Thomaston Public Schools

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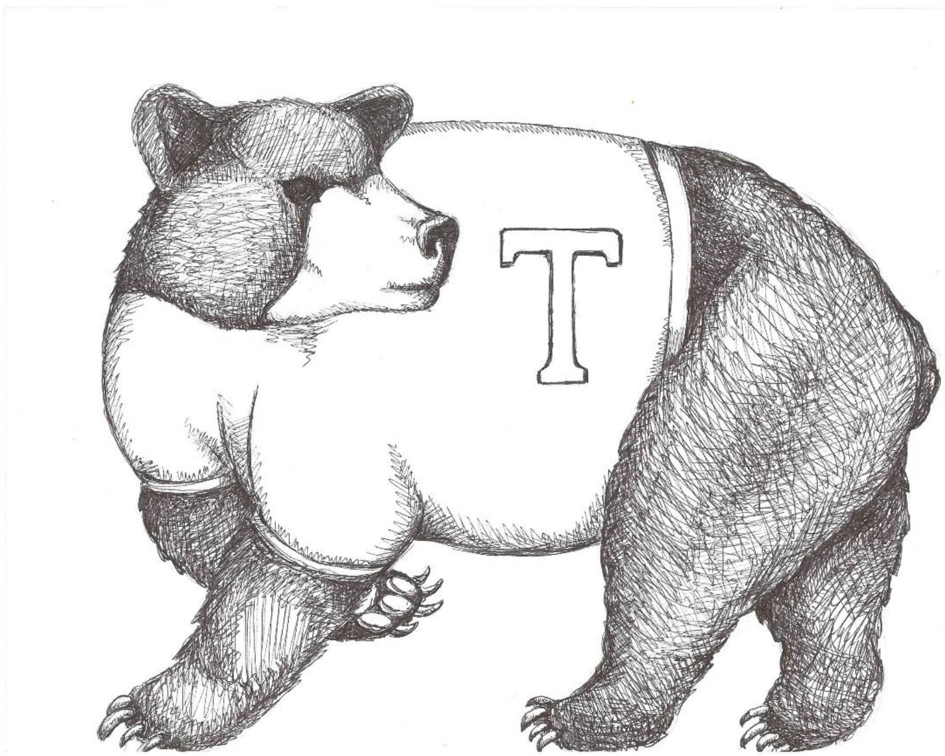

Thomaston Public Schools Curriculum
Thomaston High School
Mathematics: Algebra I   2015

Learn to Live, Live to Learn

Acknowledgements

Curriculum Writer(s):

Mark Olsen

We acknowledge and celebrate the professionalism, expertise, and diverse perspectives of these teachers. Their contributions to this curriculum enrich the educational experiences of all Thomaston students.

______*Alisha DiCorpo*________________________________
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Algebra I
Mathematics - Algebra I

Board of Education Mission Statement:

IN A PARTNERSHIP OF FAMILY, SCHOOL AND COMMUNITY, OUR MISSION IS TO EDUCATE, CHALLENGE AND INSPIRE EACH INDIVIDUAL TO EXCEL AND BECOME A CONTRIBUTING MEMBER OF SOCIETY.

Departmental Philosophy:

The Mathematics Department strives to instill in each student a conceptual understanding of and procedural skill with the basic facts, principles and methods of mathematics. We want our students to develop an ability to explore, to make conjectures, to reason logically and to communicate mathematical ideas. We expect our students to learn to think critically and creatively in applying these ideas. We recognize that individual students learn in different ways and provide a variety of course paths and learning experiences from which students may choose. We emphasize the development of good writing skills and the appropriate use of technology throughout our curriculum. We hope that our students learn to appreciate mathematics as a useful discipline in describing and interpreting the world around us.

Main Resource used when writing this curriculum:
NYS COMMON CORE MATHEMATICS CURRICULUM A Story of Functions Curriculum. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License. A Story of Functions: A Curriculum Overview for Grades 9-12 Date: 7/31/13 5 © 2013 Common Core, Inc. Some rights reserved. commoncore.org

Course Description:

Sequence of Algebra I Modules Aligned with the Standards
Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs
Module 2: Descriptive Statistics
Module 3: Linear and Exponential Functions
Module 4: Polynomial and Quadratic Expressions, Equations and Functions
Module 5: A Synthesis of Modeling with Equations and Functions

Summary of Year

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The modules deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Algebra I

Solving characteristic problems involving the analytic geometry of lines, including, writing the equation of a line given a point and a slope.
Adding, subtracting and multiplying polynomials.
Transforming expressions and chunking (seeing the parts of an expression as a single object) as used in factoring, completing the square, and other algebraic calculations.

CCLS Major Emphasis Clusters

Seeing Structure in Expressions
- Interpret the structure of expressions

Arithmetic with Polynomials and Rational Expressions
- Perform arithmetic operations on polynomials

Creating Equations
- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities
- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Represent and solve equations and inequalities graphically

Interpreting Functions
- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context

Interpreting Categorical and Quantitative Data
- Interpret linear models

Rationale for Module Sequence in Algebra I

Module 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain precisely the process of solving an equation. Students, through reasoning, develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities and make conjectures about the form that a linear equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations. They learn the terminology specific to polynomials and understand that polynomials form a system analogous to the integers.

Module 2: This module builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students display and interpret graphical representations of data, and if appropriate, choose regression techniques when building a model that approximates a linear relationship between quantities. They analyze their knowledge of the context of a situation to justify their choice of a linear model. With linear models, they plot and analyze residuals to informally assess the goodness of fit.

Module 3: In earlier grades, students defined, evaluated, and compared functions in modeling relationships between quantities. In this module, students learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on their understanding of integer exponents to consider exponential functions with integer domains. They compare and contrast linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions. In building models of relationships between two quantities, students analyze the key features of a graph or table of a function.

Module 4: In this module, students build on their knowledge from Module 3. Students strengthen their ability to discern structure in polynomial expressions. They create and solve equations involving quadratic and cubic expressions. In this module's modeling applications, students reason abstractly and quantitatively in interpreting parts of an expression that represent a quantity in terms of its context; they also learn to make sense of problems and persevere in solving them by choosing or producing equivalent forms of an expression (e.g., completing the square in a quadratic expression to reveal a maximum value). Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They learn through repeated reasoning to anticipate the graph of a quadratic function by interpreting the structure of various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function.

Module 5: In this module, students expand their experience with functions to include more specialized functions—linear, exponential, quadratic, square, and cube root, and those that are piecewise-defined, including absolute value and step. Students select from among these functions to model phenomena using the modeling cycle.

Curriculum Map / Pacing Guide

Note: Adjustments should be made to accommodate testing schedules as they are made available. Pacing is based on the testing of the 2014-2015 school year.

	Grade 9 -- Algebra I	Grade 10 -- Geometry	Grade 11 -- Algebra II	Grade 12 -- Precalculus	
20 days	M1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)	M1: Congruence, Proof, and Constructions (45 days)	M1: Polynomial, Rational, and Radical Relationships (45 days)	M1: Complex Numbers and Transformations (40 days)	20 days
20 days					20 days
20 days	M2: Descriptive Statistics (25 days)	M2: Similarity, Proof, and Trigonometry (45 days)	M2: Trigonometric Functions (20 days)	M2: Vectors and Matrices (40 days)	20 days
20 days	M3: Linear and Exponential Functions		M3: Functions (45 days)		20 days
	State Examinations	State Examinations	State Examinations	State Examinations	
20 days	(35 days)	M3: Extending to Three Dimensions (10 days)		M3: Rational and Exponential Functions (25 days)	20 days
20 days	M4: Polynomial and Quadratic Expressions, Equations and Functions (30 days)	M4: Connecting Algebra and Geometry through Coordinates (25 days)		M4: Trigonometry (20 days)	20 days
20 days			M4: Inferences and Conclusions from Data (40 days)		20 days
20 days	M5: A Synthesis of Modeling with Equations and Functions (20 days)	M5: Circles with and Without Coordinates (25 days)		M5: Probability and Statistics (25 days)	20 days
20 days	Review and Examinations	Review and Examinations	Review and Examinations	Review and Examinations	20 days

Key:	Number and Quantity and Modeling	Geometry and Modeling	Algebra and Modeling	Statistics and Probability and Modeling	Functions and Modeling

Relationships Between Quantities and Reasoning with Equations and Their Graphs

Overview

By the end of Grade 8, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students are introduced to non-linear equations and their graphs. Students formalize their understanding of equivalent algebraic expressions and begin their study of polynomial expressions. Further, they learn that there are some actions that, when applied to the expressions on both sides of an equal sign, will not result in an equation with the same solution set as the original equation. Finally, they encounter problems that induce the full modeling cycle, as it is described in the Common Core Learning Standards for Mathematics.

In Topic A, students explore the main functions that they will work with in Algebra I: linear, quadratic, and exponential. The goal is to introduce students to these functions by having them make graphs of situations (usually based upon time) in which the functions naturally arise **(A-CED.A.2)**. As they graph, they reason abstractly and quantitatively as well as choose and interpret units to solve problems related to the graphs they create **(N-Q.A.1, N-Q.A.2, N-Q.A.3)**.

In middle school, students applied the properties of operations to add, subtract, factor, and expand expressions **(6.EE.A.3, 6.EE.A.4, 7.EE.A.1, 8.EE.A.1)**. Now, in Topic B, students use the structure of expressions to define what it means for two algebraic expressions to be equivalent. In doing so, they discern that the commutative, associative, and distributive properties help link each of the expressions in the collection together, even if the expressions look very different themselves **(A-SSE.A.2).** They learn the definition of a polynomial expression and build fluency in identifying and generating polynomial expressions as well as adding, subtracting, and multiplying polynomial expressions **(A-APR.A.1).** The Mid-Module Assessment follows Topic B.

Throughout middle school, students practice the process of solving linear equations **(6.EE.B.5, 6.EE.B.7, 7.EE.B.4, 8.EE.C.7)** and systems of linear equations **(8.EE.C.8)**. Now, in Topic C, instead of just solving equations, they formalize descriptions of what they learned before (variable, solution sets, etc.) and are able to explain, justify, and evaluate their reasoning as they strategize methods for solving linear and non-linear equations **(A-REI.A.1, A-REI.B.3, A-CED.A.4)**. Students take their experience solving systems of linear equations further as they prove the validity of the addition method, learn a formal definition for the graph of an equation and use it to explain the reasoning of solving systems graphically, and represent the solution to systems of linear inequalities graphically **(A-CED.A.3, A-REI.C.5, A-REI.C.6, A-REI.D.10, A-REI.D.12)**.

In Topic D, students are formally introduced to the modeling cycle (see page 61 of the CCLS) through problems that can be solved by creating equations and inequalities in one variable, systems of equations, and graphing **(N-Q.A.1, A-SSE.A.1, A-CED.A.1, A-CED.A.2, A-REI.B.3)**. The End-of-Module Assessment follows Topic D.

Relationships Between Quantities and Reasoning with Equations and Their Graphs

Unit 1

Subject: Mathematics

Grade/Course: Grade 9 / Algebra I

Pacing: 40 days

Unit of Study: Unit 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs

Priority Standards:

Reason quantitatively and use units to solve problems.

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

Perform arithmetic operations on polynomials.

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Create equations that describe numbers or relationships.

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Foundational Standards

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3°C > -7°C$ to express the fact that $-3°C$ is warmer than $-7°C$.

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.3 Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

Reason about and solve one-variable equations and inequalities.

6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

6.EE.B.8 Write an inequality of the form x > c or x < c to represent a constraint or condition in a realworld or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Use properties of operations to generate equivalent expressions.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

Work with radicals and integer exponents.

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 32 × 3–5 = 3–3 = 1/33 = 1/27.

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form x 2 = p and x 3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √ is irrational.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Focus Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Students are presented with problems that require them to try special cases and simpler forms of the original problem to gain better understanding of the problem.

MP.2 Reason abstractly and quantitatively.
Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.

MP.3 Construct viable arguments and critique the reasoning of others.
Students reason about solving equations using "if-then" moves based on equivalent expressions and properties of equality and inequality. They analyze when an "if-then" move is not reversible.

MP.4 Model with mathematics.
Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace from modeling bacteria growth to understanding the federal progressive income tax system.

MP.6 Attend to precision.
Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (e.g., squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation.

MP.7 Look for and make use of structure.
Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves.

MP.8 Look for and express regularity in repeated reasoning.
After solving many linear equations in one variable, students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: . They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.

"Unwrapped" Standards

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in

two variables as the intersection of the corresponding half-planes.	
Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
Units.	use (L3) - units to solve problems choose (L3) - appropriate units interpret (L2) - units choose (L3) - scales and origins in graphical displays interpret (L2) - scales and origins in graphical displays
The structure of expressions.	interpret (L2) - parts of an expression interpret (L2) - complicated expressions
Arithmetic operations on polynomials.	compare (L2) - properties of polynomials to properties of integers perform (L1) - operations on polynomials
Equations that describe numbers or relationships.	create (L4) - one- and two-variable equations create (L4) - one-variable inequalities use (L1) - equations to solve one-variable problems graph (L2) - two-variable equations represent (L3) - constraints using equations and inequalities interpret (L2) - solutions rearrange (L1) - formulas
Solution of equations as a process of reasoning.	explain (L2) - steps in solving equations
One-variable equations and inequalities.	solve (L2) - one-variable equations solve (L2) - one-variable inequalities
Systems of equations.	solve (L2) - by elimination solve (L2) - by substitution solve (L2) - by graphing
Graphs of equations and inequalities.	understand (L2) - graph is a set of solutions graph (L2) - inequalities

Essential Questions	**Big ideas**
When and how is mathematics used in solving real world problems?	Mathematics can be used to solve real world problems and can be used to communicate solutions. Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations. Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities.
When and why is it necessary to follow set rules/procedures/properties when manipulating numeric or algebraic expressions?	Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. Multiple representations may be used to model given real world relationships.
What characteristics of problems would determine how to model the situation and develop a problem solving strategy?	

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary New terms Piecewise-Linear Function Numerical Symbol Variable Symbol Numerical Expression Algebraic Expression Equivalent Numerical Expressions Equivalent Algebraic	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences	Type: Mid-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addressed: N-Q.A.1, N-Q.A.2, N-Q.A.3, A-APR.A.1, A-SSE.A.2 Type: End-of-Module Assessment Task

Expressions Polynomial Expression Monomial Degree of a Monomial Standard Form of a Polynomial Expression in One Variable Degree of a Polynomial in Standard Form Leading Term and Leading Coefficient of a Polynomial in Standard Form Constant Term of a Polynomial in Standard Form Solution Solution Set Graph of an Equation in Two Variables Zero Product Property Familiar Terms and Symbols Equation Identity Inequality System of Equations Properties of Equality Properties of Inequality Solve Linear Function Formula Term Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	IXL Math	Administered: After Topic D Format: Constructed response with rubric Standards Addresses: N-Q.A.1, A-SSE.A.1, A-SSE.A.2, A-APR.A.1, A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4, A-REI.A.1, A-REI.B.3, A-REI.C.5, A-REI.C.6, A-REI.D.10, A-REI.D.12

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task detailed in Unit 3.
Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 3.

Instructional Resources
Suggested Tools and Representations Coordinate Plane Equations and Inequalities Graphing calculator ixl.com (Math) flippedmath.com (Algebra I) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/algebra-i-module-1-topic-lesson-1 (See Appendix A for an example.) http://illuminations.nctm.org/Lesson.aspx?id=1189

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Precalculus propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-

	pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..." Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are "just right" for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day's sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond. Assess by multiple means, including "show and tell" rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking

	process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities.

	Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Piecewise-Linear Function** - a function from the union of the intervals to the set of real numbers such that the function is defined by linear functions on each interval. **Numerical Symbol** - a symbol that represents a specific number. **Variable Symbol** - a symbol that is a placeholder for a number. **Numerical Expression** - an algebraic expression that contains only numerical symbols and that evaluates to a single number. **Algebraic Expression** **Equivalent Numerical Expressions** - numerical expressions that evaluate to the same number. **Equivalent Algebraic Expressions** - algebraic expressions are equivalent if we can convert one expression into the other by repeatedly applying the commutative, associative, and distributive properties and the	**Provide Multiple Means of Representation** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **Provide Multiple Means of Representation** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **Provide Multiple Means of Action and Expression**

properties of rational exponents to components of the first expression. **Polynomial Expression** **Monomial** - A monomial is a polynomial expression generated using only the multiplication operator. **Degree of a Monomial** - the sum of the exponents of the variable symbols that appear in the monomial. **Standard Form of a Polynomial Expression in One Variable** **Degree of a Polynomial in Standard Form** - the highest degree of the terms in the polynomial, namely . **Leading Term and Leading Coefficient of a Polynomial in Standard Form** - The a_nx^n term is called the leading term, and a_n is called the leading coefficient. **Constant Term of a Polynomial in Standard Form** - the value of the numerical expression found by substituting 0 into all the variable symbols of the polynomial. **Solution** - a number in the domain of the variable that, when substituted for all instances of the variable in both expressions, makes the equation a true number sentence. **Solution Set** - The set of solutions of an equation. **Graph of an Equation in Two Variables** - The set of all points in the coordinate plane that are	**<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.

solutions to an equation in two variables. **Zero Product Property** **Familiar Terms and Symbols** Equation Identity Inequality System of Equations Properties of Equality Properties of Inequality Solve Linear Function Formula Term	the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom’s Taxonomy with questions such as: “What would happen if...?”“Can you propose an alternative...?”“How would you evaluate...?”“What choice would you have made...?” Ask “Why?” and “What if?” questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Descriptive Statistics

Overview

In this module, students reconnect with and deepen their understanding of statistics and probability concepts first introduced in Grades 6, 7, and 8. There is variability in data, and this variability often makes learning from data challenging. Students develop a set of tools for understanding and interpreting variability in data and begin to make more informed decisions from data. Students work with data distributions of various shapes, centers, and spreads. Measures of center and measures of spread are developed as ways of describing distributions. The choice of appropriate measures of center and spread is tied to distribution shape. Symmetric data distributions are summarized by the mean and mean absolute deviation, or standard deviation. The median and the interquartile range summarize data distributions that are skewed. Students calculate and interpret measures of center and spread and compare data distributions using numerical measures and visual representations.

Students build on their experience with bivariate quantitative data from Grade 8; they expand their understanding of linear relationships by connecting the data distribution to a model and informally assessing the selected model using residuals and residual plots. Students explore positive and negative linear relationships and use the correlation coefficient to describe the strength and direction of linear relationships. Students also analyze bivariate categorical data using two-way frequency tables and relative frequency tables. The possible association between two categorical variables is explored by using data summarized in a table to analyze differences in conditional relative frequencies.

This module sets the stage for more extensive work with sampling and inference in later grades.

Descriptive Statistics

Unit 2

Subject: Mathematics

Grade/Course: Grade 9 / Algebra

Pacing: 25 Days

Unit of Study: Unit 2: Descriptive Statistics

Priority Standards:

Summarize, represent, and interpret data on a single count or measurement variable

S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables

S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.C.9 Distinguish between correlation and causation.

Foundational Standards

Develop understanding of statistical variability.

6.SP.A.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

6.SP.A.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.A.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots. Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Investigate patterns of association in bivariate data.

8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

8.SP-A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Focus Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation and summary for a given data distribution.

MP.2 Reason abstractly and quantitatively.
Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. Students form summaries of data using graphs, two-way tables, and other representations that are appropriate for a given context and the statistical question they are trying to answer. Students reason about whether two variables are associated by considering conditional relative frequencies.

MP.3 Construct viable arguments and critique the reasoning of others.
Students examine the shape, center, and variability of a data distribution and use characteristics of the data distribution to communicate the answer to a statistical question in the form of a poster presentation. Students also have an opportunity to critique poster presentations made by other students.

MP.4 Model with mathematics.
Students construct and interpret two-way tables to summarize bivariate categorical data. Students graph bivariate numerical data using a scatterplot and propose a linear, exponential, quadratic, or other model to describe the relationship between two numerical variables. Students use residuals and residual plots to assess if a linear model is an appropriate way to summarize the relationship between two numerical variables.

MP.5 Use appropriate tools strategically.
Students visualize data distributions and relationships between numerical variables using graphing software. They select and analyze models that are fit using appropriate technology to determine whether or not the model is appropriate. Students use visual representations of data distributions from technology to answer statistical questions.

MP.6 Attend to precision.
Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students use statistical terminology appropriately.

“Unwrapped” Standards	
S-ID.A.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

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c. Fit a linear function for a scatter plot that suggests a linear association.

S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.C.9 Distinguish between correlation and causation.

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
One variable quantitative data.	Represent (L2) - one-variable data
Measures of center and spread.	Compare (L2) - data
Differences in shape, center and spread.	Interpret ((L2) - data
Outliers.	Evaluate (L4) - effects
Categorical data.	Summarize (L2)
	Interpret (L2)
	Recognize (L1) - associations and trends
	Represent (L2) - using scatterplots
Two quantitative variables.	Describe (L2) - relationship of data
Families of functions.	Represent (L2) - data
	Choose (L3) - appropriate representation
Fit of a function.	Assess (L3)

Residual plot. Linear function. Slope and intercept of a linear function. Correlation coefficient of linear fit. Correlation and causation.	Analyze (L4) Create (L4) Interpret (L2) - in context Compute (L1) - using technology Interpret (L2) Distinguish between (compare) (L2)

Essential Questions	**Big ideas**
Why is data collected and analyzed? How do people use data to influence others? When and how is mathematics used in solving real world problems?	Data can be used to solve real world problems and can be used to communicate solutions. The way that data is collected, organized and displayed influences interpretation. Mathematics can be used to solve real world problems and can be used to communicate solutions. Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary New terms Skewed Data Distribution Sample Standard Deviation Interquartile Range Association Conditional Relative Frequency Residual Residual Plot Correlation Coefficient Familiar Terms and Symbols Mean Median Data Distribution Variability Mean Absolute Deviation Box plot Quartile Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences IXL Math	Type: Mid-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addressed: S-ID.A.1, S-ID.A.2, S-ID.A.3 Type: End-of-Module Assessment Task Administered: After Topic D Format: Constructed response with rubric Standards Addresses: S-ID.A.2, S-ID.A.3, S-ID.B.5, S-ID.B.6, S-ID.C.7, S-ID.C.8, S-ID.C.9

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task detailed in Unit 3.
Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 3.

Instructional Resources
Suggested Tools and Representations Graphing calculator Spreadsheet software Dot plot Box plot Histogram Residual plot ixl.com (Math) flippedmath.com (Algebra I) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/algebra-i-module-1-topic-lesson-1 (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students

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New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Skewed Data Distribution** - a data distribution that is not symmetric with respect to its mean. **Outlier** - a value that is greater than Q_3 by a distance of $1.5 \cdot IQR$, or a value that is less than Q_1 by a distance of $1.5 \cdot IQR$. **Sample Standard Deviation** - the principle (positive) square root of the sample variance. **Interquartile Range** - the length of the interval that contains the middle 50% of the data values. **Association** - any relationship between measures of two types of quantities so that one is statistically dependent on the other. **Conditional Relative Frequency** - compares a frequency count to the marginal total that represents the condition of interest. **Residual** - the (actual y -value) – (predicted y-value) for the given x .	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>**

Residual Plot **Correlation Coefficient** - The correlation coefficient, often denoted by ⍰, is a number between −1 and +1, inclusively, that measures the strength and direction of a linear relationship. **Familiar Terms and Symbols** Mean Median Data Distribution Variability Mean Absolute Deviation Box plot Quartile	<u>**Provide Multiple Means of Action and Expression**</u> First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. <u>**Provide Multiple Means of Engagement**</u> Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.

	the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Linear and Exponential Functions

Overview

In earlier grades, students define, evaluate, and compare functions and use them to model relationships between quantities **(8.F.A.1, 8.F.A.2, 8.F.A.3, 8.F.B.4, 8.F.B.5)**. In this module, students extend their study of functions to include function notation and the concepts of domain and range. They explore many examples of functions and their graphs, focusing on the contrast between linear and exponential functions. They interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations.

In Topic A, students explore arithmetic and geometric sequences as an introduction to the formal notation of functions **(F-IF.A.1, F-IF.A.2)**. They interpret arithmetic sequences as linear functions with integer domains and geometric sequences as exponential functions with integer domains **(F-IF.A.3, F-BF.A.1a)**. Students compare and contrast the rates of change of linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change **(F-IF.B.6, F-LE.A.1, F-LE.A.2, F-LE.A.3)**.

In Topic B, students connect their understanding of functions to their knowledge of graphing from Grade 8. They learn the formal definition of a function and how to recognize, evaluate, and interpret functions in abstract and contextual situations **(F-IF.A.1, F-IF.A.2)**. Students examine the graphs of a variety of functions and learn to interpret those graphs using precise terminology to describe such key features as domain and range, intercepts, intervals where the function is increasing or decreasing, and intervals where the function is positive or negative **(F-IF.A.1, F-IF.B.4, F-IF.B.5, F-IF.C.7a).**

In Topic C, students extend their understanding of piecewise functions and their graphs including the absolute value and step functions. They learn a graphical approach to circumventing complex algebraic solutions to equations in one variable, seeing them as ▯(▯) = ▯(▯) and recognizing that the intersection of the graphs of ▯(▯) and ▯(▯) are solutions to the original equation **(A-REI.D.11)**. Students use the absolute value function and other piecewise functions to investigate transformations of functions and draw formal conclusions about the effects of a transformation on the function’s graph **(F-IF.C.7, F-BF.B.3)**.

Finally, in Topic D, students apply and reinforce the concepts of the module as they examine and compare exponential, piecewise, and step functions in a real-world context **(F-IF.C.9)**. They create equations and functions to model situations **(A-CED.A.1, F-BF.A.1, F-LE.A.2)**, rewrite exponential expressions to reveal and relate elements of an expression to the context of the problem **(A-SSE.B.3c, F-LE.B.5)**, and examine the key features of graphs of functions, relating those features to the context of the problem **(F-IF.B.4, F-IF.B.6)**.

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Linear and Exponential Functions

Unit 3

Subject: Mathematics

Grade/Course: Grade 9 / Algebra

Pacing: 35 days

Unit of Study: Unit 3: Linear and Exponential Functions

Priority Standards:

Write expressions in equivalent forms to solve problems.

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions.

Create equations that describe numbers or relationships.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Represent and solve equations and inequalities graphically.

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Understand the concept of a function and use function notation.

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Interpret functions that arise in applications in terms of the context.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Build a function that models a relationship between two quantities.

F-BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Build new functions from existing functions

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Foundational Standards

Work with radicals and integer exponents

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Define, evaluate, and compare functions

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Reason quantitatively and use units to solve problems.

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $\square(1 + \square)^{\square}$ as the product of □ and a factor not depending on □.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $\square^4 - \square^4$ as $(\square^2)^2 - (\square^2)^2$, thus recognizing it as a difference of squares that can be factored as $(\square^2 - \square^2)(\square^2 + \square^2)$.

Create equations that describe numbers or relationships.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $\square = \square\square$ to highlight resistance □.

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Focus Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Students are presented with problems that require them to try special cases and simpler forms of the original problem to gain insight into the problem.

MP.2 Reason abstractly and quantitatively.
Students analyze graphs of non-constant rate measurements and apply reason (from the shape of the graphs) to infer the quantities being displayed and consider possible units to represent those quantities.

MP.4 Model with mathematics.
Students have numerous opportunities to solve problems that arise in everyday life, society, and the workplace (e.g., modeling bacteria growth and understanding the federal progressive income tax system).

MP.7 Look for and make use of structure.
Students reason with and analyze collections of equivalent expressions to see how they are linked through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves (e.g., 2□ + 4 = 10, 2(□ − 3) + 4 = 10, 2(3□ − 4) + 4 = 10).

MP.8 Look for and express regularity in repeated reasoning.
After solving many linear equations in one variable, students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters. They pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequalities, to find equivalent expressions and solve equations, while recognizing common ways to solve different types of equations.

"Unwrapped" Standards

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context. Build new functions from existing functions

F-BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
Properties of exponents.	Use (L1)
Expressions for exponential functions.	Transform (L4)
Equations and inequalities in one variable.	Create (L4) Solve (L2) - problems
Graphical intersection of two functions.	Explain (L2) Solve (L2)
Function notation.	Understand (L2) Evaluate (L4) Interpret (L2)
Functions from sequences whose domain is a subset of integers.	Recognize (L1)
Key features of graphs and tables for a function that models a relationship between two variables including;. intercept intervals where the function is increasing, decreasing or constant relative maximums and minimums end behavior periodicity	Identify (L1)
Domain of a function.	Relate (L2) - to graph
Average rate of change of a function.	Calculate (L1) Interpret (L2) Estimate (L2) - from a graph
Linear and quadratic functions.	Graph (L2) Evaluate (L4)
Properties of two functions.	Compare (L2)
Functions that describe a relationship between two quantities.	Create (L4) - an explicit expression Create (L4) - a recursive process

Transformations of functions, Even and odd functions. Rate of change of a linear function. Rate of change of an exponential function. Linear and exponential functions.	Explain (L2) - steps for calculation from context Identify (L2) - effects Recognize (L1) - from graph Recognize (L1) - from algebraic expressions Prove (L4) Recognize (L1) Prove (L4) Recognize (L1) Construct (L3) Compare (L2) Interpret (L2) - parameters

Essential Questions	**Big ideas**
When and how is mathematics used in solving real world problems? When and why is it necessary to follow set rules/procedures/properties when manipulating numeric or algebraic expressions? What characteristics of problems would determine how to model the situation and develop a problem solving strategy?	Mathematics can be used to solve real world problems and can be used to communicate solutions. Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations. Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities. Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. Multiple representations may be used to model given real world relationships.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary New terms Function Domain Range Linear Function Average Rate of Change Piecewise Linear Function Familiar Terms and Symbols Numerical Symbol Variable Symbol Constant Numerical Expression Algebraic Expression Number Sentence Truth Values of a Number Sentence Equation Solution Solution Set Simple Expression Factored Expression Equivalent Expressions Polynomial Expression Equivalent Polynomial Expressions Monomial Coefficient of a Monomial Terms of a Polynomial Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences IXL Math	Type: Mid-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addressed: F-IF.A.1, F-IF.A.2, F-IF.A.3, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7a, F-BF.A.1a, F-LE.A.1, F-LE.A.2, F-LE.A.3 Type: End-of-Module Assessment Task Administered: After Topic D Format: Constructed response with rubric Standards Addresses: A-CED.A.1, A-REI.D.11, A-SSE.B.3c, F-IF.A.1, F-IF.A.2, F-IF.A.3, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-IF.C.9, F-BF.A.1a, F-BF.B.3, F-LE.A.1, F-LE.A.2, F-LE.A.3, F-LE.B.5

Performance Task

Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task detailed below.

"The Benefits of Business" (The "engaging" scenario that accompanies this performance task is detailed following the tasks in the "Engaging Learning Experiences" section.)

Task 1: Students design logo for the business using lines on a coordinate plane. Students use at least two specific equations of lines and identify points on lines. Once created, the logo is transferred to plain paper.

Task 2: Given profit lines for competing companies, students analyze the y-intercepts and slopes.

Task 3: Students use calculators to discover how m and b affect the line using slope-intercept form.

Task 4: Students apply linear equations to the company's cost and price per item.

Algebra I: Benefits of Business
RUBRIC

The elements of performance required by this task are:

Construct a function to model a linear relationship between two quantities.
Graph linear functions and show slopes and intercepts.
Compare properties of two functions.
Create equations in one variable and use them to solve problems

Possible Responses Point distribution

1. Total possible points - 8
   a) Decide on product and creates appropriate original logo on coordinate plane. - 2 points
   b) Generate linear function for at least two lines making up the logo. - 4 points (1 pt for each slope and intercept)
   c) Logo is transferred to plain paper ready for company use (art, color) - 2 pts
   Partial credit
   Computational errors for part (b) - deduct 1
2. Total possible points - 11
   a) Determines the equation of the five year growth model. - 2 pt (1 pt for each slope and intercept)
   b) Interprets the meaning of the slope and intercept of the equation generated in part (a) - 3 points
   c) Determines the equations of all three sections of piecewise linear model of yearly growth model. - 6 (1 pt for each slope and intercept) - 2
   Partial credit
   Computational errors for part (a) and (c) - deduct 1
3. Total possible points - 10
   a) Explains the effect of changing the slopes of the growth equations found in part (2) using examples - 3 points
   b) Explains the effect of changing the y-intercepts of the growth equations found in part (2)

using examples - 3 points

c) Correctly determines the equation for an increase in growth of 3% for the second half of the year - 2 points

d) Correctly reports the effect on the final profit figure for the year of the increase in part (c) - 2 points

Partial credit

Correct explanation without examples in parts(a) and (b) - deduct 1

Correct examples with unclear explanation in parts (a) and (b) -deduct 1

Computational errors - deduct 1

4. Total possible points - 7

a) Graphs sales data on the coordinate plane - 2 points

b) Determines quadratic function to model sales data - 3 points

c) Determines optimum price for product - 2 points

Partial credit

Computational errors - deduct 1

TOTAL POSSiBLE POiNTS = 36

Algebra I: Benefits of Business

Rubric

Performance Level Descriptions and Cut Scores
Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0–10 points)
The student's response shows few of the elements of performance that the task demands as defined by the CCSS. The student's work shows a minimal attempt and lack of coherence. The student fails to use appropriate tools strategically. The student is unable to make sense of the problem and apply mathematical concepts in this modeling situation.

Level 2: Performance Below Standard (11–23 points)
The student's response shows some of the elements of performance that the task demands as defined by the CCSS. The student might ignore or fail to address some of the constraints of the problem. The student may occasionally make sense of quantities or relationships in the problem. The student attempts to use some appropriate tools with limited success. The student may have trouble generalizing or applying mathematical methods in this modeling situation.

Level 3: Performance at Standard (24–35 points)
For most of the task, the student's response shows the main elements of performance that the tasks demand as defined by the CCSS with few minor errors or omissions. The student explains the problem and identifies constraints. The student makes sense of quantities and their relationships in the modeling situation. The student uses appropriate tools. The student might discern patterns or structures and make connections between representations. The student is able to make sense of the problem and apply geometric concepts to this modeling situation.

Level 4: Achieves Standards at a High Level (33–36 points)
The student's response meets the demands of nearly all of the tasks as defined by the CCSS and is organized in a coherent way. The communication is clear and precise. The body of work looks at the overall situation of the problem and process, while attending to the details. The student routinely interprets the mathematical results, applies concepts in the context of the situation, reflects on whether the results make sense and uses all appropriate tools strategically.

Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included below. The scenario can be used for the performance task listed above. If possible, a businessman from the community gives the class a 10-15 minute presentation about how math affects his job and its importance to the success of his business. Following the presentation, the problem is presented: You are going to start a new business. it will be your job to choose an item to sell to the general public, create a logo for your company, and develop a selling price based on profit and cost. These are the general goals of most new companies. Be creative and research your product thoroughly. You have several tasks to perform. When you have completed the tasks and are ready to launch your business, you will make a presentation to a panel of investors (sharks). They can either back your company or back out. You need their money so you need to convince them!

Instructional Resources
Suggested Tools and Representations Coordinate Plane Equations and Inequalities Graphing Calculator ixl.com (Math) flippedmath.com (Algebra I) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/algebra-i-module-1-topic-lesson-1 (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Precalculus propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-

	pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are "just right" for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day's sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond. Assess by multiple means, including "show and tell" rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking

	process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **<u>Provide Multiple Means of Engagement</u>** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities.

	Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Function** - a correspondence between two sets, ⍰ and ⍰, in which each element of ⍰ is matched to one and only one element of ⍰. The set ⍰ is called the domain; the set ⍰ is called the range. **Domain** - Refer to the definition of function. **Range** - Refer to the definition of function. **Linear Function** - a polynomial function of degree 1. **Average Rate of Change** **Piecewise Linear Function -** a function from the union of the intervals to the set of real numbers such that the function is defined by linear functions on each interval. **Familiar Terms and Symbols** Numerical Symbol Variable Symbol Constant Numerical Expression Algebraic Expression Number Sentence	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that..." "Your solution is different from/ the same as mine because..." "My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>**

Truth Values of a Number Sentence Equation Solution Solution Set Simple Expression Factored Expression Equivalent Expressions Polynomial Expression Equivalent Polynomial Expressions Monomial Coefficient of a Monomial Terms of a Polynomial	**<u>Provide Multiple Means of Action and Expression</u>** First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.

	the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Polynomial and Quadratic Expressions, Equations, and Functions

Overview

By the end of middle school, students are familiar with linear equations in one variable **(6.EE.B.5, 6.EE.B.6, 6.EE.B.7)** and have applied graphical and algebraic methods to analyze and manipulate equations in two variables **(7.EE.A.2).** They used expressions and equations to solve real-life problems **(7.EE.B.4).** They have experience with square and cube roots, irrational numbers **(8.NS.A.1)**, and expressions with integer exponents **(8.EE.A.1)**.

In Algebra I, students have been analyzing the process of solving equations and developing fluency in writing, interpreting, and translating among various forms of linear equations (Module 1) and linear and exponential functions (Module 3). These experiences, combined with modeling with data (Module 2), set the stage for Module 4. Here, students continue to interpret expressions, create equations, rewrite equations and functions in different but equivalent forms, and graph and interpret functions using polynomial functions— more specifically quadratic functions as well as square root and cube root functions.

Topic A introduces polynomial expressions. In Module 1, students learned the definition of a polynomial and how to add, subtract, and multiply polynomials. Here, their work with multiplication is extended and connected to factoring polynomial expressions and solving basic polynomial equations **(A-APR.A.1, AREI.D.11)**. They analyze, interpret, and use the structure of polynomial expressions to multiply and factor polynomial expressions **(A-SSE.A.2)**. They understand factoring as the reverse process of multiplication. In this topic, students develop the factoring skills needed to solve quadratic equations and simple polynomial equations by using the zero product property **(A-SSE.B.3a)**. Students transform quadratic expressions from standard form, $ax^2 + bx + c$, to factored form, $f(x) = a(x - m)(x - n)$, and then solve equations involving those expressions. They identify the solutions of the equation as the zeros of the related function. Students apply symmetry to create and interpret graphs of quadratic functions **(F-IF.B.4, F-IF.C.7a)**. They use average rate of change on an interval to determine where the function is increasing or decreasing **(F-IF.B.6)**. Using area models, students explore strategies for factoring more complicated quadratic expressions, including the product-sum method and rectangular arrays. They create one- and two-variable equations from tables, graphs, and contexts and use them to solve contextual problems represented by the quadratic function **(A-CED.A.1, A-CED.A.2)**. Students then relate the domain and range for the function to its graph and the context **(F-IF.B.5)**.

Students apply their experiences from Topic A as they transform quadratic functions from standard form to vertex form, $f(x) = a(x - h)^2 + k$, in Topic B. The strategy known as completing the square is used to solve quadratic equations when the quadratic expression cannot be factored **(A-SSE.B.3b)**. Students recognize that this form reveals specific features of quadratic functions and their graphs, namely the minimum or maximum of the function (i.e., the vertex of the graph) and the line of symmetry of the graph **(A-APR.B.3, F-IF.B.4, F-IF.C.7a)**. Students derive the quadratic formula by completing the square for a general quadratic equation in standard form, $y = ax^2 + bx + c$, and use it to determine the nature and number of solutions for equations when y equals zero **(A-SSE.A.2, A-REI.B.4)**. For quadratics with irrational roots, students use the quadratic formula and explore the properties of irrational numbers **(N-RN.B.3)**. With the added technique of completing the square in their toolboxes, students come to see the structure of the equations in their various forms as useful for gaining insight into the features of the graphs of equations **(A-SSE.B.3)**. Students study business applications of quadratic functions as they create quadratic equations and graphs from tables and contexts and then use them to solve problems involving profit, loss, revenue, cost, etc. **(A-CED.A.1, A-CED.A.2, F-IF.B.6, F-IF.C.8a)**. In addition to applications in business, students solve physics-based problems involving objects in motion. In doing so, students also interpret expressions and parts of expressions in context and recognize when a single entity of an expression is dependent or independent of a given quantity **(A-SSE.A.1)**.

In Topic C, students explore the families of functions that are related to the parent functions, specifically for quadratic, square root, and cube root, to perform horizontal and vertical translations as well as shrinking and stretching **(F-IF.C.7b, F-BF.B.3).** They recognize the application of transformations in vertex form for a quadratic function and use it to expand their ability to efficiently sketch graphs of square and cube root functions. Students compare quadratic, square root, or cube root functions in context and represent each in different ways (verbally with a description, numerically in tables, algebraically, or graphically). In the final two lessons, students examine real-world problems of quadratic relationships presented as a data set, a graph, a written relationship, or an equation. They choose the most useful form for writing the function and apply the techniques learned throughout the module to analyze and solve a given problem **(A-CED.A.2)**, including calculating and interpreting the rate of change for the function over an interval **(F-IF.B.6).**

Polynomial and Quadratic Expressions, Equations and Functions

Unit 4

Subject: Mathematics

Grade/Course: Grade 9 / Algebra

Pacing: 30 days

Unit of Study: Unit 4: Polynomial and Quadratic Expressions, Equations and Functions

Priority Standards:

Use properties of rational and irrational numbers.

N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Interpret the structure of expressions

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

Write expressions in equivalent forms to solve problems.

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Perform arithmetic operations on polynomials

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials

A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Create equations that describe numbers or relationships.

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Solve equations and inequalities in one variable.

A-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Represent and solve equations and inequalities graphically.

A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Interpret functions that arise in applications in terms of the context.

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Build new functions from existing functions.

F-BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Foundational Standards

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Work with radicals and integer exponents.

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Reason quantitatively and use units to solve problems.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Create equations that describe numbers or relationships.

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law ▯ = ▯▯ to highlight resistance ▯.

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Understand the concept of a function and use function notation.

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If is a function and is an element of its domain, then () denotes the output of corresponding to the input . The graph of is the graph of the equation = ().

F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Build a function that models a relationship between two quantities.

F-BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Focus Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In Module 4, students make sense of problems by analyzing the critical components of the problem, a verbal description, data set, or graph and persevere in writing the appropriate function to describe the relationship between two quantities.

MP.2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between algebraic manipulation of expressions and equations and interpretation of the quantities in the relationship in terms of the context. Students must be able to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own without necessarily attending to their referents, and then to contextualize—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning requires the habit of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities (not just how to compute them), knowing different properties of operations, and using them with flexibility.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph (in the terms of the context), and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. In Topic C, students use the full modeling cycle. They model quadratic functions presented mathematically or in a context. They explain the reasoning used in their writing or by using appropriate tools, such as graphing paper, graphing calculator, or computer software.

MP.5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Throughout the entire module, students must decide whether to use a tool to help find a solution. They must graph functions that are sometimes difficult to sketch (e.g., cube root and square root functions) and functions that are sometimes required to perform procedures that, when performed without technology, can be tedious and distract students from thinking mathematically (e.g., completing the square with non-integer coefficients). In such cases, students must decide when to use a tool to help with the calculation or graph so they can better analyze the model.

MP.6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. When calculating and reporting quantities in all topics of Module 4, students must be precise in choosing appropriate units and use the appropriate level of precision based on the information as it is presented. When graphing, they must select an appropriate scale.

MP.7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. They can see algebraic expressions as single objects, or as a composition of several objects. In this Module, students use the structure of expressions to find ways to rewrite them in different but equivalent forms. For example, in the expression $x^2 + 9x + 14$, students must see the 14 as 2×7 and the 9 as $2 + 7$ to find the factors of the quadratic. In relating an equation to a graph, they can see $y = -3(x - 1)^2 + 5$ as 5 added to a negative number times a square and realize that its value cannot be more than 5 for any real domain value.

"Unwrapped" Standards

N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
Closure property of rational numbers.	Explain (L2)
Expressions that represent a quantity in terms of its context.	Interpret (L2)
Expressions.	Rewrite (L2) - using structure
Quadratic expressions.	Factor (L2) - to reveal zeros Complete (L2) - the square
Closure property for polynomial functions.	Explain (L2)
Zeros of polynomial functions.	Identify 9L1) Graph (L2) - polynomials using zeros

Quadratic formula.	Derive (L3)
Quadratic functions in one and two variables	Create (L4) Solve (L2)- using inspection, taking square roots, factoring, completing the square, quadratic formula
Graphical intersection of two functions.	Explain (L2) Solve (L2) - problems
Key features of graphs and tables for a function that models a relationship between two variables including;. intercept intervals where the function is increasing, decreasing or constant relative maximums and minimums end behavior periodicity	Identify (L1)
Domain of a function.	Relate (L2) - to graph
Average rate of change of a function.	Calculate (L1) Interpret (L2) Estimate (L2) - from a graph
Linear and quadratic functions.	Graph (L2) Evaluate (L4)
Square root, cube root and piecewise defined functions	Graph (L2) Evaluate (L4)
Properties of two functions.	Compare (L2)
Quadratic functions that describe a relationship between two quantities.	Create (L4) - in different but equivalent forms Interpret (L2) - properties
Transformations of functions,	Identify (L1) - effects
Even and odd functions.	Recognize (L1) - from graph Recognize (L1) - from algebraic expressions

Essential Questions	Big ideas
When and how is mathematics used in solving real world problems? When and why is it necessary to follow set rules/procedures/properties when manipulating numeric or algebraic expressions? What characteristics of problems would determine how to model the situation and develop a problem solving strategy?	Mathematics can be used to solve real world problems and can be used to communicate solutions. Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations. Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities. Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. Multiple representations may be used to model given real world relationships.

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – "Dipsticks"	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary New terms Axis of symmetry of the graph of a quadratic function Cube root function Degree of a monomial term	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge	Type: Mid-Module Assessment Task Administered: After Topic A Format: Constructed response with rubric Standards Addressed: A-SSE.A.1,

Degree of a polynomial Discriminant End behavior of a quadratic function Factored form for a quadratic function Leading coefficient Parent function Quadratic formula Quadratic function Roots of a polynomial function Square root function Standard form for a quadratic function Standard form of a polynomial in one variable Vertex form Vertex of the graph of a quadratic function Familiar Terms and Symbols Average rate of change Binomial Closed Closure Coefficient Cubic Cube root Degree of a polynomial Domain and range Explicit expression Factor Integers Irrational numbers Monomial Parabola Power Quadratic Rational numbers Real numbers Recursive process Solutions (solution set) of an equation Solution set Square root Term Trinomial Zeros of a function	Exit Ticket Student Conferences IXL Math	A-SSE.A.2, A-SSE.B.3a, A-APR.A.1, A-CED.A.1, A-CED.A.2, A-REI.B.4b, A-REI.D.11, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7a Type: End-of-Module Assessment Task Administered: After Topic C Format: Constructed response with rubric Standards Addresses: N-RN.B.3, A-SSE.A.1, A-SSE.A.2, A-SSE.B.3a, A-SSE.B.3b, A-APR.B.3, A-CED.A.1, A-CED.A.2, A-REI.B.4, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-IF.C.7b, F-IF.C.8a, F-IF.C.9, F-BF.B.3

Conduct opening exercise Use exit ticket as pre-assessment and post where applicable		

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task detailed in Unit 3.
Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 3.

Instructional Resources
Suggested Tools and Representations Coordinate Plane Equations Graphing calculator Graph paper ixl.com (Math) flippedmath.com (Algebra I) Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/algebra-i-module-1-topic-lesson-1 (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students
21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Precalculus propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match.

	Teach students how to ask questions (such as “Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because…” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…” Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust “partner share” for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or “show”) to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as “______ is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are “just right” for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day’s sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond. Assess by multiple means, including “show and tell” rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight

	each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?” Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, “How did I improve? What did I do well?” Focus on students’ mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., ‘show’). Listen intently in order to uncover the math content in the students’ speech. Use non-verbal signals, such as “thumbs-up.” Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as “Add your ones up first/Make a bundle if you can!” Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior. Allow “everyday” and first language to express math understanding. Re-teach the same concept with a variety of fluency

	games. Allow students to lead group and pair-share activities. Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Axis of symmetry of the graph of a quadratic function** - Given a quadratic function in standard form, (⍰) = ⍰⍰ 2 + ⍰⍰ + ⍰, the vertical line given by the graph of the equation, ⍰ = − ⍰/2⍰ , is called the axis of symmetry of the graph of the quadratic function. **Cube root function** - A polynomial function of degree 3. **Degree of a monomial term** - The sum of the exponents of the variables that appear in a term of a polynomial. **Degree of a polynomial** - The highest degree of the terms in the polynomial. **Discriminant** - The discriminant of a quadratic function in the form ⍰⍰^{2} + ⍰⍰ + ⍰ = 0 is ⍰^{2} − 4⍰⍰. The nature of the roots of a quadratic equation can be identified by determining if the discriminant is positive, negative, or equal to zero. **End behavior of a quadratic function** - Given a quadratic function in the form ⍰(⍰) = ⍰⍰^{2} +	<u>**Provide Multiple Means of Representation**</u> Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. "I do, we do, you do." Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. <u>**Provide Multiple Means of Representation**</u> Teach students how to ask questions (such as,"Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters," such as: "I agree because..." "Can you explain how you solved it?" "I noticed that...""Your solution is different from/ the same as mine because...""My mistake was to..." Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. <u>**Provide Multiple Means of Action and Expression**</u>

□□ + □ (or □(□) = □(□ − h)^2 + □), the quadratic function is said to open up if □ > 0 and open down if □ < 0. **Factored form for a quadratic function** - A quadratic function written in the form □(□) = □(□ − □)(□ − □). **Leading coefficient** - The coefficient of the term of highest degree. **Parent function** - The simplest function in a "family" of functions that can each be formed by one or more transformations of another. **Quadratic formula** - The formula that emerges from solving the general form of a quadratic equation by completing the square. **Quadratic function** - A polynomial function of degree 2. **Roots of a polynomial function** - The domain values for a polynomial function that make the value of the polynomial function equal zero when substituted for the variable. **Square root function** - The parent function □(□) = √□. **Standard form for a quadratic function** - A quadratic function written in the form □(□) = □□^{2} + □□ + □. **Standard form of a polynomial in one variable** **Vertex form** - Completed-square form for a quadratic function; in other words, written in the form □(□) = □(□ − h)^2 + □. **Vertex of the graph of a quadratic function** - The point	<u>**Provide Multiple Means of Action and Expression**</u> First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. <u>**Provide Multiple Means of Engagement**</u> Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.

where the graph of a quadratic function and its axis of symmetry intersect. **Familiar Terms and Symbols** Average rate of change Binomial Closed Closure Coefficient Cubic Cube root Degree of a polynomial Domain and range Explicit expression Factor Integers Irrational numbers Monomial Parabola Power Quadratic Rational numbers Real numbers Recursive process Solutions (solution set) of an equation Solution set Square root Term Trinomial Zeros of a function	the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

A Synthesis of Modeling with Equations and Functions

Overview

In Grade 8, students used functions for the first time to construct a function that models a linear relationship between two quantities **(8.F.B.4)** and to describe qualitatively the functional relationship between two quantities by analyzing a graph **(8.F.B.5)**. In the first four modules of Algebra I, students learn to create and apply linear, quadratic, and exponential functions in addition to square and cube root functions **(F-IF.C.7).** In Module 5, they synthesize what they have learned during the year by selecting the correct function type in a series of modeling problems without the benefit of a module or lesson title that includes function type to guide them in their choices. This supports the CCLS requirement that student's use the modeling cycle, in the beginning of which they must formulate a strategy. Skills and knowledge from the previous modules will support the requirements of this module, including writing, rewriting, comparing, and graphing functions **(F-IF.C.7, F-IF.C.8, F-IF.C.9)** and interpretation of the parameters of an equation **(F-LE.B.5)**. Students also draw on their study of statistics in Module 2, using graphs and functions to model a context presented with data and tables of values **(S-ID.B.6)**. In this module, we use the modeling cycle (see page 72 of the CCLS) as the organizing structure rather than function type.

Topic A focuses on the skills inherent in the modeling process: representing graphs, data sets, or verbal descriptions using explicit expressions **(F-BF.A.1a).** Information is presented in graphic form in Lesson 1, as data in Lesson 2, and as a verbal description of a contextual situation in Lesson 3. Students recognize the function type associated with the problem **(F-LE.A.1b, F-LE.A.1c)** and match to or create 1- and 2-variable equations **(A-CED.A.1, A-CED.A.2)** to model a context presented graphically, as a data set, or as a description **(F-LE.A.2)**. Function types include linear, quadratic, exponential, square root, cube root, absolute value, and other piecewise functions. Students interpret features of a graph in order to write an equation that can be used to model it and the function **(F-IF.B.4, F-BF.A.1)** and relate the domain to both representations **(F-IF.B.5)**. This topic focuses on the skills needed to complete the modeling cycle and sometimes uses purely mathematical models, sometimes real-world contexts.

Tables, graphs, and equations all represent models. We use terms such as “symbolic” or “analytic” to refer specifically to the equation form of a function model; “descriptive model” refers to a model that seeks to describe or summarize phenomena, such as a graph. In Topic B, students expand on their work in Topic A to complete the modeling cycle for a real-world contextual problem presented as a graph, a data set, or a verbal description. For each, they formulate a function model, perform computations related to solving the problem, interpret the problem and the model, and then validate through iterations of revising their models as needed, and report their results.

Students choose and define the quantities of the problem **(N-Q.A.2)** and the appropriate level of precision for the context **(N-Q.A.3).** They create 1- and 2-variable equations **(A-CED.A.1, A-CED.A.2)** to model the context when presented as a graph, as data, and as a verbal description. They can distinguish between situations that represent a linear **(F-LE.A.1b)**, quadratic, or exponential **(F-LE.A.1c)** relationship. For data, they look for first differences to be constant for linear relationships, second differences to be constant for quadratic relationships, and a common ratio for exponential relationships. When there are clear patterns in the data, students will recognize when the pattern represents a linear (arithmetic) or exponential (geometric) sequence **(F-BF.A.1a, F-LE.A.2)**. For graphic presentations, students interpret the key features of the graph, and for both data sets and verbal descriptions, students sketch a graph to show the key features **(F-IF.B.4)**. They calculate and interpret the average rate of change over an interval, estimate when using the graph **(F-IF.B.6)**, and relate the domain of the function to its graph and to its context **(F-IF.B.5)**.

A Synthesis of Modeling with Equations and Functions

Unit 5

Subject: Mathematics

Grade/Course: Grade 9 / Algebra

Pacing: 20 days

Unit of Study: Unit 5: A Synthesis of Modeling with Equations and Functions

Priority Standards:

Reason quantitatively and use units to solve problems.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Create equations that describe numbers or relationships

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Interpret functions that arise in applications in terms of the context.

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Build a function that models a relationship between two quantities.

F-BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Foundational Standards

Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (▯▯, ▯▯) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

c. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Interpret expressions for functions in terms of the situation they model.

F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Summarize, represent, and interpret data on two categorical and quantitative variables.

S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

d. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

e. Informally assess the fit of a function by plotting and analyzing residuals.

Focus Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In Module 5, students make sense of the problem by analyzing the critical components of the problem, presented as a verbal description, a data set, or a graph and persevere in writing the appropriate function that describes the relationship between two quantities. Then, they interpret the function in the context.

MP.2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between algebraic manipulation of expressions and equations and interpreting the quantities in the relationship in terms of the context. In Topic A, students develop fluency in recognizing and identifying key features of the three primary function types studied in Algebra I, as well as manipulating expressions to highlight those features. Topic B builds on these skills so that when students are given a verbal description of a situation that can be described by a function, they decontextualize it and apply the skills they learned in Topic A in order to further analyze the situation. Then, they contextualize their work so they can compare, interpret, and make predictions and claims. In the assessment, students are frequently asked to explain their solutions so that teachers have a clear understanding of the reasoning behind their results.

MP.4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph in the terms of the context, and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. In Topic B, students use the full modeling cycle with functions presented mathematically or in a context, including linear, quadratic, and exponential. They explain their mathematical thinking in writing and using appropriate tools, such as graph paper, graphing calculator, or computer software.

MP.5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a

computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Throughout the entire module students must decide whether or not to use a tool to help find solutions. They must graph functions that are sometimes difficult to sketch (e.g., cube root and square root) and sometimes are required to perform procedures that can be tedious, and sometimes distract from the mathematical thinking, when performed without technology (e.g., completing the square with non-integer coefficients). In these cases, students must decide whether to use a tool to help with the calculation or graph so they can better analyze the model. Students should have access to a graphing calculator for use on the module assessment.

MP.6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. When calculating and reporting quantities in all topics of Module 5, students must choose the appropriate units and use the appropriate level of precision based on the information as it is presented. When graphing they must select an appropriate scale.

"Unwrapped" Standards

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-BF.A.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Concepts (What Students Need to Know)	**Skills (What Students Need to Be Able to Do)**
Appropriate quantities for the purpose of descriptive modeling	Define (L1)
Level of accuracy.	Choose (L3)
Equations and inequalities in one variable.	Choose (L3) - from family of functions Solve (L2) - problems
Equations in two variables.	Create (L4) Graph (L2)
Key features of graphs and tables for a function that models a relationship between two variables including;. intercept intervals where the function is increasing, decreasing or constant relative maximums and minimums end behavior periodicity	Interpret (L2)
Domain of a function.	Relate (L2) - to graph and to quantitative relationship
Average rate of change of a function.	Calculate (L1) Interpret (L2) Estimate (L2)- from a graph
Functions that describe a relationship between two	Create (L4)- an explicit expression

quantities.	Create (L4) - a recursive process Explain (L2) - steps for calculation from context
Properties of two functions.	Compare (L2)
Rate of change of a linear function.	Prove (L4) Recognize (L1)
Rate of change of an exponential function.	Prove (L4) Recognize (L1)
Linear and exponential functions	Construct (L3)

Essential Questions	**Big ideas**
When and how is mathematics used in solving real world problems?	Mathematics can be used to solve real world problems and can be used to communicate solutions. Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations. Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities.
When and why is it necessary to follow set rules/procedures/properties when manipulating numeric or algebraic expressions?	Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. Multiple representations may be used to model given real world relationships.
What characteristics of problems would determine how to model the situation and develop a problem solving strategy?	

Assessments		
Common Formative Pre-Assessments	Progress Monitoring Checks – “Dipsticks”	Common Formative Mid and or Post-Assessments Resources
Pretest vocabulary New terms Analytic Model Descriptive Model Familiar Terms and Symbols Analytical Model Arithmetic Sequence Average Rate of Change Cube Root Function End Behavior Exponential Function First Differences Function Geometric Sequence Linear Function Parameter Parent Function Piecewise Defined Function Quadratic Function Range Recursive Process Square Root Function Second Differences Conduct opening exercise Use exit ticket as pre-assessment and post where applicable	Post-test vocabulary Opening Exercise - Give again and reflect on results from first administration prior to the unit. Exploratory Challenge Exit Ticket Student Conferences IXL Math	Type: End-of-Module Assessment Task Administered: After Topic B Format: Constructed response with rubric Standards Addresses: N-Q.A.2, N-Q.A.3, A-CED.A.1, A-CED.A.2, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-BF.A.1, F-LE.A.1, F-LE.A.2

Performance Task
Performance tasks are to be created with teacher input throughout the year. A sample of a possible performance task detailed in Unit 3.

Engaging Learning Experiences
Engaging learning experiences are to be created with teacher input throughout the year. A sample of an engaging scenario is included in Unit 3.

Instructional Resources
Suggested Tools and Representations Scientific Calculator Graphing Calculator Graph paper ixl.com (Math) flippedmath.com (Algebra I) Geometer's Sketch Pad GeoGebra Lesson plans for all modules within the unit, that are compatible with this curriculum, can be found on EngageNY. The link below allows access to all lessons in Algebra I. (Just scroll down once you get there.) https://www.engageny.org/resource/algebra-i-module-1-topic-lesson-1 (See Appendix A for an example.)

Instructional Strategies	Meeting the Needs of All Students

21st Century Skills Critical thinking and problem solving Collaboration and leadership Agility and Adaptability Effective oral and written communication Accessing and analyzing information **Marzano's Strategies** Identifying Similarities and Differences Reinforcing Effort and Providing Recognition Nonlinguistic Representations Homework and Practice Cooperative Learning Setting Objectives and Providing Feedback	The modules that make up Precalculus propose that the components of excellent math instruction do not change based on the audience. That said, there are specific resources included within this curriculum to highlight strategies that can provide critical access for all students. Researched-based Universal Design for Learning (UDL) has provided a structure for thinking about how to meet the needs of diverse learners. Broadly speaking, that structure asks teachers to consider multiple means of representation; multiple means of action and expression; and multiple means of engagement. Tables at the end of this section offer suggested scaffolds, utilizing this framework, for Students with Disabilities, Students Performing above Grade Level, and Students Performing below Grade Level. UDL offers ideal settings for multiple entry points for students and minimizes instructional barriers to learning. Teachers will note that many of the suggestions on a chart will be applicable to other students and overlapping populations. It is important to note that although the scaffolds/accommodations integrated into the course might change how a learner accesses information and demonstrates learning; they do not substantially alter the instructional level, content, or performance criteria. Rather, they provide students with choices in how they access content and demonstrate their knowledge and ability. **<u>Provide Multiple Means of Representation</u>** Teach from simple to complex, moving from concrete to abstract at the student's pace. Clarify, compare, and make connections to math words in discussion, particularly during and after practice. Partner key words with visuals and gestures. Connect language with concrete and pictorial experiences. Couple teacher-talk with "math-they-can-see," such as models. Let students use models and gestures to calculate and explain. For example, a student searching to define "multiplication" may model groups of 6 with drawings or concrete objects and write the number sentence to match. Teach students how to ask questions (such as "Do you agree?" and "Why do you think so?") to extend "think-

	pair-share" conversations. Model and post conversation "starters," such as: "I agree because…" "Can you explain how you solved it?" "I noticed that…" "Your solution is different from/ the same as mine because…" "My mistake was to…" Couple number sentences with models. Enlarge sprint print for visually impaired learners. Use student boards to work on one calculation at a time. Invest in or make math picture dictionaries or word walls. **<u>Provide Multiple Means of Action and Expression</u>** Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models; pair share; small group share. For example: Use student boards to adjust "partner share" for deaf and hard-of-hearing students. Partners can jot questions and answers to one another on slates. Use vibrations or visual signs (such as clap, rather than a snap or "show") to elicit responses from deaf/hard of hearing students. Vary choral response with written response on student boards to ease linguistic barriers. Support oral or written response with sentence frames, such as "______ is ____ hundreds, ____ tens, and ____ ones. Adjust oral fluency games by using student and teacher boards or hand signals, such as showing the sum with fingers. Use visual signals or vibrations to elicit responses. Adjust wait time for interpreters of deaf and hard-of-hearing students. Select numbers and tasks that are "just right" for learners. Model each step of the algorithm before students begin. Give students a chance to practice the next day's sprint beforehand. Give students a few extra minutes to process the information before giving the signal to respond. Assess by multiple means, including "show and tell" rather than written. Elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking

	process. Before students solve, ask questions for comprehension, such as, "What unit are we counting? What happened to the units in the story?" Teach students to use self-questioning techniques, such as, "Does my answer make sense?" Concentrate on goals for accomplishment within a time frame as opposed to a task frame. Extend time for task. Guide students to evaluate process and practice. Have students ask, "How did I improve? What did I do well?" Focus on students' mathematical reasoning (i.e., their ability to make comparisons, describe patterns, generalize, explain conclusions, specify claims, and use models), not their accuracy in language. **Provide Multiple Means of Engagement** Make eye-to-eye contact and keep teacher-talk clear and concise. Speak clearly when checking answers for sprints and problems. Check frequently for understanding (e.g., 'show'). Listen intently in order to uncover the math content in the students' speech. Use non-verbal signals, such as "thumbs-up." Assign a buddy or a group to clarify directions or process. Teach in small chunks so students get a lot of practice with one step at a time. Know, use, and make the most of Deaf culture and sign language. Use songs, rhymes, or rhythms to help students remember key concepts, such as "Add your ones up first/Make a bundle if you can!" Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words. Incorporate activity. Get students up and moving, coupling language with motion. Make the most of the fun exercises for activities like sprints and fluencies. Celebrate improvement. Intentionally highlight student math success frequently. Follow predictable routines to allow students to focus on content rather than behavior. Allow "everyday" and first language to express math understanding. Re-teach the same concept with a variety of fluency games. Allow students to lead group and pair-share activities.

	Provide learning aids, such as calculators and computers, to help students focus on conceptual understanding

New Vocabulary	Students Achieving Below Standard	Students Achieving Above Standard
New or Recently Introduced Terms **Analytic Model** - A model that seeks to explain data based on deeper theoretical ideas. **Descriptive Model** - A model that seeks to describe phenomena or summarize them in a compact form. **Familiar Terms and Symbols** Analytical Model Arithmetic Sequence Average Rate of Change Cube Root Function End Behavior Exponential Function First Differences Function Geometric Sequence Linear Function Parameter Parent Function Piecewise Defined Function Quadratic Function Range Recursive Process Square Root Function Second Differences	**<u>Provide Multiple Means of Representation</u>** Model problem-solving sets with drawings and graphic organizers (e.g., bar or tape diagram), giving many examples and visual displays. Guide students as they select and practice using their own graphic organizers and models to solve. Use direct instruction for vocabulary with visual or concrete representations. Use explicit directions with steps and procedures enumerated. Guide students through initial practice promoting gradual independence. “I do, we do, you do.” Use alternative methods of delivery of instruction such as recordings and videos that can be accessed independently or repeated if necessary. Scaffold complex concepts and provide leveled problems for multiple entry points.	The following provides a bank of suggestions within the Universal Design for Learning framework for accommodating students who are above grade level in your class. Variations on these accommodations are elaborated within lessons, demonstrating how and when they might be used. **<u>Provide Multiple Means of Representation</u>** Teach students how to ask questions (such as,“Do you agree?” and “Why do you think so?”) to extend “think-pair-share” conversations. Model and post conversation “starters,” such as: “I agree because...” “Can you explain how you solved it?” “I noticed that...”“Your solution is different from/ the same as mine because...”“My mistake was to...” Incorporate written reflection, evaluation, and synthesis. Allow creativity in expression and modeling solutions. **<u>Provide Multiple Means of Action and Expression</u>**

	<u>Provide Multiple Means of Action and Expression</u> First use manipulatives or real objects (such as dollar bills), then make transfer from concrete to pictorial to abstract. Have students restate their learning for the day. Ask for a different representation in the restatement. 'Would you restate that answer in a different way or show me by using a diagram?' Encourage students to explain their thinking and strategy for the solution. Choose numbers and tasks that are "just right" for learners but teach the same concepts. Adjust numbers in calculations to suit learner's levels. For example, change 429 divided by 2 to 400 divided by 2 or 4 divided by 2. **<u>Provide Multiple Means of Engagement</u>** Clearly model steps, procedures, and questions to ask when solving. Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling). Have students work together to solve and then check their solutions. Teach students to ask themselves questions as they solve: Do I know	Encourage students to explain their reasoning both orally and in writing. Extend exploration of math topics by means of challenging games, puzzles, and brain teasers. Offer choices of independent or group assignments for early finishers. Encourage students to notice and explore patterns and to identify rules and relationships in math. Have students share their observations in discussion and writing (e.g., journaling). Foster their curiosity about numbers and mathematical ideas. Facilitate research and exploration through discussion, experiments, internet searches, trips, etc. Have students compete in a secondary simultaneous competition, such as skip-counting by 75s, while peers are completing the sprint. Let students choose their mode of response: written, oral, concrete, pictorial, or abstract. Increase the pace. Offer two word problems to solve, rather than one. Adjust difficulty level by increasing the number of steps (e.g., change a one-step problem to a two-step problem). Adjust difficulty level by enhancing the operation (e.

	the meaning of all the words in this problem?; What is being asked?; Do I have all of the information I need?; What do I do first?; What is the order to solve this problem? What calculations do I need to make? Practice routine to ensure smooth transitions. Set goals with students regarding the type of math work students should complete in 60 seconds. Set goals with the students regarding next steps and what to focus on next. Reinforce foundational standards (listed after priority standards) for the unit.	g., addition to multiplication), increasing numbers to millions, or decreasing numbers to decimals/fractions. Let students write word problems to show mastery and/or extension of the content. **<u>Provide Multiple Means of Engagement</u>** Push student comprehension into higher levels of Bloom's Taxonomy with questions such as: "What would happen if...?""Can you propose an alternative...?""How would you evaluate...?""What choice would you have made...?" Ask "Why?" and "What if?" questions. Celebrate improvement in completion time (e.g., Sprint A completed in 45 seconds and Sprint B completed in 30 seconds). Make the most of the fun exercises for practicing skip-counting. Accept and elicit student ideas and suggestions for ways to extend games. Cultivate student persistence in problem-solving and do not neglect their need for guidance and support.

Appendix A: Lesson Plan Sample
Module 1 Lesson 1

The following is a sample lesson plan from EngageNY. The lesson in its entirety can be found at **https://www.engageny.org/resource/algebra-i-module-1-topic-lesson-1**
In addition to the lesson plan, printable worksheets, sample student answers are available online. EngageNY can be used as a resource for all modules.

Lesson 1: Graphs of Piecewise Linear Functions

Student Outcomes

Students define appropriate quantities from a situation (a "graphing story"), choose and interpret the scale and the origin for the graph, and graph the piecewise linear function described in the video. They understand the relationship between physical measurements and their representation on a graph.

Classwork

Exploratory Challenge (20 minutes)

Show the first minutes of the video below, telling the class that our goal will simply be to describe in words the motion of the man. (Note: Be sure to stop the video at 1:08 because after that the answers to the graphing questions are given.)

> Elevation vs. Time #2 [http://www.mrmeyer.com/graphingstories1/graphingstories2.mov. This is the second video under "Download Options" at the site http://blog.mrmeyer.com/?p=213 called "Elevation vs. Time # ."]

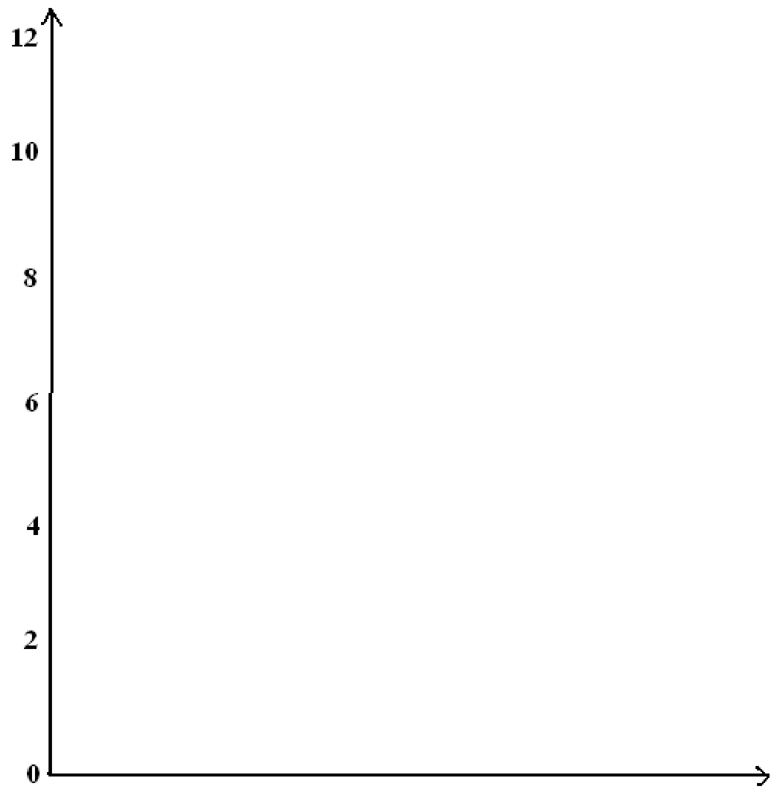
After viewing the video, have students share out loud their ideas on describing the motion. Some might speak in terms of speed, distance traveled over time, or change of elevation. All approaches are valid. Help students begin to shape their ideas with precise language. **(MP1)**

Direct the class to focus on the change of elevation of the man over time and begin to put into words specific details linking elevation with time.

> How high do you think he was at the top of the stairs? How did you estimate that elevation?
>
> Were there intervals of time when his elevation wasn't changing? Was he still moving?
>
> Did his elevation ever increase? When?

Help students discern statements relevant to the chosen variable of elevation.

If students do not naturally do so, suggest representing this information on a graph. As per the discussion that follows, display a set of axes on the board with vertical axis labeled in units relevant to the elevation.



Ask these types of questions:

How should we label the vertical axis?

What unit of measurement should we choose (feet or meters)?

How should we label the horizontal axis?

What unit of measurement should we choose?

Should we measure the man's elevation to his feet or to his head on the graph?

The man starts at the top of the stairs. Where would that be located on the graph?

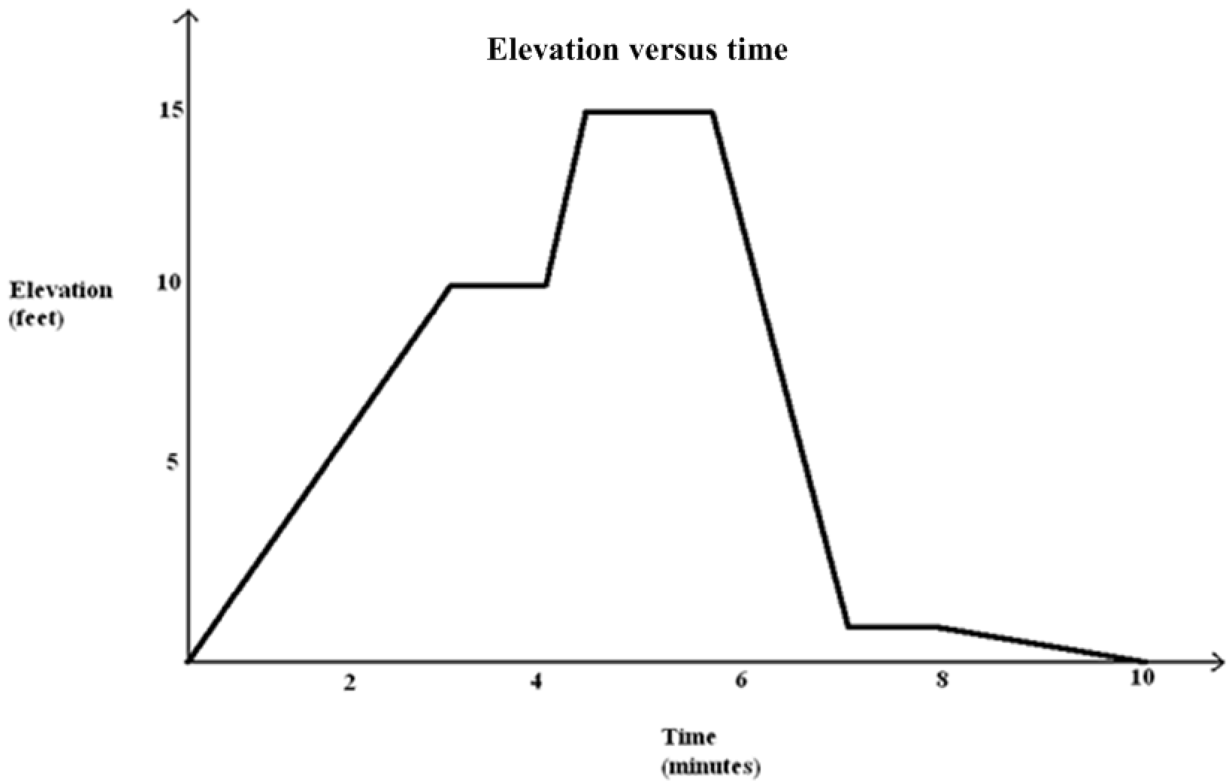
Show me with your hand what the general shape of the graph should look like.

Give time for students to draw the graph of the story (alone or in pairs). Lead a discussion through the issues of formalizing the diagram: The labels and units of the axes, a title for the graph, the meaning of a point plotted on the graph, a method for finding points to plot on the graph, and so on. **(MP6)**

Note: The graph shown at the end of the video is incorrect! The man starts at " feet above the ground," which is clearly false. You might ask students, "Can you find the error made in the video?" **(MP3)**

Example 1 (15 minutes)

Present the following graph and question.



Have students discuss this question in pairs or in small groups. It will take some imagination to create a context that matches the shape of the graph, and there will likely be debate.

Additional questions to ask:

What is happening in the story when the graph is increasing, decreasing, constant over time?

Answers will vary depending on the story: a person is "walking up a hill," etc.

What does it mean for one part of the graph to be steeper than another?

The person is climbing or descending faster than in the other part.

How does the slope of each line segment relate to the context of the person's elevation?

The slope gives the average change in elevation per minute.

Is it reasonable that a person moving up and down a vertical ladder could have produced this elevation versus time graph?

It is unlikely because the speed is too slow: feet per minute. If the same graph had units in seconds then it would be reasonable.

Is it possible for someone walking on a hill to produce this elevation versus time graph and return to her starting point at the -minute mark? If it is, describe what the hill might look like.

Yes, the hill could have a long path with a gentle slope that would zigzag back up to the top and then a shorter, slightly steeper path back down to the beginning position.

What was the average rate of change of the person's elevation between time minutes and time minutes?

10/4 ft/min, or 2.5 ft/min.

These types of questions help students understand that the graph represents only elevation, not speed or horizontal distance from the starting point. This is an important observation.

Closing (5 minutes)

Ask the following:

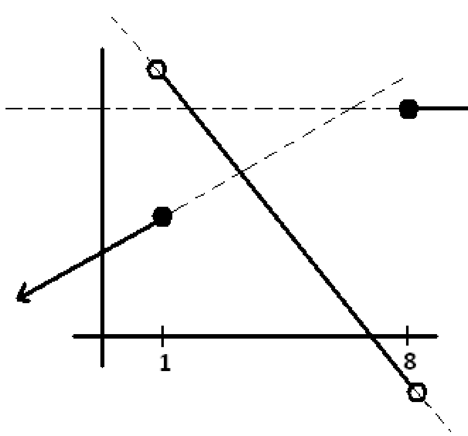
How would you describe the graph of Example 1 to a friend?

What type of equation(s) would be required to create this graph?

Introduce the following definition to your students and discuss briefly. (We will return to this definition later in the year.)

Piecewise-Defined Linear Function: Given non-overlapping intervals on the real number line, a (real) piecewise linear function is a function from the union of the intervals on the real number line that is defined by (possibly different) linear functions on each interval.

Point out that all graphs we studied today are graphs of piecewise linear functions. Remind students (see Standard 8.F.A.3) that the graphs of linear functions are straight lines, and show how each segment in one of the graphs studied today is part of a straight line as in:



Also show students the intervals on which each linear function is defined. One may wish to point out there might be ambiguity as to whether or not the endpoints of a given interval belong to that interval. For example, in the first diagram we could argue that three linear functions are defined on the intervals ,, and , or perhaps on the intervals , and instead. (Warning: Your students have not been formally introduced to interval notation.) There is no ambiguity in the second example. This point about the interval endpoints is subtle and is not an issue to focus on in a concerted way in this particular lesson.

Exit Ticket (5 minutes)