number software

Normal distribution

Two-way frequency tables

Spreadsheets

IXL Math

Instructional Strategies	Meeting the Needs of All Students
<u>21st Century Skills</u> Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **<u>Marzano's Strategies</u>** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up A Story of Functions propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Charts at the end of this section offer suggested scaffolds, utilizing this framework, for English Language Learners, Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. Additionally, individual lessons contain marginal notes to teachers (in text boxes) highlighting specific UDL information about scaffolds that might be employed with particular intentionality when working with students. These tips are strategically placed in the lesson where the teacher might use the strategy to the best advantage. It is important to note that the scaffolds/accommodations integrated into A Story of Units might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** ● Teach from simple to complex, moving from concrete to representation to abstract at the student's pace. ● Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ● Partner key words with visuals (e.g., photo of

"ticket") and gestures (e.g., for "paid"). Connect language (such as 'tens') with concrete and pictorial experiences (such as money and fingers). Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.

- Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…"
- Couple number sentences with models. For example, for equivalent fraction sprint, present 6/8 with:
- Enlarge sprint print for visually impaired learners.
- Use student boards to work on one calculation at a time.
- Invest in or make math picture dictionaries or word walls.

<u>Provide Multiple Means of Action and Expression</u>

- Provide a variety of ways to respond: oral; choral; student boards; concrete models (e.g., fingers), pictorial models (e.g., ten-frame); pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students.
- Vary choral response with written response (number sentences and models) on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones.
- Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses, such as

	hand pointed downward means count backwards in “Happy Counting.” ● Adjust wait time for interpreters of deaf and hard-of-hearing students. ● Select numbers and tasks that are “just right” for learners. ● Model each step of the algorithm before students begin. ● Give students a chance to practice the next day’s sprint beforehand. (At home, for example.) ● Give students a few extra minutes to process the information before giving the signal to respond. ● Assess by multiple means, including “show and tell” rather than written. ● Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” ● Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” ● Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** ● Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. ● Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. ● Teach in small chunks so students get a lot of

	practice with one step at a time. ● Know, use, and make the most of Deaf culture and sign language. ● Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" ● Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. ● Incorporate activity. Get students up and moving, coupling language with motion, such as "Say 'right angle' and show me a right angle with your legs," and "Make groups of 5 right now!" Make the most of the fun exercises for activities like sprints and fluencies. Conduct simple oral games, such as "Happy Counting." Celebrate improvement. Intentionally highlight student math success frequently. ● Follow predictable routines to allow students to focus on content rather than behavior. ● Allow "everyday" and first language to express math understanding. ● Re-teach the same concept with a variety of fluency games. ● Allow students to lead group and pair-share activities. ● Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	**Students Achieving Below Standard**	**Students Achieving Above Standard**
<u>**New or Recently Introduced Terms**</u> **Complement of an Event** (The *complement of an event*, A, denoted by A^C, is the event that A does not occur.) **Conditional Probability** (The probability of an event given that some other event occurs. The conditional probability	<u>**Provide Multiple Means of Representation**</u> Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. <u>**Provide Multiple Means of Representation**</u> Teach students how to ask questions (such as,"Do you agree?" and "Why

of ▯ given ▯ is denoted by ▯(▯).) **Experiment** (An *experiment* is a study in which subjects are assigned to treatments for the purpose of seeing what effect the treatments have on some response.) **Hypothetical 1000 Table** (A *hypothetical 1000 table* is a two-way table that is constructed using given probability information. It represents a hypothetical population of 1000 individuals that is consistent with the given probability distribution and also allows calculation of other probabilities of interest.) **Independent Events** (Two events ▯ and ▯ are independent if ▯(▯) = ▯(▯). This implies that knowing that ▯ has occurred does not change the probability that ▯ has occurred.) **Intersection of Two Events** (The *intersection of two events*, ▯ and ▯, denoted by ▯ ∩ ▯, is the event that ▯ and ▯ both occur.) **Lurking Variable** (A *lurking variable* is one that causes two variables to have a high relationship even though there is no	representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points. **<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. ’Would you restate that answer in a different way or show me by using a diagram?’ Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are “just right” for learners but teach the same concepts. Adjust numbers in calculations to suit learner’s levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures,	do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>** Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of

real direct relationship between the two variables.) **Margin of Error** (The *margin of error* is the maximum likely error when data from a sample are used to estimate a population characteristic, such as a population proportion or a population mean.) **Normal Distribution** (A *normal distribution* is a distribution that is bell-shaped and symmetric.) **Observational Study** (An *observational study* is one in which the values of one or more variables are observed with no attempt to affect the outcomes.) **Random Assignment** (*Random assignment* is the process of using a chance mechanism to assign individuals to treatments in an experiment.) **Random Selection** (*Random selection* is the process of selecting individuals for a sample using a chance mechanism that ensures that every individual in the population has the same chance of being selected.) **Sample Survey** (A *sample survey* is an observational study in which people respond to one or more	and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next	response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect

questions.) **Treatment** (A *treatment* is something administered in an experimental study.) **Union of Two Events** (The *union of two events*, □ and □, denoted by □ ∪ □, is the event that either □ or □ or both occur.) **Familiar Terms and Symbols**[77] Association Chance experiment Conditional relative frequency Distribution shape (skewed, symmetric) Event Mean Sample space Sampling variability Standard deviation		their need for guidance and support.

[77] These are terms and symbols students have seen previously.

Appendix A: Lesson Plan Examples Link to Module 1 Lessons

https://www.engageny.org/resource/algebra-ii-module-1

Appendix B: Assessment Examples: Module 1: Exit Tickets, Mid and End of Unit Assessments

Mid Module Assessment Link: https://www.engageny.org/file/121731/download/algebra-ii-m1-mid-module-assessment.docx?token=0xOMrAqaOmredjMTgmiF2nBDlx-XQRFJTlZhd5tLUkI

End of Module Assessment Link: https://www.engageny.org/file/121721/download/algebra-ii-m1-end-of-module-assessment.docx?token=_PYq13o1X8PJs2DGZCVA92-dtRUbcn9I1O3SXN2sB5o

Exit Ticket Embedded in Each Lesson:
https://www.engageny.org/file/120616/download/algebra-ii-m1-teacher-materials.zip?token=x6fQ-yt2tLdTjfJNSzmNSoQbRAKWFGgDVXF1aIWbhiA